3D - NUMERICAL SIMULATIONS OF MAGNETIC FIELD EVOLUTION IN THE TURBULENT INTERSTELLAR GAS

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Abstract. A fully three-dimensional kinematic model is applied to simulate the evolution of the magnetic field in a small (120 pc in size) volume of the turbulent interstellar medium (ISM) in the presence of the field diffusion. The turbulent motions are approximated by a sequence of non-overlapping in time vortices, which have the form of the rotating (with a non-zero helicity) columns much longer than the parcel size. They moved vertically and at the inclination of $\pm 30^0$ to the galactic plane. The resulting magnetic field structure closely resembles a fragment of a classical twisted flux tube, well-ordered over the whole parcel of the ISM.

Key words: -galaxies – magnetic fields – turbulent gas – numerical simulations

1. 3D - Numerical Model of Magnetic Field Evolution

In our work we studied the evolution of the interstellar magnetic field permanently twisted by turbulent gas motions and smoothed by the field diffusion in a small volume of the ISM. The parcel, which was 120 pc in size, was situated in the galactic disk above the galactic plane. Its local coordinate system had the X-axis pointing away from the galactic center (parallel to the galactic radius), the Y-axis tangential to the azimuthal direction and the Z-axis directed towards the North Galactic Pole. The initial magnetic field was assumed to be uniform and parallel to the Y-axis which was a sufficient approximation of the azimuthal field at galactocentric distances much larger than the parcel size. Its strength was 1 μG.

The parcel was perturbed by a sequence of non-overlapping in time vortices appearing within its volume at random positions. All the vortices had the form of rotating columns much longer than the parcel size. They moved vertically and at the inclination of $\pm\, 30^0$ to the galactic plane, which was a first approximation of the full distribution of the inclination angles. Due to the joint action of Coriolis force and density stratification all they had a non-zero helicity of their motions. This means that the vortices going down and up rotated always like a right-handed screw. At the inclination of $\pm 30^0$ to the galactic plane the Coriolis force had one-half of its previous strength and the non-zero helicity of motions can still be expected. Within each vortex the angular speed ω and the velocity v parallel to its axis of rotation decreased with the distance r from this axis as:

$$\omega = \omega_0 \exp\left(-0.5\,(r/r_v)^2\right) \qquad (1a)$$

$$v = v_0 \exp\left(-0.5\,(r/r_v)^2\right), \qquad (1b)$$

where r_v is the adopted vortex radius ($r_v = 10$ pc), v_0 and ω_0 are the maximum values of the velocities in the vortex center ($v_0 = 10$ km/s and $\omega_0 = 10^{-4}$deg/yr). The rotational velocity was chosen in a such way that the value of the linear velocity

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within the vortex never exceeded 10 km/s (the sound speed in the ISM, Ruzmaikin et al., 1988). All velocities were truncated at $3r_v$. In order to minimize a possible flux transport through the system boundaries vortices were not allowed to appear at the distance to the parcel walls smaller than $3r_v$.

The magnetic pressure was assumed to be smaller than the gaseous one thus, a kinematic approach in which the magnetic field followed passively the gas motions was possible. The vortices twisted and shifted the lines of force, then the resulting small scale perturbations of magnetic field were smoothed by diffusion. During the system evolution no diffusion through the parcel boundaries was permitted and the magnetic lines of force were allowed to slide freely along its walls.

The evolution of the magnetic field was analyzed using the time dependent solution of the field transport equation with diffusion:

$$\partial \vec{B}/\partial t = rot\,(\vec{v} \times \vec{B}) + \eta\, rot\,(rot\,\vec{B}), \tag{2}$$

where η is the diffusion coefficient and $\vec{v}$ is the velocity field corresponding to the described above sequence of vortices. For the value of η we adopted the turbulent diffusion coefficient ($\eta_{\mathrm{turb}} = 5 \cdot 10^{24}$ cm^2/s) since the Ohmic one is inefficient in the ISM (Ruzmaikin et al., 1988). The action of each vortex was computed in discrete time steps $\Delta t = 10^5$ yrs and with the distance between the grid points of 3 pc. For each step the "pulsed-flow method" (Bayly and Childress, 1989) has been used, which means that we considered first the vertical distortion and rotation of a perfectly frozen-in field then, the effects of the field diffusion were computed. At each grid point the magnetic field modification due to the gas motion was obtained using the Cauchy solutions of the diffusionless ($\eta = 0$) form of the transport equation (2). Effects of the field diffusion were introduced using the solution of the diffusion equation with the application of Green's functions. The whole process was repeated until the assumed vortex lifetime $\tau = 9 \cdot 10^5$ or $\tau = 15 \cdot 10^5$ yrs, has been reached, then the next one started at another randomly selected place. The calculations ended when the evolutionary time (the sum of vortex lifetimes) has reached about 10^8 yrs.

At selected evolutionary stages the field geometry was visualized by integrating numerically a set of magnetic lines within the volume analyzed. Such a set of lines, for $\tau = 15 \cdot 10^5$ yrs and the evolutionary age $t = 1.2 \cdot 10^8$ yrs, projected onto the XZ-plane is shown in Fig. 1. The magnetic lines start from positions distributed along two perpendicular lines parallel to the X and Z-axes. All lines of force were found to deviate counterclockwise from their original Y-direction. It means that all magnetic lines in our volume form a fragment of a twisted flux tube with left-handed helicity. Preliminary calculations with a two-dimensional velocity field (Otmianowska-Mazur et al., 1992) yielded the ordered magnetic field configuration twisted in the Z-direction only, which was the consequence of the assumed gas flow geometry. In this work, we demonstrated that the inclusion of vortices inclined at some angle to the galactic plane allows to obtain the field structure twisted in both x and z coordinates, thus a configuration much closer to a classical, three-dimensional helical flux tube.

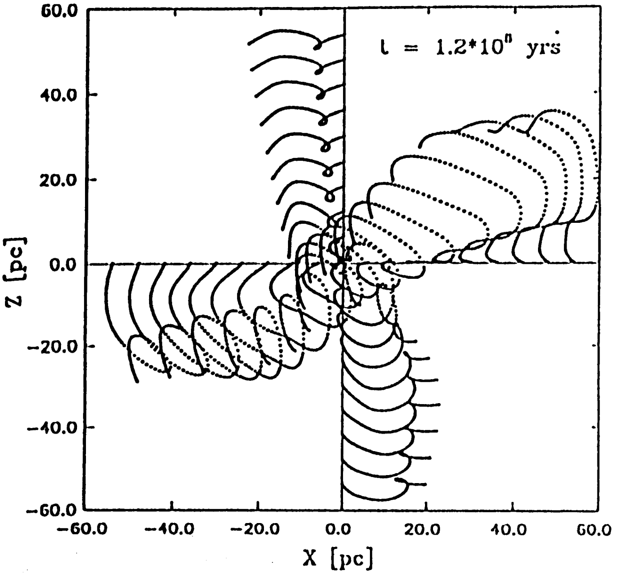

Fig. 1. The projection of magnetic field lines on the XZ-plane for the model with $\tau = 15 \cdot 10^5$ yrs and the evolutionary age $t = 1.2 \cdot 10^8$ yrs. The lines of force start from the points distributed along two perpendicular lines.

Over all the evolutionary time we made some qualitative analyses of the field structure by computing some integral parameters. These were: the magnetic field components B_x and B_z averaged over the whole parcel, their r.m.s. values and the helicity index defined as:

$$\chi = \frac{1}{N} \sum_{i=1}^{N} |(\vec{r}_i \times \vec{B}_i)| / (|\vec{r}_i| \cdot |\vec{B}_i|) , \tag{3}$$

where $\vec{r}_i$ is the radius of a given grid point relative to the parcel center and $\vec{B}_i$ is the magnetic field vector at the same point. The index χ characterizes directly the degree of helicity of the magnetic field. Its value is zero if the magnetic field is parallel to the Y-axis, while $\chi = 1$ corresponds to magnetic lines making full circles around this direction.

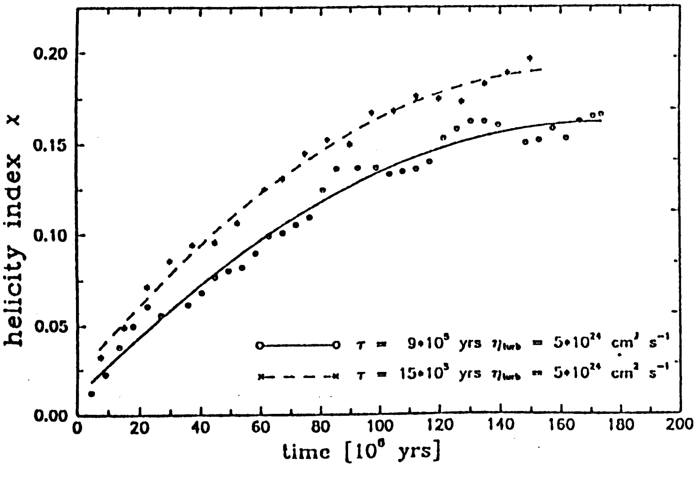

Fig. 2. The helicity index χ (see text for definition) as a function of time t, for models with $\tau = 9 \cdot 10^5$ yrs and $\tau = 15 \cdot 10^5$ yrs. The values of χ were computed at selected values of t (as indicated by symbols). The smooth curves were fitted using the second-order polynomials.

There is no significant growth of B_x and B_z components averaged over the whole parcel, thus the field evolution in our model does not lead to the generation of strong vertical or radial components, uniform over the whole ISM parcel. In contrast to that the growth of their r.m.s. values up to 30% of the initial field strength has been obtained. This means that both these components underwent a significant time evolution. The development of a globally twisted structure is clearly visible from the time changes of the helicity index χ (Fig. 2).

Its value underwent significant changes, reaching 0.2 for the simulation with $\tau = 15 \cdot 10^5$ yrs. This means that the degree of magnetic field twisting grows during all the evolutionary time. Stronger evolution was obtained for longer living vortices. The longer vortex lifetime causes the lines of force to be more twisted by a single turbulence which in turn leads to a faster magnetic field evolution.

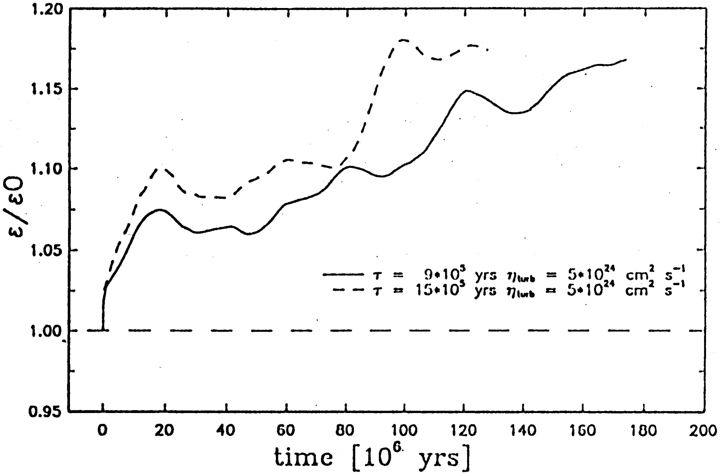

Fig. 3. The magnetic field energy density ϵ *normalized to the initial energy density* ϵ_0 *as a function of time for* $\tau = 9 \cdot 10^5$ *(the solid line) and* $\tau = 15 \cdot 10^5$ *(the dashed line). The horizontal dashed line denotes the level* $\epsilon = \epsilon_0$.

The time changes of the mean magnetic energy density within the parcel are not very strong. During the field evolution we obtained a small increase of its value up to the level of 1.2 ϵ_0 (where ϵ_0 is the energy density of the initial field) for longer living turbulences only (see Fig. 3).

We found that the observed growth was due to the twisting of magnetic lines of force and not to a systematic generation of the magnetic flux.

Fig. 4 shows the values of the B_x component of magnetic field projected onto the YZ-plane (integrated along the X-axis), for $\tau = 9 \cdot 10^5$ yrs and for the evolutionary age $t = 1.35 \cdot 10^8$ yrs. This picture corresponds to that which would be seen by an observer measuring the line-of sight component of the magnetic field e.g. by Zeeman effect. Such observations would show a reversal of this component across the turbulent region. An observational detection of such reversals would provide a good observational test of the model validity.

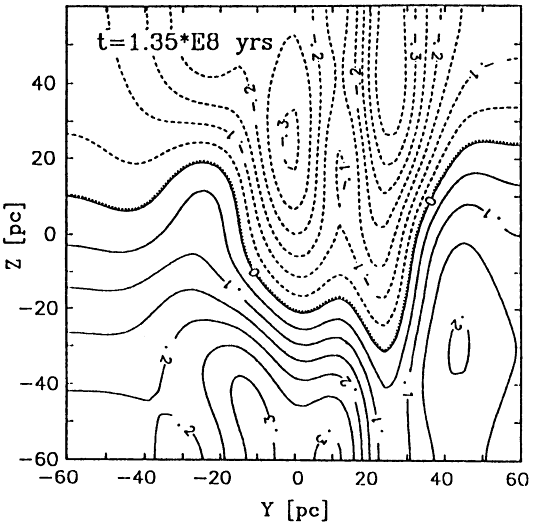

Fig. 4. The isoline map of the distribution of the B_x magnetic field component projected onto YZ-plane for the model with $\tau = 10^5$ yrs and the evolutionary age $t = 1.35 \cdot 10^8$ yrs. Dashed and solid contours delineate negative and positive values of B_x respectively. Ticked contours mark the division line between regions with $B_x < 0$ and $B_x > 0$.

2. Conclusions

Our three-dimensional simulations of magnetic field evolution in a small parcel of the interstellar gas have shown that small-scale turbulent motions with a non-zero helicity accompanied by the field diffusion give rise to a magnetic field structure resembling a fragment of a twisted magnetic flux tube. The resulting magnetic field configuration expands into the whole volume analyzed and has the spatial scale much larger than that of a single vortex.

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