not, however, throw any doubt upon the theory of the subject. The only conclusion I have been able to arrive at, which appears to explain away the difficulty, is this—the rate of interest realized is usually in excess of that assumed in the valuations; and this surplus rate, operating on very large investments, creates an unanticipated fund, which continues to increase during the interval between any two successive divisions of profits—thus supplying the deficiency which existed immediately after the last distribution of surplus, from the omission of the premiums on lapsed policies. A Company with large investments might therefore spend all such premiums, and yet flourish: but this in no way upsets the theoretical view, it merely shows that, while the Society spends certain premiums that are *not* profit, it compensates that error by using money that really *is* profit in the payment of policy claims. If an Office realized precisely the rate of interest assumed in the calculation of the premiums, and the marginal additions to the net premiums were entirely absorbed in expenses of management and bonuses—and if that Office, adopting the present customary mode of valuation at each division of profits, were to reserve merely the excess of the value of the risks over that of the future premiums, giving to the future expenditure and bonus funds whatever it possessed over and above the reserve so calculated—such an Office, it is certain, would be ruined in the long run, and the cause of its downfall would be the habitual neglect to include in the various estimated reserves the proper proportion of premiums on lapsed and surrendered policies.

I believe I have now fully answered all your correspondent's objections; but before closing this letter, I would respectfully remind him that the cause of truth—if that prompted him to write—is not advanced by the use of strong and unseemly language; and I regret that his communication should have been couched in such an uncharitable tone as to savour more of a desire to find fault than of a wish to place the several matters in their true light.

I am, Sir, your obedient Servant,

SAMUEL YOUNGER.

Engineers' Life Office,
1st March, 1858.

ON THE COMMUTATION TABLES RECENTLY PUBLISHED BY MR DAVID CHISHOLM.

To the Editor of the Assurance Magazine.

SIR,—Permit me to bring under the notice of your readers one of the many facilities which will be now afforded to the actuary by the publication of Mr. Chisholm's Tables of Survivorship Assurances. Besides introducing many new kinds of transactions, they greatly abbreviate the labour of the calculation of those transactions with which we are already acquainted. In "post obits," however, they afford a still greater advantage; they enable the actuary to ascertain *more correctly* the amount which should be charged the heir of entail in repayment of the sums or annuity advanced to him. The obit charged contains, besides the sum advanced and the redemption of the interest during the joint lives, the redemption of the premiums of insurance to assure the obit on the death of the heir, should he predecease the "life in possession." But while the amount of the outlay is *progressive*, the sum usually assured is maintained at its maximum from the beginning; and thus considerable injustice is done to the heir, by making

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him pay for an assurance of the full obit in the early years, when the outlay or sum at risk is small. Mr. Chisholm's tables, however, enable the actuary to make the sum assured increase yearly in proportion to the outlay; and, by lowering the premiums, enable him to fix upon a sum, in repayment of the advances of the Office, smaller than that hitherto charged.

In illustration, and taking the case contained in Mr. Tucker's letter, p. 165 of *Magazine*, Jan. 1855, and transferring the data from Equitable $3\frac{1}{2}$ per cent. and Northampton 3 per cent. to Carlisle 3 per cent.—

p 25 v 65 North. 3 per cent.$=$·01637$=$Carl. 3 per cent.$+$81·06 per cent.
$1+ab$ 25/65 Equit. $3\frac{1}{2}$ „ $=$ 9·331 $=$Carl. 3 „ $=$9·329 „
d at 5 per cent.$=$·04762 $s=$£1,000;

then, by formula $\frac{s}{1-(p+d)(1+ab)}$, obit$=$£2,482, in consideration of £1,000 advanced. The obit would have been the same if the advance had been an annuity of $\frac{£1,000}{9·331}=$£107·17 during the joint lives.

The following exhibits the yearly amount of the outlay—1st, when £1,000 is advanced; 2nd, when £107·17 is yearly advanced—compared with the *sum assured.*

Year.	Amount of Outlay at 5 per cent.		Sum assured in both cases.	Year.	Amount of Outlay at 5 per cent.		Sum assured in both cases
	If £1,000 is advanced.	If £107 17 is advanced yearly.			If £1,000 is advanced.	If £107 17 is advanced yearly.	
	£.	£.	£.		£.	£.	£.
1	1,093	155	2,482	7	1,754	1,263	2,482
2	1,190	318	2,482	8	1,885	1,482	2,482
3	1,292	489	2,482	9	2,022	1,711	2,482
4	1,399	669	2,482	10	2,165	1,951	2,482
5	1,512	857	2,482	11	2,316	2,205	2,482
6	1,630	1,055	2,482	12	2,475	2,470	2,482

The transaction, either way, lasts 12 years—one year more than the Carlisle joint expectation of 25 and 65—after which the outlay exceeds the sum assured.

With the aid of Mr. Chisholm's tables, however, the transaction may be made to assume a different aspect. If s be the *sum advanced* and O the obit, $\frac{1}{12}$(O$-s$) will be the average increase in each of the 12 years; then the premium to assure s constant and $\frac{1}{12}$(O$-s$), with an addition of ϵ per cent., will be (25$=y$, 65$=n$)

$$\left\{s\,M_{\frac{1}{x.y}}+\frac{O-s}{12}\left(R_{\frac{1}{xy}}-R_{\frac{1}{x+12\,y+12}}\right)\right\}\frac{(1+\epsilon)}{N_{xy}}.$$

Inserting this in the formula $O=\frac{s}{1-(p+d)(1+ab)}$.

$$O=\frac{s\left\{1+\left(\frac{M_{\frac{1}{xy}}-\frac{1}{12}\left(R_{\frac{1}{xy}}-R_{\frac{1}{x+12\;y+12}}\right)}{N_{xy}}\right)(1+\epsilon)\frac{N_{xy}}{D_{xy}}(1+e)\right\}}{1-\left\{\frac{R_{\frac{1}{xy}}-R_{\frac{1}{x+12\;y+12}}}{12\,N_{xy}}(1+\epsilon)+d\right\}\frac{N_{xy}}{D_{xy}}(1+e)}$$

(The above formula is not abridged, that its relation to the original formula may be apparent.)

ϵ being any addition to the net premium=in this case to 81·06 per cent.; e being any addition to the value of the joint lives annuity=o in this case.

Again, if A=the *annuity advanced*$=\frac{s(D_{xy})}{N_{xy}(1+e)}$, the average yearly increase to the outlay will be $\frac{O}{12}$, and the premium to assure this increasing sum will be

$$\frac{R_{\frac{1}{xy}} - R_{\frac{1}{x+12\,.\,y+12}}}{12\,N_{xy}}(1+\epsilon);$$

inserting this for p in the formula $O=\frac{A(1+ab)}{1-(p+d)(1+ab)}$,

$$O=\frac{s}{1-\left\{\frac{R_{\frac{1}{xy}}-R_{\frac{1}{x+12\ y+12}}}{12\,N_{xy}}(1+\epsilon)+d\right\}\frac{N_{xy}}{D_{xy}}(1+e)}.$$

Calculating the value of these formulæ upon the same data as the former—that is, p=Carl. 3 per cent+81·06 per cent., and ab=Carl. 3 per cent. (e=o per cent.), and assuming the transaction to last 12 years—the following exhibits the yearly progress of the outlay and the assurance; 1st, when the advance is £1,000, the obit being £2,284; and, 2nd, when the advance is an annuity of £107·17, the obit being in this case only £2,151.

Year.	When the Advance is a Sum of £1,000.		When the Advance is an Annuity of £107·17.	
	Amount of Outlay at 5 per cent.	Assurance.	Amount of Outlay at 5 per cent.	Assurance.
	£.	£.	£.	£.
1	1,080	1,107	135	179
2	1,165	1,214	276	359
3	1,253	1,321	424	538
4	1,346	1,428	580	717
5	1,444	1,535	743	896
6	1,546	1,642	915	1,076
7	1,654	1,749	1,096	1,255
8	1,767	1,856	1,285	1,434
9	1,886	1,963	1,484	1,614
10	2,010	2,070	1,692	1,793
11	2,141	2,177	1,912	1,972
12	2,280	2,284	2,142	2,151

The result is, that on a transaction of £1,000 a saving to the heir of entail is effected of £200 or £350, with equal rates of premium, &c. to the Office, and all through the instrumentality of Mr. Chisholm's Tables.

I hope you will excuse my illustrating at such length the greater advantage of a *progressive assurance* than the usual form of an *assurance constant*, when intended to cover *progressive advances*.

I am, Sir,

Your most obedient Servant,

JAMES MEIKLE.

Scottish Provident Institution,
Edinburgh, 6th March, 1858.

[Note.

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[NOTE.—Our correspondent does well to point out the desirableness of reducing the heavy charge which necessarily arises under the mode of assurance usually adopted in cases such as those to which his letter refers. The effecting an assurance at once, for the probable amount of advances to be made, presses with great severity on persons having to raise money on these contingent securities. But the remedy does not altogether depend, as Mr. Meikle seems to think, upon the removal of any difficulty which may be found to attend the calculation of the increasing assurance: it has to do rather with the unwillingness of Assurance Companies to bind themselves to undertake an increasing risk, with or without limit, on a life, it may be, deteriorating as the amount at risk increases. The objection on the part of the Companies to enter into an engagement of this kind will be found to be very strong and very general; and they have, no doubt, reason on their side. It seems to us, nevertheless, that, under certain restrictions, our correspondent's suggestion might be sometimes acted upon, and he has certainly done good service in drawing attention to the matter. In his remarks on Mr. Chisholm's work we very cordially concur, and hope to take an early opportunity of dwelling more at large upon its merits.— ED. *A. M.*]

FORMULA FOR AN APPROXIMATE VALUE OF ANNUITIES AT SIMPLE INTEREST.

To the Editor of the Assurance Magazine.

SIR,—I beg to submit the following series for finding the present value of an annuity at simple interest. It is

$$\frac{\log._{\epsilon}(1+nr)}{r}+\frac{1}{2}\left(1+\frac{1}{1+nr}\right)+\frac{r}{12}\left(1-\frac{1}{\overline{1+nr}|^{2}}\right)$$
$$-\frac{r^3}{120}\left(1-\frac{1}{\overline{1+nr}|^{4}}\right)+\ldots-1.$$

It is obtained by applying a well known formula of the differential calculus (*De Morgan's Differential Calculus*, p. 311) to the summation of

$\frac{1}{1+r}+\frac{1}{1+2r}+\ldots\frac{1}{1+nr}$, and taking the limits from o to n.

Upon reference, I find that a similar, although not so convenient an approximation, has been given in Vol. V., page 256, of the *Assurance Magazine*. By a misprint, the modulus of the Napierian logarithms has been put down in it as 2·3205851, instead of 2·3025851.

I am, Sir,

Your obedient servant,

MARCUS N. ADLER.

4, *Crosby Square*,
12*th March*, 1858.