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PROVABLE CONDITIONS IN COMPUTATIONAL COMPLEXITY THEORY

DARYEL SACHSE-ÅKERLIND

Computational complexity measures and indexings of algorithms are considered within a formal axiomatic system S . S is meant to mimic the formal system within which the study of computational complexity is (implicitly) carried out - so, for example, S can be a conventional axiomatization of set theory.

The main thrust of the thesis is that for many natural questions about the complexity of algorithms, what can be formally proved falls unpleasantly short of what is actually true.

We consider abstract Blum measures over indexings of the partial recursive functions. Our results fall into three categories.

First we consider complexity questions involving some arbitrary given partial recursive function f . Associated with f will be an algorithm used to define f . Before any other algorithm can be admitted as a means of calculating f , it must be proved equivalent to our defining algorithm for f . The requirement of being provably equivalent defines an equivalence relation on the set of all algorithms. We call the equivalence classes provable equivalence classes. We show that for natural complexity questions about f , what can be proved about f depends on the provable equivalence class to which the defining algorithm for f belongs.

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Having had our attention focussed on provable equivalence classes, we next investigate the relationship between provable equivalence and the complexity of algorithms. This relationship is complex and not readily summarized, but it is closely involved with provable containment between the domains over which algorithms are defined. A general conclusion we can draw is that as the difference between the complexities of algorithms increases, what can be proved about the relationship between the algorithms decreases.

Finally, we consider provable analogues of complexity classes. Two possible definitions for provable complexity classes are proposed, based on different bounding conditions:

(1) the usual almost-everywhere bounding used to define complexity classes, and

(2) almost-everywhere bounding with the additional requirement that an explicit starting-point for the bounding be given.

Various results are developed relating the two types of provable complexity classes to each other and to ordinary complexity classes. In particular, we show that for infinitely many recursive functions f the provable complexity class of f defined using bounding conditions (1) is equal to the ordinary complexity class of f and is strictly larger than the provable complexity class of f defined using bounding conditions (2).

Department of Computer Science,
University of Indiana,
Lindley Hall,
101 Bloomington,
Indiana 47405,
USA.