A FLOW MODEL AND A TIME SCALE FOR THE ICE CORE FROM CAMP CENTURY, GREENLAND

By W. Dansgaard *and* S. J. Johnsen

(Fysisk Laboratorium II, H.C. Ørsted Institutet, Københavns Universitet, København, Denmark)

ABSTRACT. A flow model is described for the Camp Century area in Greenland. The horizontal velocity profile along the core is assumed to be uniform from the surface down to $y = 400$ m above the bottom. Below this level, the horizontal velocity v_x is assumed to decrease proportionally to y. Furthermore, at a given y, v_x is assumed to be proportional to the distance x from the ice divide. The resulting vertical strain-rate under steady-state conditions gives the age of the ice as a function of y. The flow model has explained the measured temperature profile, and the time scale has been verified by comparison between observed stable isotope variations and past climatic changes (at least 70 000 years back in time) estimated by other methods.

RÉSUMÉ. *Un modèle d'écoulement et une échelle de temps pour la carotte de glace du Camp Century, Groenland.* Les auteurs décrivent un modèle d'écoulement de la glace pour la zone du Camp Century au Groenland. Le profil horizontal des vitesses tout au mond de la carotte de glace est supposé être uniforme de la surface jusqu'à une profondeur de $y = 400$ m au-dessus de socle rocheux. En-dessous de ce niveau, la vitesse horizontale v_x est supposée décroître proportionnellement à y. De plus, à une profondeur y donnée, v_x est supposée être proportionnelle à la distance x de la ligne de séparation des glaces. La vitesse de déformation verticale qui en résulte sous des conditions stationnaires donne l'âge de la glace en fonction de y. Le modèle d'écoulement a expliqué le profil mesuré des températures et l'échelle des temps a été vérifiée par comparaison des variations observées des concentrations d'isotope stable et les changements climatiques passées (jusqu'à 70 000 ans en arrière) estimés par d'autres méthodes.

ZUSAMMENFASSUNG. *Ein Fliessmodell und eine Zeitskala für den Bohrkern von Camp Century, Grönland.* Für das Gebiet von Camp Century in Grönland wird ein Fliessmodell beschrieben. Das Horizontalgeschwindig-keitsprofil entlang des Kerns wird von der Oberfläche bis zu $y = 400$ m über dem Boden als gleichförmig angenommen. Darunter wird eine Abnahme der Horizontalgeschwindigkeit v_x proportional zu y voraus-gesetzt. Weiterhin wird unterstellt, dass sich für ein gegebenes y v_x proportional zur Entfernung x von der Eisscheide verhält. Die resultierende vertikale Verformungsgeschwindigkeit unter stetigen Bedingungen ergibt das Alter des Eises als eine Funktion von y. Das Fliessmodell erklärt das gemessene Temperaturprofil; die Zeitskala wird durch einen Vergleich zwischen beobachteten Variationen stabiler Isotope und früheren Klimaschwankungen (zumindest in den letzten 70 000 Jahren), die mit anderen Methoden abgeschätzt werden, bestätigt.

INTRODUCTION

Ice cores contain information on climatic and geochemical conditions of the past, because the isotopic and chemical composition of falling snow remains unchanged in glacier ice for long periods of time. However, the value of isotopic and chemical analysis is, of course, closely related to a knowledge of the age of the ice in question. This emphasizes the necessity for establishing a time versus depth relation.

Macroscopic stratigraphic studies have been used by many investigators on the upper strata of ice which are no more than a few hundred years old. However, the technique based on seasonal variation in light transmissivity (Langway, 1967) might be used for ice several thousand years old (Johnsen and others, unpublished).

Another possible way of establishing a time scale along an ice core would seem to be the measurement of seasonal oscillations in the isotopic composition of the ice (Epstein and Sharp, 1959) and counting isotopic maxima from the surface. However, various processes tend to diminish the isotopic gradients in snow and ice. For example, molecular diffusion in the solid ice, accelerated by the thinning of the layers, gradually obliterates the stable isotope oscillations that remain after firnification (Johnsen and Dansgaard, unpublished).

Four radioactive isotopes have been used for dating ice. Tritium (Aegerter and others, 1969) and ^{210}Pb (Goldberg, 1963; Crozaz and others, 1964; Crozaz and Langway, 1966) reach only 100 years back in time. The other two, ^{32}Si (Dansgaard and others, 1966) and ^{14}C (Scholander and others, 1962), may be detected in 3 000 and 20 000–30 000 year-old ice, respectively, but both of these techniques require several tons of ice. The technique for sampling ^{14}C from ice *in situ*, developed by Oeschger and others (1967), might in the future be perfected to be applied to deep bore holes.

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But at present a time scale for the Camp Century core must be based on calculations on a chosen flow model.

THE NYE MODEL

Most of the previously evaluated flow models imply a uniform vertical strain-rate along any vertical line in an ice cap (Nye, 1951, 1957, 1959). Furthermore, assuming that the horizontal velocity components are parallel and there is negligible melting at the bottom, Nye (1963) expressed the ratio between the reduced and initial thicknesses, λ and λ_H, of an annual layer (e.g. in cm of ice) by

$$\frac{\lambda}{\lambda_H} = \frac{y}{H},$$ (1)

y and H being its present and initial distance from the bed.

With this model, we get the change of λ per unit of time

$$\frac{d\lambda}{dt} = \frac{\lambda_H}{H} \frac{dy}{dt} = -\frac{\lambda_H \lambda}{H\tau},$$

τ being 1 year. Integration gives λ as a function of time

$$\lambda = \lambda_H \exp\left(-\frac{\lambda_H t}{H\tau}\right)$$ (2)

and from Equation (1) we get the depth of a layer age t formed at a height H above the bottom

$$y = \frac{H\lambda}{\lambda_H} = H \exp\left(-\frac{\lambda_H t}{H\tau}\right).$$ (3)

Hence, the age of a layer $(H-y)$ below surface (cf. Haefeli, 1961)

$$t = -\frac{H\tau}{\lambda_H} \ln \frac{y}{H}.$$ (4)

Figure 1 shows λ as a function of time (Equation (2)) for $H = 1\,400$ m and $\lambda_H = 0.2$ and 0.35 m. Paradoxically, the highest value of λ_H corresponds to the thinnest layers after 5 000 years. The explanation is that, for a given value of H, the horizontal movement and, therefore, the vertical strain-rate will be the faster the higher the rate of accumulation.
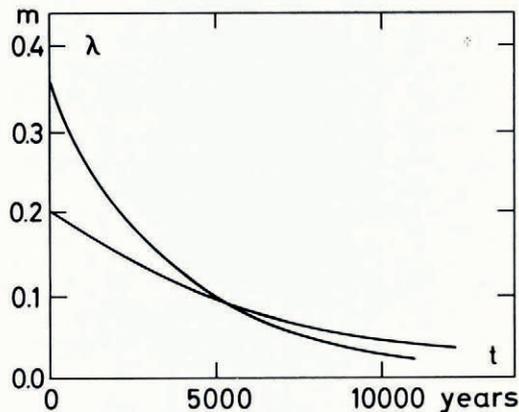

Fig. 1. *Thickness λ of an annual layer as a function of age t, according to the Nye flow model. Initial thicknesses 0.35 and 0.20 m. Thickness of the ice sheet $H = 1\,400$ m.*

The Nye model describes the flow conditions at the bedrock by sliding and/or by rapid shear strain-rates being concentrated in a thin bottom layer (Nye, 1963). However, at Camp Century the sliding is probably negligible, because the temperature at the bottom is as low as $-13°C$ (Hansen and Langway, 1966). Furthermore, the idea of relative motion being essentially concentrated in a thin bottom layer was advanced in view of the fact that the flow law of ice depends critically upon the temperature, at the same time as the temperature gradient at the bottom was assumed to be as high as $10°C/100$ m (Nye, 1959). However, at Camp Century the temperature gradient was later measured to be only $1.8°C/100$ m (Hansen and Langway, 1966). Consequently, Nye's flow model is not necessarily applicable in the case of the Camp Century area.

Non-uniform Vertical Strain-rate Model

Another approach would be to assume, like Nye (1963), that the horizontal velocity profile along a vertical line in the distance x from the ice divide may be written as

$$v_x(y) = k \cdot f(y) \cdot x$$

and also to calculate $v_x(y)$ by integrating Glen's law (Haefeli, 1961)

$$v_x(y) = \int_0^y k' \, \sigma^n \, \mathrm{d}y, \tag{5}$$

σ being the shear stress, k' and n being constants that depend on the temperature and therefore on y. This integration is justified only if the longitudinal strain-rate is small compared with the shear strain-rate at any y, which is only the case in the lower part of the ice sheet. Nevertheless, we consider Weertman's (1968) calculation of Equation (5), shown as the full curve in Figure 2, as an improvement compared with calculations based on a uniform vertical strain-rate, because Weertman's procedure accounts for the important fact, that $v_x(y) = 0$ for $y = 0$. In this work, we use the simple approximation shown as the dashed curve in Figure 2, i.e. v_x proportional to y from $y = 0$ to $y = h$, and v_x independent of y from $y = h$ to $y = H$ or, in other words

$$\left.\begin{aligned} f(y) &= y/h, & 0 \leqslant y \leqslant h, \\ f(y) &= 1 & h \leqslant y \leqslant H. \end{aligned}\right\} \tag{6}$$
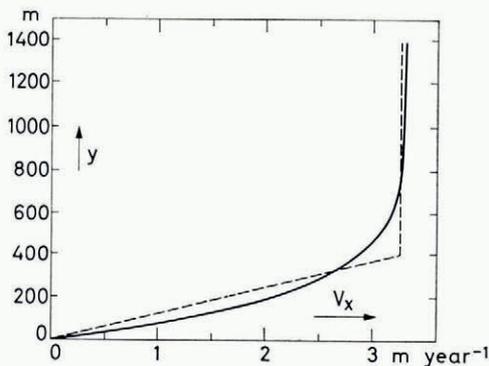

Fig. 2. *Full curve: horizontal velocity profile calculated by integration of Glen's law, assuming $v_x(0) = 0$ and $v_x(H) = 3.3$ m year^{-1} (from Weertman, 1968). Dashed curve: adopted approximation.*

The incompressibility of the ice is expressed by

$$\frac{\partial v_y}{\partial y} + \frac{\partial v_x}{\partial x} = 0,$$

and the vertical velocity component is calculated as

$$\frac{\partial v_y}{\partial y} = -\frac{\partial v_x}{\partial x} = -k f(y),$$

$$v_y = -k \int_0^y f(y)\, dy$$

and, according to Equation (6)

$$v_y = \begin{cases} -ky^2/2h, & 0 \leqslant y \leqslant h \\[2mm] -\dfrac{k}{2}(2y-h), & h \leqslant y \leqslant H. \end{cases} \tag{7}$$

For $y = h$:

$$v_h = -\lambda_h/\tau = -\frac{k}{2}h; \qquad k = 2\lambda_h/h\tau. \tag{8}$$

For $y = H$:

$$\left. \begin{aligned} v_H &= -\lambda_H/\tau = -\frac{k}{2}(2H-h) = -\lambda_h(2H-h)/h\tau, \\[2mm] \lambda_h &= \frac{h}{2H-h}\lambda_H. \end{aligned} \right\} \tag{9}$$

Inserting Equations (8) and (9) into Equation (7) gives

$$v_y = \left. \begin{cases} -\dfrac{\lambda_h}{h^2\tau}y^2 = -\dfrac{\lambda_H}{h(2H-h)\tau}y^2, & 0 \leqslant y \leqslant h \\[3mm] -\dfrac{(2y-h)\,\lambda_h}{h\tau} = -\dfrac{2y-h}{2H-h}\dfrac{\lambda_H}{\tau}, & h \leqslant y \leqslant H, \end{cases} \right\} \tag{10}$$
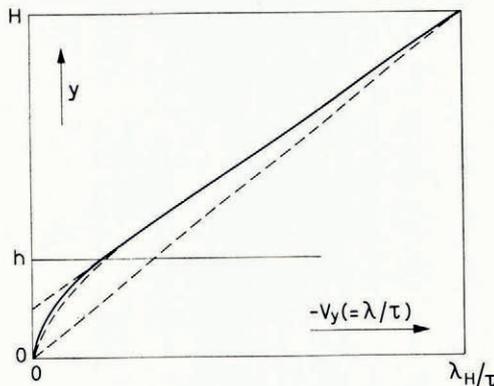

Fig. 3. *Vertical velocity v_y as a function of the distance y from the bottom. Dashed line: the Nye flow model; full curve: the flow model described on p. 217. The dashed curve close to the full curve corresponds to the horizontal velocity profile calculated by Weertman (1968); cf. Fig. 2.*

which is shown in Figure 3 (cf. discussion on p. 217-18). The straight line in the upper range intersects the y-axis at $y = h/2$. Consequently, in the same range, $h \leqslant y \leqslant H$, we can use Nye's model and Equations (1), (2) and (3), if H and y are replaced by $H - \dfrac{h}{2}$ and $y - \dfrac{h}{2}$

$$\left.\begin{aligned}
\lambda &= \frac{2y-h}{2H-h}\,\lambda_H = (2y-h)\,\frac{\lambda_h}{h}, \\
\lambda &= \lambda_H \exp\left[-\frac{2\lambda_H}{(2H-h)\,\tau}t\right] = \lambda_H \exp\left(-\frac{\lambda_h}{h\tau}t\right),
\end{aligned}\right\}\tag{11}$$

$$t = -\frac{(2H-h)\,\tau}{2\lambda_H}\ln\frac{2y-h}{2H-h} = -\frac{h\tau}{2\lambda_h}\ln\frac{2y-h}{2H-h}, \qquad h \leqslant y \leqslant H. \tag{12}$$

In the range $0 \leqslant y \leqslant h$, Equation (10) gives

$$v_y \equiv \frac{\mathrm{d}y}{\mathrm{d}t} = -\frac{\lambda}{\tau} = -\frac{\lambda_h}{h^2\tau}y^2. \tag{13}$$

Integration over the intervals $h \to y$ and $t_h \to t$ (t_h being the age of ice at $y = h$ given by Equation (12)) leads to

$$y = \frac{h}{1+\lambda_h(t-t_h)/h\tau} \tag{14}$$

or

$$t-t_h = \frac{h^2\tau}{\lambda_h}\left(\frac{1}{y}-\frac{1}{h}\right) = \frac{(2H-h)\tau}{\lambda_H}\left(\frac{h}{y}-1\right), \qquad 0 \leqslant y \leqslant h \tag{15}$$

and from Equations (13) and (14)

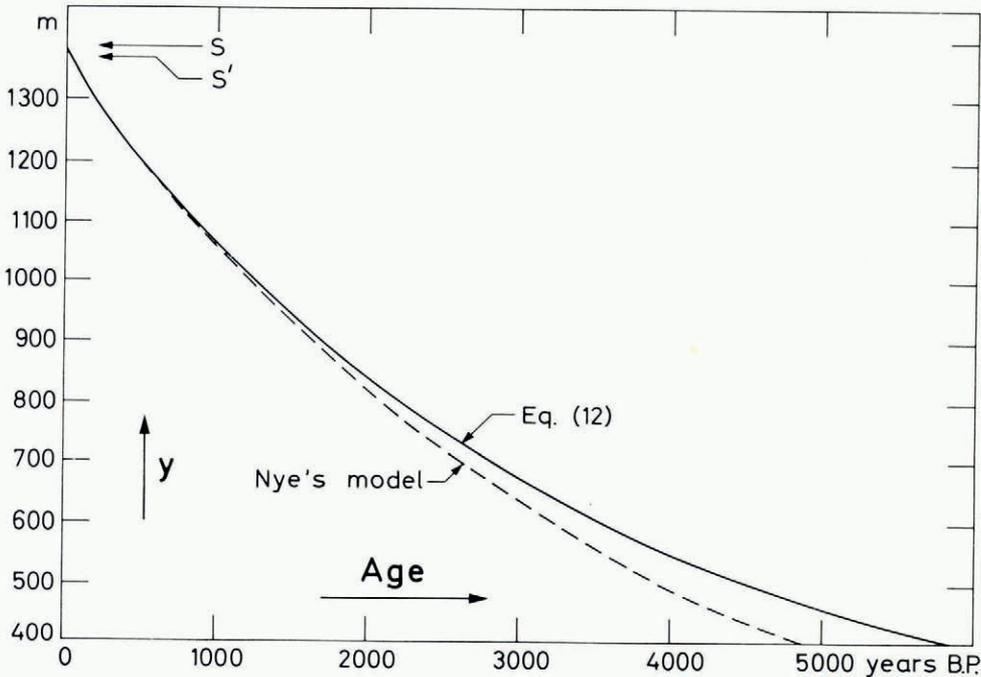$$\lambda = \lambda_h[1+\lambda_h(t-t_h)/h\tau]^{-2}. \tag{16}$$

Fig. 4. *Age of the ice in the Camp Century core as a function of the distance y from the bottom, y ⩾ h. S is the 1966 surface (y = 1 387.5 m) and S′ is the 1966 surface corrected for low densities in the upper layers (y′ = H = 1 367.5 m of ice).*

Using the equations way back in time implies that H and λ_H are independent of x and, furthermore, steady-state conditions in a broad sense, i.e. H, λ_H and ice temperature T are constant in time.

The former assumption seems justified (i) by the direction of the movement at Camp Century being approximately parallel to the iso-accumulation curves in the area (Mock, 1968), and (ii) by the site of formation of even a 15 000 year old section of the core being barely more than 50 km from Camp Century (the surface velocity is 3.3 m/year (Mock, referred to by Weertman (1968)). As to the changes in the non-independent parameters, H, λ_H and T, many thousand years back in time, it might be reasonable to assume lower λ_H and T, but higher H during the glaciation. Lower λ_H in that period would correspond to a closer packing of the annual layers, which, on the other hand, would be counteracted by slower thinning of the layers due to lower temperature (i.e. higher viscosity of the ice) and higher H. However, we have no means of verifying such assumptions, so all that we can do at present is to calculate the time scale by using the present constants and try, by other means, to check if and when an error enters (these remarks, of course, should also be considered in connection with the thinning of the layers (Fig. 3)).

The mean value of λ_H over the last 100 years has been 0.35 ± 0.03 m of ice, as shown by Crozaz and Langway (1966) by the ^{210}Pb method. The surface to bottom distance, H, was measured as 1 387.5 m in 1966 (Hansen and Langway, 1966), corresponding to 1 367.5 m
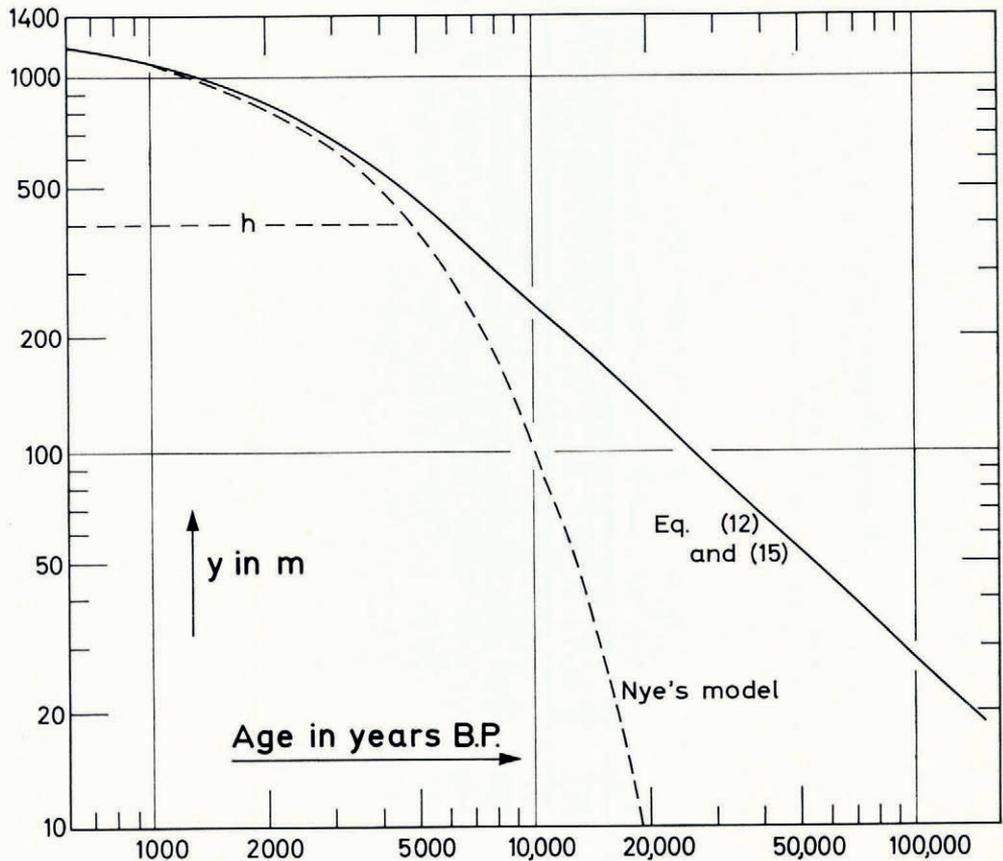

Fig. 5. Age of the ice in the Camp Century core as a function of the distance y from the bottom (1 160 m ⩾ y ⩾ 20 m), both on a logarithmic scale.

of ice in view of the low densities in the upper layers (personal communication from C. C. Langway, Jr.).

Thus, we can use Equations (12) and (15) with $H = 1\,367.5$ m, $h = 400$ m, $\lambda_H = 0.35$ m, and $\tau = 1$ year. For $t > 6\,000$ years, we can write

$$t = \left(\frac{2.670 \times 10^6}{y}\right) - 800 \text{ years B.P.}$$

The result is shown by the full curves in Figures 4 and 5. According to the Nye model (dashed curve; Equation (4)), only the lowest few metres of ice should be more than 20 000 years old, whereas in our model the ice reaches this age 130 m above the bottom.
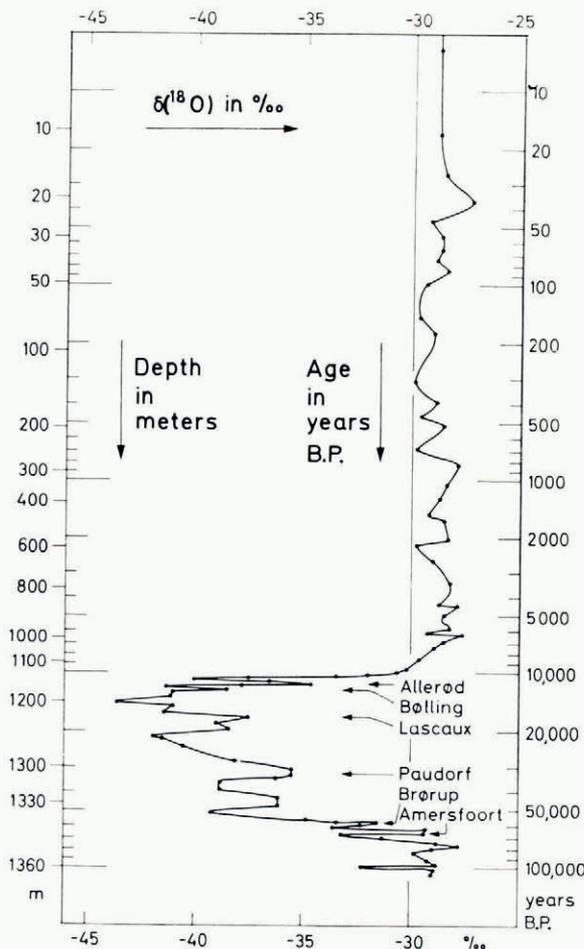

Fig. 6. *The heavy oxygen isotope concentration, $\delta(^{18}O)$, of sections of the Camp Century core versus the age of the ice (Equations (12) and (15)) plotted on a logarithmic scale to the right. The outer scale to the left gives the corresponding depth below the 1966 surface. The isotope data are given as relative deviation of the $^{18}O/^{16}O$ ratio from that of standard mean ocean water. (From Dansgaard and others, unpublished.)*

EXPERIMENTAL EVIDENCE

The validity of the flow model described above has been checked in two independent ways. First, the flow model was used as a basis for calculating the temperature profile down the bore hole (Dansgaard and Johnsen, 1969). The result fitted the profile measured by

Hansen and Langway (1966) within $\pm 0.6°C$. The calculated temperature difference between surface and bottom deviated only $0.3°C$ from the measured difference. Using the Nye model, Weertman (1968) found a deviation of $2.7°C$.

Secondly, when stable oxygen-isotope data for about 1 600 samples from the core were plotted against the age of the ice in our time scale (Fig. 6), they showed a variation which was in complete agreement with practically all known climatic changes within the last 70 000 or 100 000 years (for further details, cf. Dansgaard and others, in press). The geochemical explanation for this is the fact that the isotopic composition of precipitation at high latitudes is mainly determined by the temperature of formation (Dansgaard, 1954, 1964; Gonfiantini and Picciotto, 1959), which causes seasonal isotopic oscillations in the ice strata, as well as long-period isotopic variations in phase with climatic changes.

The agreement with other quite independent climatological estimates, covering nearly 100 000 years, leads us to the conclusion that the time scale and therefore our flow model is basically correct down to 30 or 35 m above the bottom. Below that level, the flow pattern is probably influenced by the irregular topography of the bottom, which is also indicated by visual observation of the lowest silty core sections.

Moreover, unless the influences of the various parameters, which determine the flow pattern, quite accidentally counterbalance each other (e.g. as outlined on p. 220), it would seem as if λ_H and H have not deviated considerably from their present values in the Camp Century area.

One might ask how the time scale is influenced by the choice of h (cf. Fig. 2). To check this, we consider a horizon $y = 209.0$ m above the bottom. In our time scale ($h = 400$ m) the age is $t_{209} = 12\,000$ years with an uncertainty of barely more than 1%,* whereas, for $h = 300$ m (500 m), t_{209} would be $10\,400$ years B.P. ($13\,600$ years B.P.). Thus, if $f(y)$ is accepted in accordance with Equation (6), h must be very close to 400 m.

ACKNOWLEDGEMENT

MS. received 15 December 1968

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* Because, in the same range, the late interstadials (Allerød and Bølling) are reflected by the stable-isotope curve in time intervals (in our time scale) only 100 years from the ^{14}C dates (11 800–11 000 years B.P. and 12 400–12 000 years B.P.).

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