THE RISE AND FALL OF α-MODEL VISCOSITY

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Abstract. For nearly two decades, our only useful model for the viscosity in accretion disks has been the so-called 'α-model'. However, it has become clear that the simplest models – in which α is constant – are inadequate to explain the range of behaviours seen in real disks. We show that the properties of steady-state, vertically-averaged models can be determined without any assumptions other than that the disks obey the classical Navier-Stokes equation. These solutions have derived values of α which vary with radius by many orders of magnitude even in small CV disks.

1. A short history of α-viscosity

The α-ansatz was first formulated by Shakura & Sunyaev (1973), both as a plausible physical model (the stresses are proportional to pressure in, for example, turbulent viscosity) as well as an attempt to close the system of algebraic equations determining the structure of vertically-averaged disks. This remarkably simple model was immediately applied to a host of situations and the properties of so-called α-disks were studied in great detail. α-disks ceased to be a mere theoretical construct when Hoshi (1979) realized that α-disks could suffer a limit-cycle instability. If the disks could cycle between periods of high and low accretion rates, one might explain the dwarf nova eruptions. Detailed calculations needed for the study of the dynamical evolution of such disks were then made (e.g. explicit vertical structure calculations and the inclusion of convection). The many and varied additions and improvements to the α-disk models are described by Cannizzo (1993). As the other difficult physical and numerical details behind the limit-cycle models were worked out, detailed comparisons between observed dwarf novae light curves and theoretical models began to place constraints on the value of α. It became clear that there could not be a *universal* and *single* value of α, since α-disk models needed roughly a factor of

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10 difference between the values of α in quiescent (i.e. cool) and outburst (i.e. hot) states. As the same numerical technology began to be applied to other systems, the signs that α must be a rather difficult function of something important like radius or temperature became even more obvious: one needs a value of α which is many orders of magnitude smaller in FU Ori variables (e.g. Meyer 1990); and in symbiotic stars, the value of α is apparently affected by the size of the outer disk (Duschl 1986).

2. Positive things to say about α-viscosity

Despite the blank transparency I showed at the conference on this topic (which did not elicit the desired comic relief but only an "Aw come now!" from Joe Smak), I must admit that the α prescription has taken us a long way. Although we have learned practically nothing about viscosity, we have learned a tremendous amount about accretion physics in general and cataclysmic variables in particular. All of the theoretical studies, the observational campaigns, and the detailed comparisons between theory and reality would not have been possible had we not had the vague feeling that we might have a good physical idea of what is really going on.

3. Negative things to say about α-viscosity

Though one is tempted to sing paeons, we should admit to ourselves that our reliance on α has been dangerous. Imagine having to understand stars using only the assumption that the (unknown) energy production rate in stars is a simple function of pressure: we might be able to construct a qualitative model for stellar evolution and would now be calculating 3-D numerical models of rapidly rotating stars with magnetic fields but we would have no idea about how most stars really work. The values of α in CV's disks have been 'constrained', but no real increase in physical insight has occurred. Certainly, the generally accepted values of 0.01–0.1 are too high to be explained by physical processes like spiral shocks or non-local waves. Another danger of taking α-models too seriously is the natural tendency to ignore other possibilities because of a 'fixed mind-set'. For example, observations of quiescent disks suggest that they are optically thin near the central star, whereas the presence of a hot spot suggests that there is lots of mass in the outer disk. However, constant values of α produce surface densities which decrease rather than increase with radius. Thus, clear differences between empirical evidence and theoretical predictions exist: should we add new features or change the paradigm?

4. Further constraints on viscosity?

In an effort to see if we cannot better constrain the viscous mechanisms *without the help of an α-ansatz*, let us attempt to construct a 'detailed balance' argument very similar in spirit to that used with such success in atomic physics. Assume that we have a geometrically thin, axisymmetric and steady disk in which an unknown but *local* viscous process, dependent only upon the shear in a (nearly) Keplerian velocity field, is at work: assuming $\frac{\partial}{\partial t} = \frac{\partial}{\partial \phi} = v_{\rm Z} = 0$, $v_{\rm R} \ll c_{\rm S}$, and $\sigma_{\rm visc} = \sigma_{R\phi}$, the radial component of the Navier-Stokes equation reduces to

$$\frac{1}{\rho}\frac{\partial P}{\partial R} \approx \frac{v_\phi^2}{R} - \frac{GM}{R^2} \tag{1}$$

(the missing advection term is negligible as long as the radial velocities are subsonic). This is just the equation for what are normally called 'slim disks', in which the radial pressure gradients are taken into account without actually calculating the full 2-D disk structure. The time-scale for radial pressure equilibrium is very short compared with the viscous time-scale

$$\frac{\tau_{\rm visc}}{\tau_{\rm R}} \approx \frac{2\pi R^2 \Sigma}{\dot{\rm M}} \frac{c_{\rm S}}{R} \approx 600\ r_{10}\ \dot{m}_{17}^{-1}\ \Sigma_3\ T_4^{-1/2} \tag{2}$$

where, e.g. $r_{10} \equiv R/10^{10}\,{\rm cm}$. Reasonably steady accretion disks will have had *lots* of time to come into radial pressure equilibrium.

The implication of equation (1) is that the large body of all Keplerian disks must have constant pressures. Traditional α-disks, however, do not, unless one is willing to let α be a strong function of radius. This new constraint on the structure of steady accretion disks tells us *absolutely nothing* about viscosity. Indeed, *this is exactly the point!* If there is a viscous process conforming to the few requirements we have made at work in real disks, they will produce disks with this type of structure.

5. Analytic models

Since the deviations from Keplerian motion are small and are highly localized in the dynamical boundary layer, and since the deviations from constant pressure are felt largely within the thermal boundary layer, one can construct simple analytical models for the large body of vertically-averaged accretion disks simply by assuming that the disk has a constant pressure. For the high accretion rates typical of dwarf novae in eruption, $\kappa \propto \rho T^{-5/2}$, one can solve for the properties of the disk:

$$\begin{array}{lllllll} T & \approx & 5.7\,10^5\,{\rm K} & m_0^{+0.06} & r_9^{-0.19} & \dot{m}_{17}^{+0.13} & p_9^{+0.25} \\ \Sigma & \approx & 620\,{\rm g\,cm^{-2}} & m_0^{-0.53} & r_9^{+3.19} & \dot{m}_{17}^{-0.06} & p_9^{+0.88} \\ \alpha & \approx & 7.9\,10^{-2} & m_0^{+0.97} & r_9^{-2.91} & \dot{m}_{17}^{+0.94} & p_9^{-1.13} \end{array} \tag{3}$$

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For comparison: the standard model in which α is implicity assumed to be constant yields:

$$\begin{array}{lllllll} T & \approx & 1.5\,10^5\,K & m_0^{0.25} & r_9^{-0.75} & \dot{m}_{17}^{0.30} & \alpha^{-0.20} \\ \Sigma & \approx & 150\,\mathrm{g\,cm^{-2}} & m_0^{0.25} & r_9^{-0.75} & \dot{m}_{17}^{0.70} & \alpha^{-0.80} \end{array} \qquad (4)$$

Note the drastic difference in the properties of the disks, particularly in the dependence of the surface density Σ: this difference is reconciled by the steep dependence of the viscosity with radius in the constant pressure models.

6. Realistic models

We have constructed more self-consistent models for non-α-disks, including radial energy transport and full radial pressure equilibrium with a boundary layer. The thermal width of the boundary layer is always much larger than its dynamical width. Its structure depends upon the viscosity parameter, the stellar rotation rate, and the white dwarf boundary pressure. Away from the boundary layer, the solutions are in good agreement with the constant pressure analytic solutions. We have also derived analytic disk solutions for optically thin, constant pressure disks and fully wavelength-dependent solutions using realisitic opacities and cooling. These solutions have the nice property that the surface density *increases* rather than *decreases* with radius – just what one needs to explain the structure of quiescent dwarf novae disks.

7. Conclusions

It is clear that *simple*, *local*, and *steady* viscous processes in disks require that α be a strongly decreasing function of radius – at least! Since standard α-models are inconsistent with this requirement, non-physical α-models have ceased to be useful. This means that much more work must be expended in the production of truly physical models for the viscosity [see the contribution by Tout (1996)].

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