Bull. Aust. Math. Soc. **108** (2023), 339–340
doi:10.1017/S0004972723000680

SHORT CHARACTER SUMS AND THEIR APPLICATIONS

FORREST FRANCIS

(Received 9 May 2023; first published online 11 July 2023)

2020 *Mathematics subject classification*: primary 11L40; secondary 11M06, 11M20, 11Y35.

Keywords and phrases: character sums, Burgess' bound, Pólya–Vinogradov inequality, *L*-function, explicit computation.

In analytic number theory, the most natural generalisations of the famous Riemann zeta function are the Dirichlet *L*-functions. Each Dirichlet *L*-function is attached to a q-periodic arithmetic function for some natural number q, known as a Dirichlet character modulo q. Dirichlet characters and *L*-functions encode with them information about the primes, especially in reference to their remainders modulo q. For this reason, number theorists are often interested in bounds on short sums of a Dirichlet character over the integers. For one, such sums appear as an intermediate step in partial summation bounds for Dirichlet *L*-functions. However, short character sum estimates may also be used directly to tackle other number theoretic problems.

In this thesis, we wish to examine the application of character sum estimates in some specific settings. There are three main estimates in which we are interested: the trivial bound, Burgess' bound and the Pólya–Vinogradov inequality. Of these, we will focus primarily on Burgess' bound. The main result of this thesis will be the computation of versions of Burgess' bound with explicit constants for a variety of parameters, improving upon the work of Treviño [5].

The interplay between Burgess' bound and the Pólya–Vinogradov inequality is vital in this explicit setting, and we will dedicate a portion of this thesis to investigating these interactions. This makes precise the work of Fromm and Goldmakher [4], who demonstrated a counter-intuitive influence the Pólya–Vinogradov inequality has on Burgess' bound.

Once we have established improvements to Burgess' bound in the explicit setting, we will 'test' these improvements by tackling several applications where Burgess' bound has been used previously. Primary among these is an explicit bound for

Thesis submitted to the University of New South Wales in November 2022; degree approved on 2 March 2023; supervisor Timothy Trudgian.

L-functions across the critical strip. We also include applications to norm-Euclidean cyclic fields and least *k*th power nonresidues.

Some of this research has been published in [1–3].

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FORREST FRANCIS,
School of Physical, Environmental and Mathematical Sciences,
University of New South Wales, Canberra ACT 2610, Australia
e-mail: forrestj7@gmail.com

https://doi.org/10.1017/S0004972723000680 Published online by Cambridge University Press