J. Fluid Mech. (2023), *vol.* 964, F1, doi:10.1017/jfm.2023.305

JFM FOCUS ON FLUIDS

Large-scale impacts of small-scale ocean topography

A. Mashayek†

Department of Earth Sciences, University of Cambridge, Cambridge CB2 3EQ, UK

(Received 7 April 2023; accepted 8 April 2023)

While large-scale seafloor features (e.g. continental slopes, mid-ocean ridges) help shape the broad patterns of ocean circulation, small-scale rough topography (e.g. seamounts, abyssal hills) can also impact the large-scale dynamics in two significant ways. First, they impact the momentum budget through flow steering, flow blocking and drag force, non-local momentum transfer via the generation of radiating internal waves and momentum dissipation by topographically induced turbulence. Second, they impact the density budget. Turbulence induced by flow–topography interactions facilitates ocean overturning circulation by upwelling dense, deep waters which form in polar oceans and sink to the abyss. Rough topography and its associated dynamics are of subgrid scale in Earth systems models (ESMs) and need parameterization for the foreseeable future. A parameterization of the intertwined impacts of flow–topography interactions on momentum and density budgets is currently non-existent, although its importance is well established. Radko (*J. Fluid Mech.*, vol. 961, 2023, A24) provides a novel analytical model for representing the impact of rough seafloor on larger-scale flows, spanning slow to fast flow-speed regimes in homogeneous and multilayer models. The proposed 'closure' is in remarkable agreement with high-resolution numerical simulations and provides a crucial step forward in parameterizing the impact of rough topography in coarse-resolution ocean models. Extending the model to the full Navier–Stokes equations, linking it to the density budget and considering more realistic ocean topography are three critical future steps towards implementing Radko's theory in operational ESMs.

Key words: ocean processes, topographic effects, quasi-geostrophic flows

1. Introduction

To leading order, and assuming a quasi-stationary ocean circulation, the work on the ocean by wind stress is balanced by the negative work at the seafloor due to pressure

† Email address for correspondence: mashayek@esc.cam.ac.uk

build-up by bottom topography (Naveira Garabato *et al.* 2013). As a specific example, the strength of the current system circumnavigating the globe in the Southern Ocean, where lack of continental zonal blocking allows for the formation of zonal jets analogous to the atmosphere, may be calculated as the net zonal momentum input via wind stress at the surface and the topographic form stress across ocean ridges (Masich, Chereskin & Mazloff 2015). Earth systems models (ESMs), typically having horizontal resolutions of $O(100\ \text{km})$, are capable of capturing the pressure gradient (and thus the associated form drag) across major topographic features, such as continental slopes and prominent ridges and plateaus with typical horizontal scales of 500–1000 km. Operational and regional ocean models, with typical horizontal grid spacing of $O(10\text{–}100\ \text{km})$, further resolve flow steering and blocking by smaller topographic features such as minor ridges, seamounts, constrictions, fracture zones, etc. For horizontal scales <10 km, the interaction of tides, eddies and currents with rough topography generates both radiating waves (such as internal tides or lee waves) and non-propagating processes (e.g. hydraulically controlled overturns, trapped waves, boundary layer turbulence; see Meredith & Garabato (2021)). This latter category is almost entirely of subgrid scale in ESMs and regional ocean models. Nevertheless, such processes can have important implications for the dynamical balance of the large-scale flow. For example, while the wave drag and frictional drag associated with small-scale topography are insignificant as compared with the local wind stress momentum input over large areas of the ocean, in certain regions, such as the Southern Ocean, equatorial ocean and over the ridge systems of the Indian and Atlantic basins, they can play a significant role in the momentum budget (Naveira Garabato *et al.* 2013). Of particular importance is the leading-order impact of wave drag induced by the interaction of strong circumpolar currents with rough topography in the Southern Ocean over the deep density stratification in the global ocean as a result of the inter-connectedness of all ocean basins through the Southern Ocean (Marshall *et al.* 2017).

In addition to the momentum balance of the ocean, small-scale topography plays an essential role in ocean circulation through the density budget. The deep branch of ocean circulation is fed by dense waters that form in the North Atlantic and around Antarctica, sinking to the abyssal ocean, carrying anthropogenic heat and carbon (among other tracers) away from the atmosphere (Talley 2013). Turbulence, induced by the interaction of tides, currents and eddies with rough topography, helps mix dense waters with overlying lighter waters, facilitating their upwelling and thus the ocean's meridional overturning circulation. Such turbulence consists of bottom-enhanced boundary turbulence (Polzin & McDougall 2022) and interior mixing generated by topographically induced internal waves that can transfer energy and momentum upward all the way to the surface mixed layer (Shakespeare & Hogg 2019; de Lavergne *et al.* 2020; Baker & Mashayek 2021). While the energy associated with mixing is three orders of magnitude smaller than the total energy associated with the ocean meridional overturning circulation, mixing is essential for the overturning circulation as, in its absence, the deep ocean would remain filled (and stuck) with old dense waters of polar origin (Wunsch & Ferrari 2004; Ferrari 2014).

Due to the practical limitations of deep ocean observations, our understanding of the topographic processes that impact the ocean's large-scale momentum and density budgets is limited but rapidly growing (Meredith & Garabato 2021). Parameterizations of such subgrid-scale processes in ESMs are in their infancy, and hence reliable turbulence closure models for flow–topography interactions are much needed. Radko (2023) provides a useful step forward in this direction.

2. Overview

Building on earlier works (Radko 2022*a*,*b*), Radko (2023) extends his sandpaper theory for flow–topography interaction, which previously only included the high-Reynolds-number (fast-flow) regime, to also account for slow flows. He then provides an analytical form that bridges the two asymptotic limits, resulting in a holistic model spanning slow-to fast-flow regimes. Implementing the model in coarse-resolution simulations remarkably reproduces detailed features of flow–topography interactions. The theory is based on three key elements: multiscale analysis, spectral representation of rough topography and quasi-geostrophic rigid-lid framework in single-layer and multilayer configurations.

Within the quasi-geostrophic framework, which allows for balanced dynamics, internal waves are excluded, the impact of ageostrophic submesoscale dynamics is mimicked via an eddy viscosity and frictional bottom Ekman flow is represented through a drag coefficient (although it is not explored as the coefficient is set to zero throughout the analysis). Thus, the model accounts for horizontal scales $>O(10\ \text{km})$. The examples discussed are also aptly aligned with parameters relevant to the Southern Ocean, where flow–rough topography interactions have the most far-reaching global influence (Naveira Garabato *et al.* 2013; Marshall *et al.* 2017).

Figure 1(*a*) shows the topography at the model's heart (hence the term 'sandpaper'). Such bathymetric representation, built upon the observationally derived spectrum of seafloor roughness (Goff & Jordan 1988), mimics the statistics of abyssal hills and has been widely used to study topographically induced internal waves and boundary turbulence (Baker & Mashayek 2022; Polzin & McDougall 2022). Such statistical representation allows for describing the topography in terms of Fourier modes. It thus facilitates the separation of the bathymetric wavenumbers into slowly varying bathymetry (low wavenumbers) and small-scale topography (high wavenumbers). The splitting of topographic height, η, into large-scale (η_L) and small-scale (η_s) regimes naturally lends to multiscale mechanics analysis. The key parameter for such analysis in Radko's work is $\varepsilon = L_C/L_{LS}$, where L_C represents the cut-off wavelength formally separating small and large scales and L_{LS} represents the scale of the background flow on which the influence of topography is investigated (i.e. mesoscales and larger).

In the fast-flow regime, where $Re \sim O(\varepsilon^{-1}) \gg 1$, Radko (2022*a*,*b*) had shown that the dynamics of the perturbations (due to flow–topography interactions) was controlled by the homogenization of small-scale vorticity through the diffusive influence of rough topography. Such diffusion, mimicking the steering of flow by topographic features (see figure 1*b*, top row), is inferred using the Fourier image of the statistically isotropic small-scale topography component. Importantly, in this regime, the topographic form drag is not consequential for the large-scale flow (more on this in the next section). The shortcoming of the fast-flow regime of Radko (2022*a*,*b*) was that once the flow speeds were reduced ($Re \lesssim 1$), the topographic forcing would blow up. While this was dealt with by an *ad hoc* constraining of the forcing in the earlier works, in Radko (2023) this issue is addressed by introducing a slow-flow limit and a bridge between the two. In the slow-flow regime corresponding to η_L, where $Re \sim O(\varepsilon) \ll 1$, Radko shows that the leading-order balance lies between vortex stretching/squeezing due to advection of large-scale flow into deeper/shallower regions and the frictional spin-down of vortical motions (see figure 1*b*, bottom row). Radko manages to bridge the two limits by considering the crossing point between the two asymptotic models, represented by the transitional velocity $V_C \sim \nu\kappa_\eta$, where ν is the eddy viscosity representing the unresolved smaller scales and κ_η is the dominant wavenumber of rough topography. Using this crossing point, Radko successfully

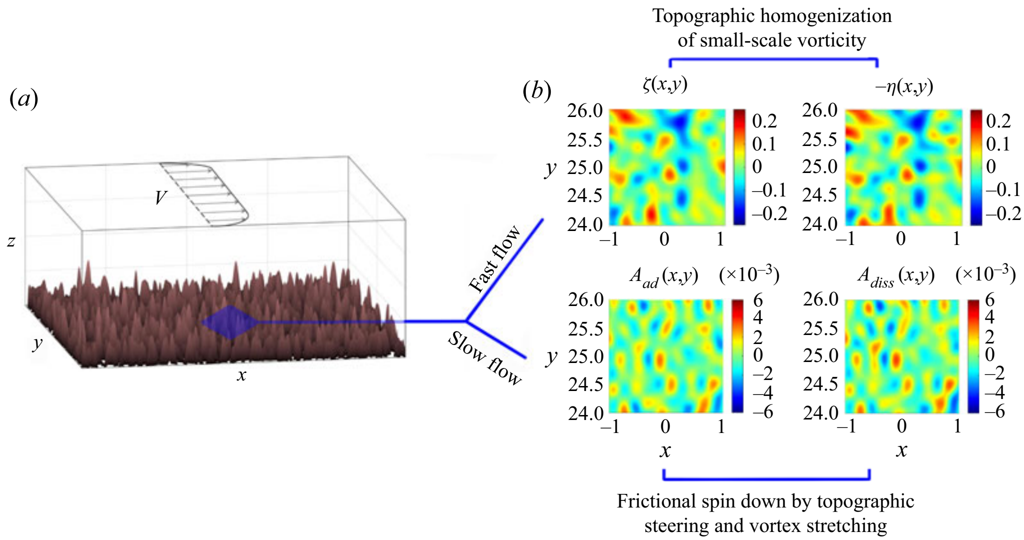

Figure 1. (*a*) A schematic diagram, representing the topography employed in the sandpaper model of Radko (2023). (*b*) An enlarged view of the solution over a small part of the domain, showing the dominant impacts of topography on the flow in the fast-flow (top row) and slow-flow (bottom row) regimes. Adapted from Radko (2023).

introduces a simple functional form that bridges the two limits, is well-defined across the slow-to-fast scales and is in remarkable agreement with high-resolution numerical simulations performed for verification.

The strength of Radko's approach is that (*a*) the two limits of the parameterization are derived directly from governing equations without any of the empirical assumptions common in other closure models, and (*b*) the bridge between the two limits works so well that the final composite model agrees with simulations in which no explicit assumption was made about the existence of separation of scales. From a physical standpoint, Radko's model predictions have important implications: in fast flows, topographic drag decreases with increasing speed (somewhat counter-intuitive but with potentially important global implications; see also Marshall *et al.* (2017)), whereas in slow flows, drag increases with increasing speed. This non-monotonic momentum forcing could have observable consequences as it suggests a preferential suppression of flows with speeds close to V_C.

3. Future

The model of Radko (2023) provides a promising prospect for representing the unresolved complex topography in ESMs. Several extensions are nevertheless required beforehand. The following considerations are essential as theories of flow–topography interaction advance:

1. The spectral representations of seafloor roughness (e.g. the widely used one of Goff & Jordan (1988), also the choice in Radko (2023)) have significant shortcomings. While the abyssal hills, consistent with a fractal process (Goff & Jordan 1988), host the rate-controlling dynamics that facilitate upwelling (Polzin & McDougall 2022), they lack larger-scale topographic features that (a) can host nonlinear non-propagating dynamics and (b) are essential for prescription of correct boundary conditions for lee wave generation in models. These issues impact the drag and

mixing representation adversely (Klymak *et al.* 2021; Baker & Mashayek 2022; Polzin & McDougall 2022).

2. The quasi-geostrophic framework employed in Radko (2023) systematically prohibits accounting for many ageostrophic submesoscale and radiating processes. Propagating internal tides and lee waves can significantly impact the vertical momentum distribution in the water column. They can also have a non-local impact on momentum and density budgets across the globe (Shakespeare & Hogg 2019; de Lavergne *et al.* 2020; Baker & Mashayek 2021). A host of other processes at the bottom boundary, such as a zoo of submesoscale instabilities and hydraulically controlled flow through narrow constrictions and passages, also have important implications for drag and mixing (Polzin & McDougall 2022). While the internal gravity waves are not accounted for by Radko (2023), the eddy viscosity mimics the impact of other bottom-dwelling processes. Extension of the model to primitive equations, as suggested by the author, will go a long way towards allowing for these processes by extending the range of scales permitted and accounting for the density stratification.
3. Finally, flow–topography interactions can also prove significant for budgets of tracers such as carbon, nutrients and heat. For example, tracers have been found to concentrate around rough topography in the Southern Ocean in patterns similar to those in figure 1 (Mashayek *et al.* 2017; Prend *et al.* 2019). Including tracer budgets in the theory seems straightforward and can prove quite insightful.

In summary, Radko's work provides a solid base upon which the representation of rough ocean topography in ESMs can evolve.

Acknowledgements. A.M. thanks L. Cimoli for constructive comments.

Funding. The author acknowledges NERC grant NE/P018319/1 and ONR grant N00014-22-1-2082.

Declaration of interest. The author reports no conflict of interest.

Author ORCIDs.
A. Mashayek https://orcid.org/0000-0002-8202-3294.

References

Baker, L.E. & Mashayek, A. 2021 Surface reflection of bottom generated oceanic lee waves. *J. Fluid Mech.* **924**, A17.

Baker, L.E. & Mashayek, A. 2022 The impact of representations of realistic topography on parameterized oceanic lee wave energy flux. *J. Geophys. Res.: Oceans* **127** (10), e2022JC018995.

Ferrari, R. 2014 What goes down must come up. *Nature* **513** (7517), 179–180.

Goff, J.A. & Jordan, T.H. 1988 Stochastic modeling of seafloor morphology: inversion of sea beam data for second-order statistics. *J. Geophys. Res.: Solid* **93** (B11), 13589–13608.

Klymak, J.M., Balwada, D., Garabato, A.N. & Abernathey, R. 2021 Parameterizing nonpropagating form drag over rough bathymetry. *J. Phys. Oceanogr.* **51** (5), 1489–1501.

de Lavergne, C., Vic, C., Madec, G., Roquet, F., Waterhouse, A.F., Whalen, C.B., Cuypers, Y., Bouruet-Aubertot, P., Ferron, B. & Hibiya, T. 2020 A parameterization of local and remote tidal mixing. *J. Adv. Model. Earth Sy.* **12** (5), e2020MS002065.

Marshall, D.P., Ambaum, M.H.P., Maddison, J.R., Munday, D.R. & Novak, L. 2017 Eddy saturation and frictional control of the Antarctic circumpolar current. *Geophys. Res. lett.* **44** (1), 286–292.

Mashayek, A., Ferrari, R., Merrifield, S., Ledwell, J.R., St Laurent, L. & Garabato, A.N. 2017 Topographic enhancement of vertical turbulent mixing in the southern ocean. *Nat. Commun.* **8** (1), 14197.

Masich, J., Chereskin, T.K. & Mazloff, M.R. 2015 Topographic form stress in the southern ocean state estimate. *J. Geophys. Res.: Oceans* **120** (12), 7919–7933.

Meredith, M. & Garabato, A.N. 2021 *Ocean Mixing: Drivers, Mechanisms and Impacts*. Elsevier.

NAVEIRA GARABATO, A.C., NURSER, A.J.G., SCOTT, R.B. & GOFF, J.A. 2013 The impact of small-scale topography on the dynamical balance of the ocean. *J. Phys. Oceanogr.* **43** (3), 647–668.

POLZIN, K.L. & MCDOUGALL, T.J. 2022 Mixing at the ocean's bottom boundary. In *Ocean Mixing* (ed. M. Meredith & A.N. Garabato), pp. 145–180. Elsevier.

PREND, C.J., GILLE, S.T., TALLEY, L.D., MITCHELL, B.G., ROSSO, I. & MAZLOFF, M.R. 2019 Physical drivers of phytoplankton bloom initiation in the southern ocean's Scotia sea. *J. Geophys. Res.: Oceans* **124** (8), 5811–5826.

RADKO, T. 2022*a* Spin-down of a baroclinic vortex by irregular small-scale topography. *J. Fluid Mech.* **953**, A7.

RADKO, T. 2022*b* Spin-down of a barotropic vortex by irregular small-scale topography. *J. Fluid Mech.* **944**, A5.

RADKO, T. 2023 A generalized theory of flow forcing by rough topography. *J. Fluid Mech.* **961**, A24.

SHAKESPEARE, C.J. & HOGG, A.MCC. 2019 On the momentum flux of internal tides. *J. Phys. Oceanogr.* **49** (4), 993–1013.

TALLEY, L.D. 2013 Closure of the global overturning circulation through the Indian, Pacific, and southern oceans: schematics and transports. *Oceanography* **26** (1), 80–97.

WUNSCH, C. & FERRARI, R. 2004 Vertical mixing, energy, and the general circulation of the oceans. *Annu. Rev. Fluid Mech.* **36**, 281–314.