is a subgroup of G acting naturally on G/H. If G is nilpotent, or solvable, the homogeneous space is called a nilmanifold or a solvmanifold respectively. A basic knowledge of Lie groups and Lie algebras is assumed, but there is a useful summary of known results on nilmanifolds (including the work of Malcev), solvmanifolds, ergodic flows and group representations.

Flows on some compact three-dimensional homogeneous spaces are considered in detail. Familiar properties of flows on tori are generalised to flows on compact nilmanifolds, and using group representation theory the ergodic flows are identified and shown to be minimal (all orbits dense). These results are applied to diophantine approximations, giving a generalisation of Kronecker's theorem. Flows on solvmanifolds are studied in some detail. The final section is on discrete groups with dense orbits, and a conjecture of Mahler on diophantine approximations is proved.

D. J. SIMMS

MILNOR, J., *Morse Theory*, Annals of Mathematics Studies 51 (Princeton University Press, 1963), vi + 153 pp., 24s.

This very welcome publication of a lecture course given in Princeton makes generally available for the first time a connected account in modern terms of Marston Morse's theory relating the nature of critical points of a differentiable function on a manifold to the topology of the manifold. Morse's work was done over thirty years ago, but has recently been in the limelight as a result of Bott's use of it in determining the stable homotopy groups of the unitary and orthogonal groups, and Smale's proof in dimensions greater than four of the generalised Poincaré conjecture that a compact n-manifold with the same homotopy type as an n-sphere is homeomorphic to an n-sphere.

The general theory obtains information about the topological cell structure of manifolds by studying critical points of C^∞ functions, and about the space of paths joining two points by studying Jacobi fields along the geodesics joining the two points. Some previous elementary knowledge of homotopy theory, Riemannian geometry, and the calculus of variations would therefore be helpful. The applications are mainly to Lie groups.

The elegance and conciseness of the account and the skill with which the proofs are presented make this a very rewarding book to read. The way in which the calculus of variations is used to yield results on the homotopy properties of the unitary and orthogonal groups is a striking example of cross-fertilisation in mathematics.

D. J. SIMMS