FORUM

A Study of a Species of Short-Method Table

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THIS paper discusses navigation tables based on the decomposition of the astronomical triangle into two right-angled spherical triangles by a great circle arc extending from the zenith to the meridian of an observed celestial body. In a recent fairly comprehensive study of some thirty short-method tables in which the division of the PZX-triangle forms the principle of construction, nineteen are of the species to be discussed.

Following the introduction in 1871 of the first short-method table by Thomson, some twenty years were to pass before any real advance was made in this field. Thomson's table was in fact re-issued by Kortazzi in 1880 and by Collet in 1891, in modified forms, but it was Professor F. Souillagouët[1] of France who is to be credited for introducing something novel and decidedly better than Thomson's table. Unlike the earlier ones it was designed specifically for the Marcq Saint Hilaire method of sight reduction and, in contrast to Thomson's table which was based on the division of the PZX-triangle by a perpendicular from X, Souillagouët's was based on division by a perpendicular from Z.

In the following discussion we shall denote the sides and angles of the two right-angled triangles which are produced from the division of the PZX-triangle in accordance with the attached diagram and confine our attention (for the sake of simplicity) to the single case in which latitude and declination have the same name; the meridian angle is less than 6 hr and the foot of the perpendicular, denoted by M, lies between P and X.

Souillagouët's table for altitude is based on the formulae:

$$\tan a = \cot \phi \cos p \tag{1}$$
$$\sin x = \cos \phi \sin p \tag{2}$$
$$\sin h = \cos x \cos b \tag{3}$$
$$\cos x = \sin \phi / \cos a \tag{4}$$

From equations (3) and (4):

$$\sin h = (\sin \phi / \cos a) \cos b \tag{5}$$

Hence:

$$\log \sin h = \log (\sin \phi / \cos a) + \log \cos b$$

Or:

$$\log \sin h = \log x + \log \cos b \tag{6}$$

Souillagouët provided a double-entry table giving (against arguments ϕ and p) arc a computed from equation (1) and log X computed from equation (4) using values of x computed from equation (2). Arc b is then found from the relationship $b = 90° - (a + d)$; finally, a log sine table is entered with the sum of log x and log cos b to find altitude h, as in equation (6).

The process of finding altitude using Souillagouët's method is very rapid, requiring only three table entries and two simple additions, and free from tedious interpolations provided that the *point auxiliaire*, to use Souillagouët's term, corresponds to integral degrees of ϕ and p.

Two years after the publication of Souillagouët's table his compatriot, Lieutenant R. Delafon of the French Navy, brought out the first table of its type to give azimuths as well as altitudes.[2]

Delafon's table is based on the formulae:

$$\begin{cases} \sin \phi = \cos x \cos a & (7) \\ \sin h = \cos x \cos b & (8) \end{cases}$$

$$\begin{cases} \tan p = \tan x / \sin a & (9) \\ \tan z_1 = \tan a / \sin x & (10) \\ \tan z_2 = \tan b / \sin x & (11) \end{cases}$$

Delafon recognized that equations (7) and (8) have the same form, as also do equations (9), (10) and (11). His table gives, in essence, values of p, $\phi(h)$, and $z_1(z_2)$ against arguments $a(b)$ and x, each at degree intervals from 0° to 90°. Compared with Souillagouët's method Delafon's was tedious, largely on account of the interpolation required and the need for auxiliary tables to facilitate this. Only one edition of Delafon's table was published, but a second edition of Souillagouët's table appeared in 1900.

The next significant contribution to short-method tables was made by Charles Bertin—not only Professor of Hydrography at Saint Malo but also 'ex-Commandant Pilote aviateur'. Bertin designed his table,[3] which was first published in 1919, for aviators as well as mariners. The table is based on the formulae:

$$\tan a = \cot \phi \cos p \quad (1)$$

$$\sin x = \cos \phi \sin p \quad (2)$$

$$\sin h = \cos x \cos b \quad (3)$$

$$\cot z_2 = \sin x / \tan b \quad (12)$$

The first three equations are identical with those used by Souillagouët. Bertin's method suffered in the same way as Delafon's from troublesome interpolations, in this instance facilitated by means of a graphical method. Nevertheless Bertin's table was extensively used in the French Navy and Merchant Service and a second edition was published in 1929.

In the year following the first publication of Bertin's table the Japanese edition of Ogura's table[4] appeared and, four years later in 1924, an English edition[5] was published. Sinkiti Ogura (1884–1937) whose table had a profound effect on many later table-makers entered the Japanese Imperial Naval Service, after completing a course in astronomy at the Tokyo Imperial University, in 1910. In 1927 he was placed in charge of the Section at the Hydrographic Department devoted to navigation and oceanography. Ogura made valuable contributions in the field of oceanography, especially tidal work in Japanese waters, but to navigators he is best known for his *Altitude Tables* first published in 1920.

In the preface to the English edition we are told that in 1919 the Naval College of Japan requested the Hydrographic Department for criticism of a proposed appendix to the nautical tables that had been compiled at the College. It appears to have been fortuitous that at the same time Ogura had devised a new tabular method for finding altitude. It was decided, therefore, to compile a volume of tables to include Ogura's table and the necessary calculations were made under Ogura's supervision. It is interesting to note that the volume[4] which appeared in 1920 included a direct method of sight reduction, based on haversines, invented by Commander Yonemura I.J.N., an instructor at the Naval College.

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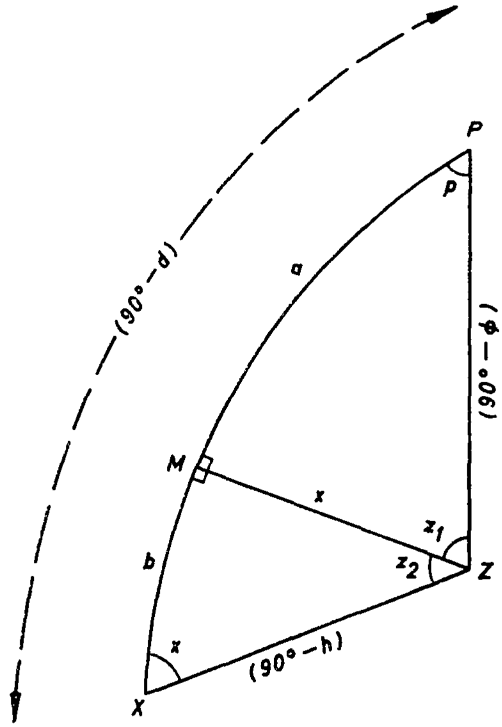

FIG. 1

The English version was ready for publication in August 1923 but, when it was on the point of distribution, the headquarters of the Hydrographic Department was totally destroyed by the earthquake and fire of 1 September 1923 and all the printed tables were reduced to ashes; despite the set-back the tables appeared in 1924.

Ogura did not base his table on the decomposition of the PZX-triangle although his method is usually explained by so doing. He starts with the spherical cosine formula applied to the PZX-triangle and his treatment is as follows:

$$\sin h = \sin \phi \sin d + \cos \phi \cos d \cos p$$

Assume that:

$$H \sin K = \sin \phi \qquad (13)$$

$$H \cos K = \cos \phi \cos p \qquad (14)$$

Then:

$$\sin h = H \sin K \sin d + H \cos K \cos d$$

and:

$$\sin h = H \cos (K + d) \qquad (15)$$

From (13) and (14) we have:

$$\tan K = \tan \phi \sec p \qquad (16)$$

For a given latitude ϕ and meridian angle p, K can readily be found from equa-

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tion (16). Knowing K, H can then be found from equation (13). By combining K with d and using equation (15) h may be found thus:

$$\log \sin h = \log H + \log \cos (K + d) \qquad (17)$$

For the purposes of his table Ogura expressed equation (15) in the inverted form:

$$\operatorname{cosec} h = (1/H) \sec (K + d) \qquad (18)$$

The auxiliary angle K was calculated to the nearest minute of arc; since ϕ and p are represented in the table at integral degrees this precision is always possible. An accurate value of K having thus been found an equally precise value results from the equation:

$$H = \sin \phi \operatorname{cosec} K \qquad (13)$$

or:

$$1/H = \operatorname{cosec} \phi \sin K \qquad (19)$$

since it is the reciprocal of H which is tabulated.

Values of K and log $(1/H)$ are arranged in Ogura's table (which occupies only 18 small pages) for each integral degree of latitude from 0° to 65° and meridian angle from 0° to 90°. The method is very similar to Souillagouët's, but Ogura based his solution on secants and cosecants whereas Souillagouët's was based on sines and cosines. Ogura's K corresponds to $(b + d)$ in the diagram, that is to say it is the declination of the foot of the perpendicular, and $1/H$ is equivalent to log sec x.

H. B. Goodwin in his notice[6] of Ogura's table in *The Nautical Magazine* of 1921 made the suggestion that a table on the Ogura model might well be included in Inman's or Norie's Tables. No doubt as a result of this suggestion a table entitled 'Short Method for Zenith Distance' made its first appearance in Norie's Tables in 1924.[7] This gives values of quantities called A and K; A corresponding to Ogura's $1/H$. The table became known as the 'A and K Table' and has been a feature of Norie's Tables since 1924. A small point of difference is that Norie's table gives 'log secants of zenith distance' instead of the 'log cosecants of altitude' given by Ogura.

Although no hard evidence exists it is not unlikely that Ogura's table was better known, at least to British navigators of the Merchant Navy, than any other table of its type.

Ogura's countryman Captain Toshi-Ichi Arimitsi of the Japanese Merchant Service brought out a set of tables[8] based on Ogura's method in 1921. Arimitsi's table is of 90 pages compared with Ogura's 18, the longer table being based on an interval of 15 minutes for latitude and meridian angle instead of one degree.

Almost contemporaneously with the introduction of Ogura's table Professor W. M. Smart and Commander F. N. Shearme R.N. introduced their *Position Line Tables*[9] in 1922. They used formulae (1) and (2) of Souillagouët's method and:

$$\sin h = \cos x \sin (a + d) \qquad (20)$$

The arc a and the log cosine of x were denoted by U and V respectively, so that:

$$\log \sin h = V + \log \sin (U + d) \qquad (21)$$

The *Position Line Tables* were included in the Inman collection of nautical tables in the 1940 reprint of the second edition, by H. B. Goodwin, of William Hall's rearrangement. They occupy 34 pages and require the use of a log sine table as well as the 'U and V Tables'.

In 1924 Commander (later Vice-Admiral) I. A. Newton of the Portuguese Navy and Captain J. C. Pinto of the Portuguese Merchant Service collaborated in producing a navigation table[10] based on the formulae:

$$\begin{cases} \sin x = \cos\phi \sin p & (22) \\ \cot Z_1 = \sin\phi \tan p & (23) \\ \cot(90^\circ - a) = \cot\phi \cos p & (24) \end{cases}$$

$$\begin{cases} \sin h = \cos b \sin(90 - x) & (25) \\ \cot Z_2 = \sin x \tan(90 - b) & (26) \\ \cot X = \cot x \cos(90 - b) & (27) \end{cases}$$

Newton and Pinto based their method on the similarity of equations (22), (23) and (24) to equations (25), (26) and (27) respectively. Although their table was thoughtfully designed and the rules are given, according to the authors, 'in an elegant abridged manner' the solution is complex and suffers from the need to interpolate.

In 1927 Lieut. Com. P. V. H. Weems U.S.N. brought out his neat and compact *Line of Position Book* in 44 pages.[11] Weems employed Ogura's method, having obtained the latter's permission to do so, and within a short time all the 1000 copies printed had been distributed to the ships of the United States Navy. In the following year a new edition appeared in which the table was extended from 65° to 90°. Weems informs us that the tables were rushed to completion for the possible use of:

> '. . . my friend, Mr. Lincoln Ellsworth and of my classmate, Commander R. E. Byrd, in their projected polar flights. It is also hoped that these tables will find use in what many believe to be the era of trans-polar flights . . .'

The United States Navy Publication No. 208,[12] first published in 1928, was the work of Lieut. Com. J. Y. Dreisonstok. The basis of Dreisonstok's table is similar to Ogura's, the principal difference being that Ogura worked with *K*, the declination of the foot of the perpendicular ZM, whereas Dreisonstok worked with the complement of *K*. Within a year of publication E. B. Collins of the U.S. Hydrographic Office extended the computations for the table from 65° to 90° for the 1929 Byrd expedition to the Antarctic, for which Weems had already extended his table.

In 1931 Lieut. John E. Gingrich U.S.N. produced a set of navigation tables[13] which he had arranged four years earlier whilst a teacher of navigation at the U.S. Naval Academy at Annapolis. Gingrich employed Ogura's method (Ogura having given him permission) in a rearranged form. Two tables are used for finding altitude: the first gives 10^5 log secant *x* (*x* is designated *A* in the table) against arguments of latitude and meridian angle at one-degree intervals, from 0° to 65° and 0° to 90° respectively. The second table gives 10^5 log cosecants of all angles from 0° to 90° at one-minute intervals.

Gingrich denoted the declination of the foot of the perpendicular ZM by *K* and the arc ZM by *A*. His tables I and II are based on:

$$10^5 \log \operatorname{cosec} h = 10^5 \log \sec A + 10^5 \log \sec (K + d)$$

Captain Pinto, who as we have noted collaborated with Newton, was the sole author of a set of tables published in 1933.[14] Pinto's 'Simplex' tables were based on Souillagouët's formulae (1) and (2) and the inverted form of formula (3). Pinto's first table gives respondents p (declination of M corresponding to Ogura's K), and log secant x (corresponding to Ogura's $1/H$) against arguments of latitude at $\frac{1}{4}°$ intervals from 0° to 60° and meridian angle for each minute of time from 0 hr to 12 hr. The second table gives log secants (and cosecants) at minute-of-arc intervals from 0° to 90°. The solution for altitude is very similar to Ogura's.

The Deutsche Seewarte issued its Publication No. 2154, familiarly known as 'F-Tafel', in 1937.[15] The method of finding altitude is similar to the 'Sine Method' introduced by Smart and Shearme and is based on the formula:

$$\log \sin a = V + \log \sin (U + d)$$

in which a, V, d and U denote, respectively, altitude (h), declination (d), log cos ZM (x) and arc ZM (x).

Hughes' tables,[16] designed by Dr. L. J. Comrie and first published in 1938, were beautifully produced and are according to D. H. Sadler in a tribute to Comrie in 1950, '. . . probably the finest book of navigational tables of their type'. Comrie's method for finding altitude is similar to Ogura's and the table is based on the formula:

$$\log \operatorname{cosec} a = A + \log \sec (K + d)$$

K having the same meaning as Ogura's K, and A corresponding to Ogura's $1/H$.

In 1939 Captains W. M. Myerscough and W. Hamilton of the L.C.C. School of Engineering and Navigation brought out their *Rapid Navigation Tables*.[17] Their table for solving altitude is similar to Ogura's table.

In the 1943 edition of his tables[18] Aquino abandoned the basis of his earlier tables in favour of the method adopted by Souillagouët and Ogura; these fine tables by Aquino have been discussed in a recent paper.[19] In 1945 E. E. Benest and E. M. Timberlake brought out tables[20] based on Ogura's formulae. The latest short-method tables of the same species are those of Lieuwen, author of two sets of tables; the first[21] published in 1949 and the second[22]—an improved version of the first—in 1953. The basis of Lieuwen's table for altitude is Ogura's method.

The type of short-method table described above is efficient for finding altitude: it is only when provision is made for finding azimuth simultaneously that the use of the tables becomes complicated; and this is probably the main reason why navigators generally have not favoured the use of short-method tables.

Thomson, whose table was recently discussed,[23] circumvented a tabular or computational method of finding azimuth by a relatively cumbersome plotting method which, interestingly enough, was also adopted by Benest and Timberlake for their method published in 1945. In 1891 Souillagouët provided a separate short-method table for finding azimuth, based on dividing the PZX-triangle by a perpendicular from the observed body on to the observer's celestial meridian. But before the end of the nineteenth century time-azimuth, altitude-azimuth and ABC tables were readily available, and navigators were accustomed to and skilled at using them. The case for a special provision for finding azimuth from short-method tables was therefore weak.

Souillagouët and Ogura, in contrast to most table-makers, provided tables specifically for finding altitude. Each gave separate tables for finding azimuth—Souillagouët a short-method table and Ogura an ABC table. But this provision was not really necessary and its only virtue seems to have been to have everything necessary for sight reduction in one book. Most other table-makers provided a simultaneous solution for altitude and azimuth, but the prodigious amount of time and energy that went into the construction of such tables, and the remarkable ingenuity of their authors, seem not to have been warranted; if the general opinion of practical navigators on their usefulness is to be the criterion.

It is well known that to derive maximum benefit from a short-method table the user must be absolutely familiar with it and thoroughly skilled in its use. Who more so than the inventor of the table has this familiarity and skill? Perhaps it was this that led many a table-maker to the belief that his was superior to every other, for it certainly appears that the history of short-method tables is a history of exaggerated claims.

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