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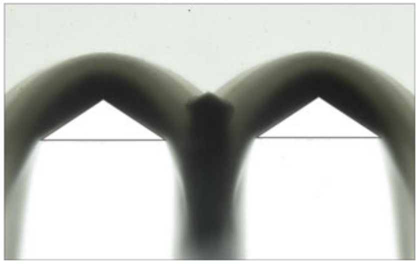
Shocking granular flows



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When a lightning bolt darts across the sky, the thunderclap that reaches our ears a few seconds later is an example of a fluid dynamical shock: a wave across which flow properties such as pressure and density change almost discontinuously. In compressible fluids these shocks are associated with high-energy supersonic flows and so require specialist equipment to realise in steady state. But in granular media, shocks occur much more readily and at flow speeds easily obtainable in the laboratory. In the featured article, Khan *et al.* (*J. Fluid Mech.*, vol. 884, 2020, R4) exploit this to explore a remarkable range of steady and oscillatory shocks and shock interactions, which demonstrate many of the unique rheological complexities of granular flow.

Key words: granular media, shock waves

1. Introduction

Shock waves can occur in a fluid whenever the speed of a flow, or of an obstacle moving through it, exceeds the local propagation speed of waves within that fluid. Shocks were first described by Mach & Salcher (1887) in the context of a compressible flow of air, where the relevant wave propagation speed is the speed of sound. The ratio of flow speed to acoustic wave speed is known accordingly as the Mach number.

Shock waves of a different sort are found in shallow free-surface flows, where they are called bores or hydraulic jumps. Such a hydraulic jump occurs when thin rapid flow of water down the face of a weir meets the river below and suddenly increases in depth. The relevant wave speed in this flow is the speed of surface gravity waves, and the ratio of flow speed to this wave speed is denoted by the Froude number.

While physically very different, these compression and free-surface shocks arise from the same underlying conservation of mass and momentum across the shock, and can be described by similar systems of hyperbolic conservation laws, namely the compressible Euler equations and the shallow-water equations, respectively.

In granular flows, the speed of acoustic waves is considerably less than that in a homogeneous fluid or solid, and is comparable to the surface gravity wave

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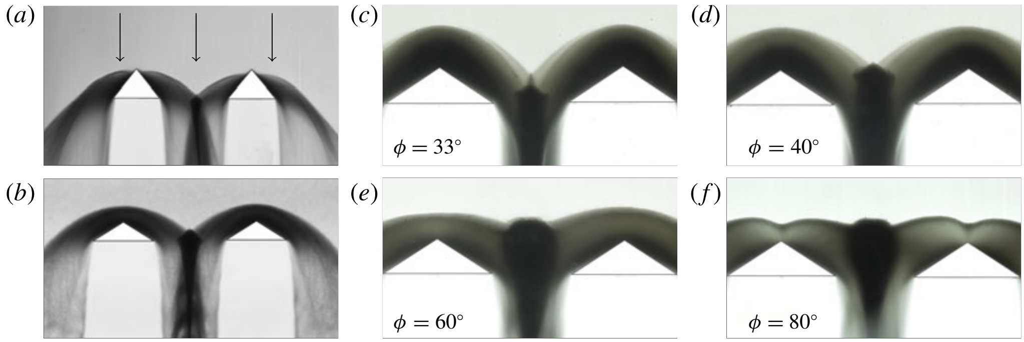

FIGURE 1. Adapted from Khan *et al.* (2020). (*a*) A rapid granular flow (direction indicated by arrows) impinging onto two triangular obstacles. Oblique granular shocks separate undisturbed flow from denser and thicker flow (which appears darker). (*b*) Shocks from blunter obstacles are detached (i.e. they do not contact the obstacle apex) and collide to form a very dense granular streak. (*c–f*) The changing morphology of granular shocks and dense streak as the chute inclination angle ϕ is increased from 33° to 80°.

speed. For example, in the highlighted paper, Khan *et al.* (2020) estimate a speed of sound through flowing glass beads of 0.22 m s^{-1}, at least three orders of magnitude slower than the speed of sound in air or glass. Consequently, a free-surface granular avalanche may readily exhibit both compression shocks and free-surface shocks, perhaps simultaneously (Eglit, Kulibaba & Naaim 2007). Both types of shock have been observed in the flow of granular snow avalanches around obstacles, and the design of structures to mitigate the effects of such avalanches (Hákonardóttir *et al.* 2003) has been a significant motivation for the study of free-surface granular shocks (e.g. Gray, Tai & Noelle 2003).

The interactions between shocks in a granular flow have received comparatively little study. As in fluids, when two granular compression or free-surface shocks collide, they reflect off one another, sometimes forming a third, stronger shock, known as a Mach stem (Rericha 2004; Vreman *et al.* 2007; Johnson & Gray 2011). The highlighted paper by Khan *et al.* reveals how granular rheology can make these shock interactions significantly more complex.

2. Overview

Khan *et al.* (2020) study granular shocks experimentally, using a gravity-driven flow of grains through a thin glass channel, 900 mm long by 310 mm wide, 5 mm deep, and inclined at $\phi \approx 33°–80°$ to the horizontal. During an experiment, ∼7 kg of 0.125 mm diameter glass beads are released uniformly across the chute width, forming a thin accelerating sheet of granular flow, about 2.5 mm deep and reaching speeds of 1–3 m s^{-1}. At the midpoint of the chute, two triangular obstacles alongside one another, with vertices pointing towards the oncoming flow, obstruct and divert the flowing grains, creating shocks in the granular flow (figure 1*a*).

Across these shocks, the flow rapidly becomes both denser and thicker, thus combining aspects of both classical free-surface and compression shocks. Depending on the shape and spacing of the obstacles, and inclination angle of the chute, which determines the speed of the incoming flow, Khan *et al.* demonstrate that a wide variety of steady and unsteady shock structures can form.

When the obstacles are sufficiently widely spaced, the flow around each can be treated separately. As in supersonic compressible fluid flows, the flow depends qualitatively on the angle at the apex of each obstacle. When this angle is small, oblique shocks form at this apex (figure 1*a*), whereas for larger angles the shock is detached from the obstacle, forming some distance upstream of the apex (figure 1*b*).

When the chute inclination angle ϕ is small (figure 1*c*), a region of slowly moving densely packed grains collects around the apex of each obstacle (Amarouchene, Boudet & Kellay 2001). The focus of Khan *et al.*'s work, however, is the flow in between two relatively closely spaced obstacles, where the shocks and associated thicker stream of grains diverted from each obstacle collide. The collision of these two streams forms a region of very dense, near-incompressible granular flow (with grain volume fraction close to the maximum $\approx$0.6) that is in frictional contact with both the lower and upper surfaces of the chute. The energy loss associated with the inelastic collision of grains in this region promotes the formation of a persistent dense cluster, within an otherwise energetic granular flow (Kudrolli, Wolpert & Gollub 1997). Grains at the top of this dense region rest on those by those below, forming a granular pile (which is likely further supported by friction with the chute walls, as in Taberlet *et al.* (2003)) but grains at the very bottom of the dense region fall under gravity to form a dense granular streak. As the inclination angle of the chute is increased (figure 1*c*–*f*), this shock interaction region becomes the densest and slowest-moving part of the flow. The presence of such a dense, liquid-like region, suspended away from the obstacle by dilute gas-like flow, is reminiscent of the so-called granular Leidenfrost effect (Eshuis *et al.* 2005), in which a dense granular flow is suspended above a vibrating boundary by energetic grains in a dilute gas-like state.

As the inclination angle ϕ increases, the upstream edge of this dense region of grains changes in shape from pointed (figure 1*c*,*d*) to a blunt, flat-topped shape (figure 1*e*,*f*). For parameters close to the transition between these two regimes (figure 1*e*), Khan *et al.* (2020) observe that the dense collision region oscillates from side to side, bouncing between the two obstacles with a frequency of approximately 3 Hz. This instability leaves oscillations in the dense streak of grains downstream.

3. Future

The granular flows observed by Khan *et al.* (2020) present a significant challenge for theoretical continuum models for granular flow, exhibiting both dilute and dense regions of flow, a strong influence of boundaries, and flow features potentially comparable in size to the mean free path of grains. Some insight into the role of granular rheology in these flows can be gained by comparing the experimental results with solutions of possibly the simplest analogous shock-forming system, the non-dimensional shallow-water equations (Tan 1992). These hyperbolic equations express the conservation of mass, $\partial_t h + \nabla \cdot (h\bar{\boldsymbol{u}}) = 0$, and the conservation of momentum, $\partial_t(h\bar{\boldsymbol{u}}) + \nabla \cdot (h\bar{\boldsymbol{u}}\bar{\boldsymbol{u}} + \boldsymbol{I}h^2/2) = 0$, of a thin layer of inviscid, incompressible fluid, of thickness h and depth-averaged (two-dimensional) velocity $\bar{\boldsymbol{u}}$. Granular physics is entirely neglected in these equations, yet numerical solutions of flows past triangular obstacles (figure 2) show some qualitatively similar shock interactions to those observed by Khan *et al.*, illustrating the fundamental importance of mass and momentum conservation in determining these structures. But the simple shallow-water model fails, of course, to capture much of the complexity of Khan *et al.*'s flows, including large density variations, oscillatory shocks and the 'indent' in the detached

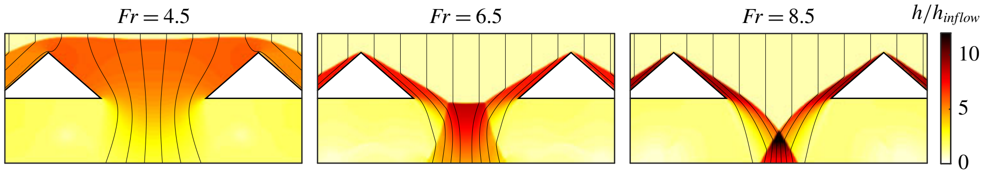

FIGURE 2. Steady numerical solutions of the shallow-water equations for uniform flow past triangular obstacles, showing flow thickness (shading) and streamlines. Discontinuities in thickness indicate the formation of shocks. The shock structures depend strongly on the inflow Froude number $Fr = |\bar{\boldsymbol{u}}|/\sqrt{h}$, and are reminiscent of several regimes observed by Khan *et al.* (2020).

shock near the apex of each obstacle at high inclinations (figure 1*f*), which must be attributed to specifics of granular compression and shear rheology. A unified theoretical description of these observations, and more generally of granular flows with both gas-like and liquid-like regions, remains an important challenge for the granular modelling community.

Declaration of interests

The author reports no conflict of interest.

References

AMAROUCHENE, Y., BOUDET, J. F. & KELLAY, H. 2001 Dynamic sand dunes. *Phys. Rev. Lett.* **86**, 4286–4289.

EGLIT, M. E., KULIBABA, V. S. & NAAIM, M. 2007 Impact of a snow avalanche against an obstacle. Formation of shock waves. *Cold Reg. Sci. Technol.* **50** (1–3), 86–96.

ESHUIS, P., VAN DER WEELE, K., VAN DER MEER, D. & LOHSE, D. 2005 Granular Leidenfrost effect: experiment and theory of floating particle clusters. *Phys. Rev. Lett.* **95**, 258001.

GRAY, J. M. N. T., TAI, Y.-C. & NOELLE, S. 2003 Shock waves, dead zones and particle-free regions in rapid granular free-surface flows. *J. Fluid Mech.* **491**, 161–181.

HÁKONARDÓTTIR, K. M., HOGG, A. J., JÓHANNESSON, T. & TÓMASSON, G. G. 2003 A laboratory study of the retarding effects of braking mounds on snow avalanches. *J. Glaciol.* **49** (165), 191–200.

JOHNSON, C. G. & GRAY, J. M. N. T. 2011 Granular jets and hydraulic jumps on an inclined plane. *J. Fluid Mech.* **675**, 87–116.

KHAN, A., VERMA, S., HANKARE, P., KUMAR, R. & KUMAR, S. 2020 Shock–shock interactions in granular flows. *J. Fluid Mech.* **884**, R4.

KUDROLLI, A., WOLPERT, M. & GOLLUB, J. P. 1997 Cluster formation due to collisions in granular material. *Phys. Rev. Lett.* **78**, 1383–1386.

MACH, E. & SALCHER, P. 1887 Photographische Fixierung der durch Projektile in der Luft eingeleiteten Vorgänge. *Sitzungsber. Akad. Wiss. Wien* **95**, 764–780.

RERICHA, E. 2004 Shocks in rapid granular flows. PhD thesis, University of Texas at Austin, TX.

TABERLET, N., RICHARD, P., VALANCE, A., LOSERT, W., PASINI, J. M., JENKINS, J. T. & DELANNAY, R. 2003 Superstable granular heap in a thin channel. *Phys. Rev. Lett.* **91**, 264301.

TAN, W. Y. 1992 *Shallow Water Hydrodynamics*. Elsevier Science.

VREMAN, A. W., AL-TARAZI, M., KUIPERS, J. A. M., VAN SINT ANNALAND, M. & BOKHOVE, O. 2007 Supercritical shallow granular flow through a contraction: experiment, theory and simulation. *J. Fluid Mech.* **578**, 233–269.