DISCUSSION OF THE CALCULATION OF THE DENSITY LAW (Dz)

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Abstract : A matrix method involving eigenvector expansion is used to solve the "fundamental equation" of stellar statistics. This method is applied to M dwarfs and K giants.

I - INTRODUCTION

We investigate a new matrix method for solving the "fundamental equation" of stellar statistics. This equation is a linear integral equation of the first kind (Fredholm's type) and is difficult to solve because of the inherent ill-conditioning of the problem. Classical methods for the determination of D(r) have been reviewed for example by van Rhijn (1965) and Mihalas (1968). None of these "classical" methods allow a rigorous determination of the resultant stellar space density and of the uncertainties.

A new approach to this problem was made by Dolan (1974). He used a matrix method which allows a rapid and easy reduction but also requires an analytic form for the luminosity function. Its main advantage is that it provides an explicit measure of the uncertainty in D(r).

Solving this type of integral equation is a common problem in many fields of applied mathematics and physics and new methods using eigenvector expansions have been developped in the last few years : Varah (1973), Ekström and Rhoads (1974), Anderssen and Bloomfield (1974). The approximate and physically acceptable solution is obtained by a generalized Fourier expansion, the coefficients of which are weighted so as to minimize the effects of the higher order Fourier modes of the density law. This method is rigorous, allows a direct and rapid numerical computation of the solution and of its uncertainty and does not require any restriction regarding the shape of the luminosity function. It is even possible, and very easy, to introduce into the equation the variation of the luminosity function with the distance of the galactic plane.

Edith A. Müller (ed.), Highlights of Astronomy, Vol. 4, Part II, 79-82.

https://doi.org/10.1017/S1539299600003129 Published online by Cambridge University Press

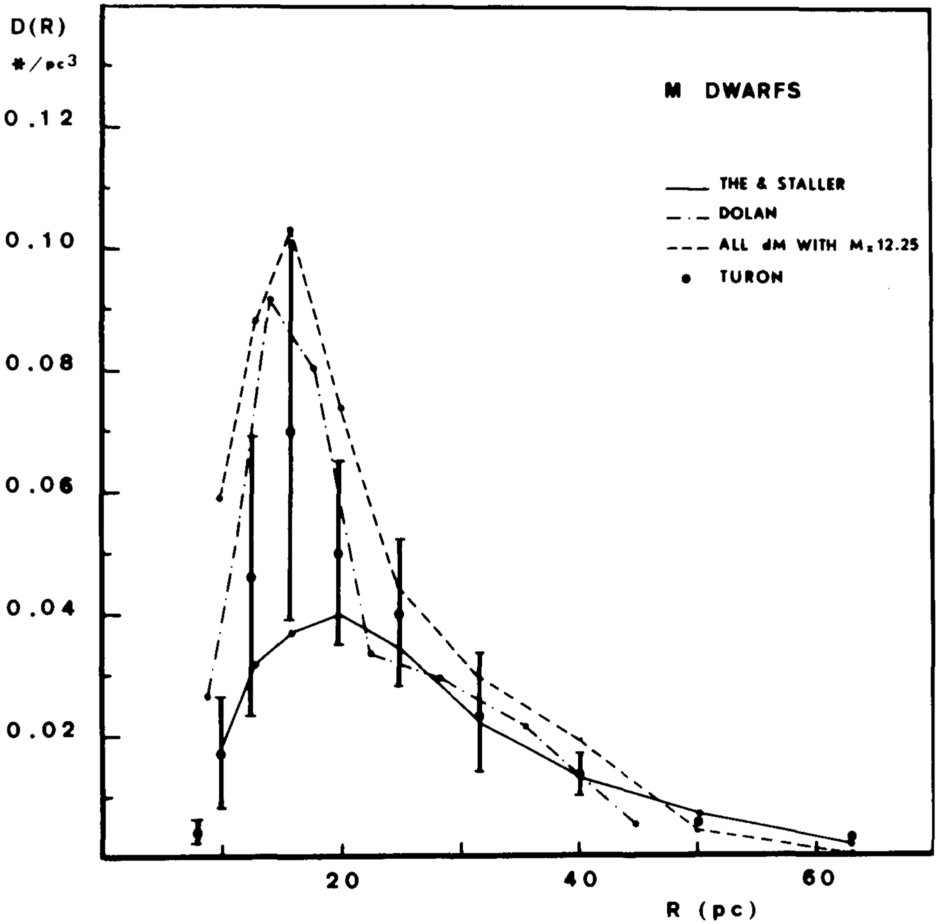

Fig. 1 - Density law for M dwarfs

II - APPLICATIONS

Until now, we have applied the method to two types of stars : the M dwarfs and the K giants.

1) The M dwarfs

The sample of Thé and Staller (1974) is used to derive a solution for the space density of M2 to M4 dwarfs near the South Galactic Pole. The same assumptions are made for the luminosity function (gaussian distribution with $\bar{M}_{pg} = 12.25$ and $\sigma = 0.5$ magnitude) and the same magnitude interval ($\delta m = 0.5$) is used as in the paper of Thé and Staller in order to facilitate the comparison of the results. The results are shown in Fig. 1 with the solution given by Thé and Staller who used the Malmquist method and that of Dolan (1975). Dolan's results, values of D(r) and the uncertainties in them, are confirmed. It should

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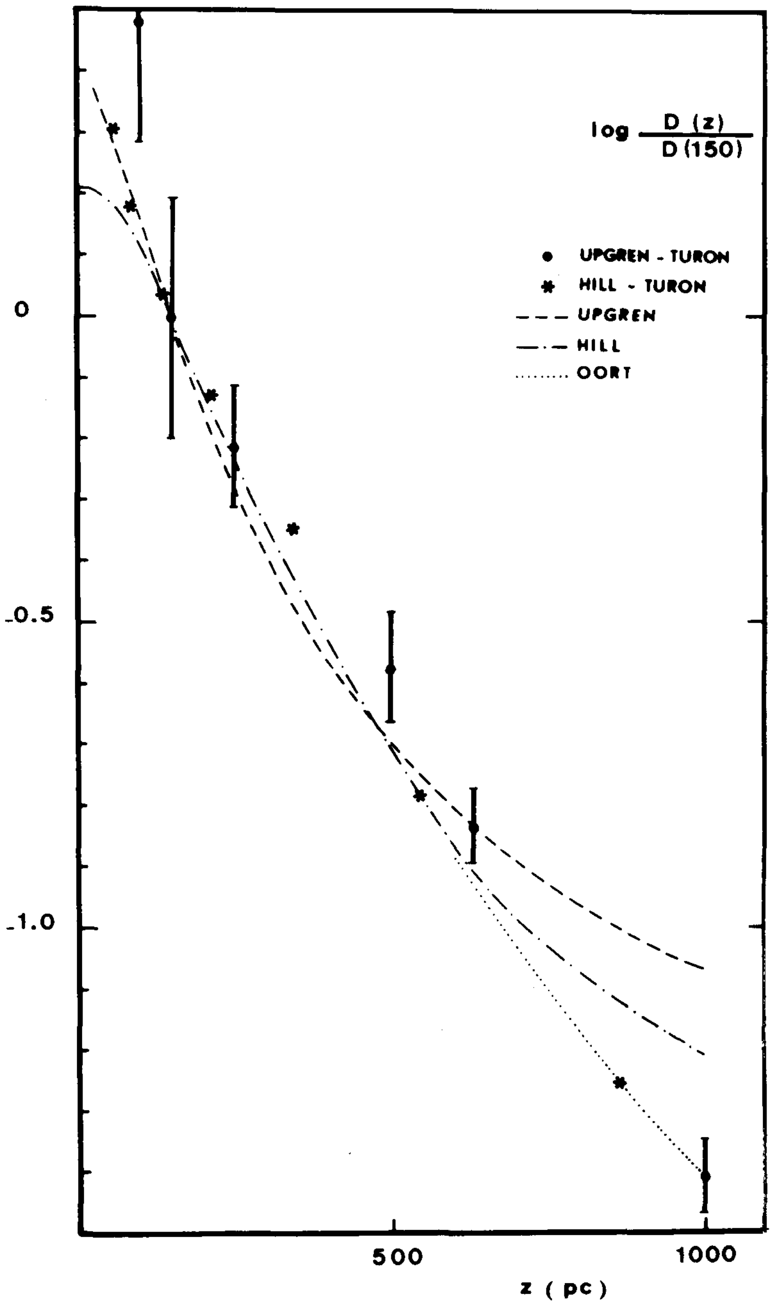

Fig. 2 - Density law for K giants

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be noted that the error bars only represent the effect of the statistical uncertainties in the data and not the possible systematic effects such as incompleteness.

2) The K giants

The method has been applied to two samples of K giants : those of Hill (1960) and of Upgren (1962), both towards the North Galactic Pole. The results are shown in Fig. 2 with the solutions given by Hill and Upgren and with the dynamical solution given by Oort (1960). It is quite remarkable that the present solutions, for both samples, are in very nice agreement with Oort's solution, justifying his assumption that the z-distribution of the total stellar density is the same as that of the K giants. The slope of our solutions for z smaller than 500 pc is a little different from that of the other authors. This slope is confirmed by Grenon (1976) from a new sample of K giants which has been very carefully chosen and which is quite complete in the zone under consideration. These results will have some implications on the run of the force law Kz.

In this application, Oort's luminosity function (1960) has been used. An important advantage of this method is that it allows the use of a numerical form for the luminosity function.

Acknowledgments : I should like to thank Prof. L. Martinet for suggesting this work and for the hospitality of the Geneva Observatory.

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To be published in Astronomy and Astrophysics