A TWO-STREAM INSTABILITY IN STREAMS OF CHARGED GRAINS

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ABSTRACT

We study the conditions for the onset of two-stream instabilities in interacting streams of charged grains. These conditions are such that instabilities cannot be important in the solar system of today and have probably been of very limited importance in the protoplanetary cloud.

INTRODUCTION

Charge on grains is found to be important in interstellar high-velocity clouds (Spitzer 1976, Shull 1978) and for small grains under solar system conditions (Grün and Morfill 1980). In ordinary plasmas one finds that the evolution is severely influenced by instabilities if such develop. For this reason we examine if two-stream instabilities involving grains may occur at conditions in the solar system of today or in the protoplanetary cloud. Two-stream instabilities may develop when charged particles drift relative to each other and they grow at the expense of the drift-velocity. The momentum exchange between the particle populations, when an instability is present, may be greatly enhanced compared to that resulting from ordinary Coulomb collisions (e.g. Hasegawa 1975).

COMPUTATIONS

We may at the outset put rather severe limits on the grain sizes and the relative velocities between the grains in the two streams as we have to require that the Coulomb collision frequency between grains is much larger than the collision frequency based on the geometrical cross-section of the grains. This is a necessary condition for the grains

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to behave like plasma particles. The electric potential on grains in the solar system varies from between 2 and 20 V for the zodiacal dust (Grün and Morfill 1980) and up to 800 V for grains in Jupiter's magnetosphere (Mendis and Hill 1979). With a rather high potential of 40 V, which determines the grain charge through its radius, we find by the use of the expression for the Coulomb collision frequency ν_C in $\nu_C >> n_g \pi s^2 V_{dg}$ that this leads to the condition

$$sV_{dg} << 0.1 \tag{1}$$

Thus, even for such small grains as $s = 10^{-6}$ cm the relative velocity V_{dg} between the two grain streams must be considerably less than 1 km/s.

To use the standard dispersion relation for longitudinal electrostatic waves (Ichimaru 1973) we have to assume that the distribution in velocity of each of the grain streams, being a result of a spread in eccentricity and semimajor axis in their orbits around a central body (sun or a planet), is Maxwellian. We further assign the same charge, density and radius to all grains in each stream. Our dispersion equation is a generalization of Ichimaru's (1973, p. 139) dispersion equation for a drift-plasma to our 4-component plasma. We simplify the dispersion equation by assuming the drift-velocity of the gas to be much less than their kinetic velocities

$$V_{de} \; (= V_{di}) << V_i << V_e$$

and

$$\omega/k \sim V_{dg} \; .$$

We may now reduce the dispersion equation to

$$-\frac{2k_i^{\,2}}{k_{g2}^2} - \left(\frac{k_{g1}}{k_{g2}}\right)^2 W\left(\frac{\omega - kV_{dg}}{kV_{t1}}\right) = W\left(\frac{\omega}{kV_{t2}}\right) \; . \tag{2}$$

Here k_{gj}, V_{tj} $(j=1,2)$ are the Debye wavenumber and "thermal" velocity of the two grain streams respectively.

Depending on the conditions in the plasma the disturbance which we study may either be damped out or increase its amplitude. In the latter case we have an instability. The condition for marginal stability, when the imaginary part of the frequency $\gamma = 0$, determines the boundary between growing and damped waves. This may be described by a curve $V_{dg}(k_i^{\,2}/k_{g2}^2)$ in the V_{dg}, $(k_i^{\,2}/k_{g2}^2)$ plane which is found by solving (2) for a real frequency. We solve this graphically by the

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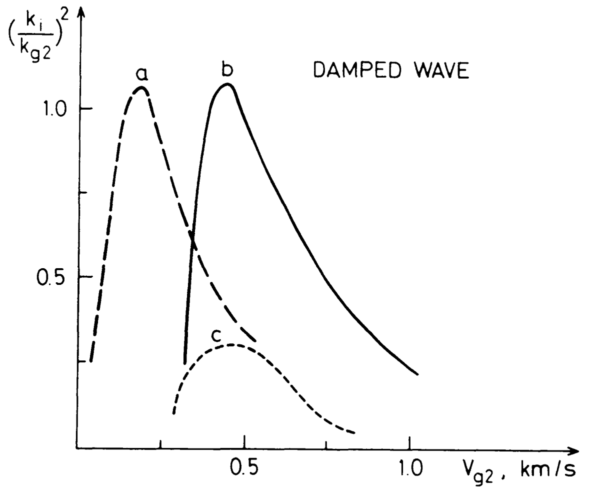

Fig. 1. The boundary between growing waves (inside the curves) and damped waves in two interacting grain streams surrounded by an ion-electron gas. We have shown three cases with different properties of the grain streams listed in the text.

method described by e.g. Ichimaru (1973). The solution is dependent on the conditions in the grain streams, that is, on V_{t1}, V_{t2} and $(k_{g1}/k_{g2})^2$. We have chosen three cases with

a) $(k_{g1}/k_{g2})^2 = 10$, $V_{t1} = 0.01$ km/s and $V_{t2} = 0.1$ km/s

b) " = 10, " = 0.1 " " " = 0.1 "

c) " = 1, " = 0.1 " " " = 0.1 "

The resulting curves are shown in fig. 1. If we assign the same charge, density and radius to the grains in both streams this would correspond to grain number density ratios n_{g1}/n_{g2} for the two streams of 0.1, 10 and 1 for the cases a), b) and c) respectively.

It is apparent from fig. 1 that small drift velocities are favoured for the onset of instability. This is not in conflict with (1). We may obtain an idea of the required density ratio between the grain streams and the surrounding gas by noting from fig. 1 that a necessary condition for an instability to develop can be put as

$$(k_i/k_{g2})^2 < 1 \quad (3)$$

We express the grain mass as $m_{g2} = 4\pi\rho s^3/3$ g, the grain

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charge as $q = s\phi/300$ e.s.u. and the grain number density as $m_{g2} = n_{gH}m_H/m_{g2}$. In these expressions ϕ is the voltage on the grain surface, m_H is the proton mass while n_{gH} is the grain equivalent number density measured in protons per cm^3. If we use $V_i = 10^6$ cm/s and a grain material density $\rho = 3$ g/cm^3 we may now write (3) as

$$n_i V_{g2}^2/n_{gH}\phi^2 < 8.5\ 10^{-25}\ s^{-4} \quad (4)$$

With $V_{g2} = 10^4$ cm/s and $\phi = 40$ Volt we have

$$n_1/n_{gH} < 1.4\ 10^{-29}\ s^{-4} \quad (5)$$

Inequalities (4) or (5) show that even for grain sizes as small as $s = 10^{-6}$ cm there must be 10^5 times more mass in such grains than in the gas. Further, the interacting grain streams must be collimated to a very high degree ($V_{g1} \sim V_{g2} \lesssim 0.1$ km/s) and have a small relative velocity ($V_{dg} << 1$ km/s). Such conditions are apparently not found in the solar system of today. If they were present in the protoplanetary cloud it probably must have been in the last stages of merging of grain streams.

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DISCUSSION

Morfill: Is the physical reason for the high grain density required for instability growth the fact that charge shielding by plasma is too effective?
Havnes: That is one way of expressing why we need $(k_i/k_{g2})^2 < 1$. If the grain density were less than required by this expression the shielding by the electrons would not allow "communication" between a sufficiently large number of grains to make the wave grow.