whence $\epsilon^{-nz} = \frac{dv}{dz}$;

giving,

$$\int 10^{-10^z} \epsilon^{-nz} dz = -10^{-10^z} \frac{\epsilon^{-nz}}{n} - \frac{(\log_e 10)^2}{n} \int 10^{-10^z} \epsilon^{-(n-\log_e 10)z} dz.$$

I am, Sir,

Your very obedient servant,

London, 12 June 1873.

W. M. MAKEHAM.

ERRATUM.—In the paper “On the Integral of Gompertz’s Function,” above referred to, there is a misprint. For the second formula on p. 309,

$$\log \frac{1}{g^{qx} \epsilon^{-(a+\delta)x}} \int_x^\infty g^{qx} \epsilon^{-(a+\delta)x} dx$$

should be read,

$$\log \frac{1}{g^{qx} \epsilon^{-(a+\delta)x}} \int_x^\infty g^{qx} \epsilon^{-(a+\delta)x} dx.$$

ON THE RELATION BETWEEN THE NET PREMIUM AND THE RATE OF INTEREST.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—In the current volume of the *Journal*, p. 227, I gave a demonstration in reference to the relation between the value of a policy and the rate of interest according to which it is calculated, and in the present letter I propose to examine in a similar way the relation between the net premium and the rate of interest.

$$\text{We have} \quad P_x = \frac{1}{1+a_x} - (1-v),$$

$$\begin{aligned} \therefore \frac{dP}{dv} &= \frac{-\frac{da}{dv}}{(1+a)^2} + 1 \text{ (omitting the subscript } x), \\ &= \frac{(1+a)^2 - \frac{da}{dv}}{(1+a)^2}. \end{aligned}$$

Thus, since P_x increases or decreases, when v increases, according as $\frac{dP}{dv}$ is positive or negative, we have only to examine whether

$$(1+a)^2 > \text{ or } < \frac{da}{dv}.$$

$$\text{Now, } (1+a)^2 = (1+a_x) + p_x v (1+a_x) + {}_2p_x v^2 (1+a_x) + \dots$$

and as shown in my former letter,

$$v \frac{da}{dv} = a_x + p_x v a_{x+1} + {}_2p_x v^2 a_{x+2} + \dots$$

or
$$\frac{da}{dv} = \frac{a_x}{v} + \frac{p_x v a_{x+1}}{v} + \frac{{}_2p_x v^2 a_{x+2}}{v} + \dots$$

If, now, $a_{x+1}, a_{x+2}, \dots$ are none of them greater than a_x ,

then
$$\frac{da}{dv} < \frac{a_x}{v} + \frac{p_x v a_x}{v} + \frac{{}_2p_x v^2 a_x}{v} + \dots$$

Hence, under the same condition, we shall certainly have

$$(1+a)^2 > \frac{da}{dv},$$

if
$$(1+a_x) + p_x v(1+a_x) + {}_2p_x v^2(1+a_x) + \dots > \frac{a_x}{v} + \frac{p_x v a_x}{v} + \frac{{}_2p_x v^2 a_x}{v} + \dots$$

that is, if
$$1 + a_x > \frac{a_x}{v},$$

or
$$i \cdot \frac{1}{i} + a_x > a_x + i a_x,$$

or
$$\frac{1}{i} > a_x;$$

that is, if the value of a perpetuity of 1 is greater than the value of a life annuity of 1, the rate of interest being the same in both cases.

In other words, since the value of the perpetuity is necessarily the greater, $\frac{dP}{dv}$ is positive; therefore P_x , the net premium, increases as the rate of interest decreases, provided that a_x is not less than $a_{x+1}, a_{x+2}, \dots$

I am, Sir,

Your obedient servant,

18 *Lincoln's Inn Fields*,
1 *March* 1873.

W. SUTTON.

ON THE FORMULA FOR THE MARKET VALUE OF A COMPLETE ANNUITY.

To the Editor of the Journal of the Institute of Actuaries.

SIR,—The usefulness of the expression for the value of a life annuity in terms of d and p , first proposed by the late Griffith Davies, is obvious, whether from a theoretical or practical point of view. From the theoretical, in that it shows the elements of which the value consists; and from the practical, in that it is of universal application, equally valid whether p and d be based on the same rate of interest or not, or when p is a purely arbitrary quantity.