ROTATION AND EVOLUTION OF Be STARS

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Abstract. The evidence in favor of the hypothesis that Be stars owe their emission properties to material rotationally ejected from the equator is reviewed. The evolutionary state of Be Stars is then discussed with reference to evolutionary sequences of stellar models. It is concluded that (i) Be stars are not confined to the secondary contraction phase as previously proposed (ii) evolution probably proceeds with uniform rotation at least outside the initial convective core. Mechanisms for transporting angular momentum are briefly discussed.

1. Be Stars as Rapid Rotators

The original motivation for this study was to examine how much may be inferred from observational material about the changes in angular momentum distribution inside stars during their evolution from the main sequence. Our discussion here will be concerned in the main with emission-line Be stars and must be considered a preliminary report.

It has long been conjectured that the Be stars owe their emission properties to the presence of a disc distribution of hot gas rotationally ejected from the star. Since such an assumption is crucial to the subsequent discussion we will first of all review the evidence in favor of this hypothesis.

When a star rotates so rapidly that the centrifugal force at the equator balances gravity, the outermost matter loses pressure connection with the underlying layers. This may appear to be a stable configuration. Both radiation pressure and pressure arising from material ejected subsequently will, however, tend to force the matter further outwards and may lead to the formation of a gaseous disc in the equatorial plane which gives rise to the emission. In this paper the velocity at which centrifugal force at the equator balances gravity will, for brevity, also be called 'critical velocity' or even 'break-up velocity', although we don't mean to imply that the star is really breaking apart.

We wish to convince ourselves, by looking at the observed distribution of projected rotational velocities, that Be stars, as a group, are indeed rotating at their critical velocity. We take the values of $v \sin i$ from the work of Slettebak (1954) and Slettebak and Howard (1955), but arrange them according to the MK rather than the Draper spectral type. In Figure 1b the distribution of rotational velocities for B6–B9 stars of

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luminosity classes III, IV, and V, brighter than $V=5^{m}.5$ and north of $-20°$ declination is shown; emission or shell stars are treated separately from the remainder. There are altogether 87 stars, 6 of which, or 7%, show emission or shell characteristics. The observed velocities are here subdivided into intervals of 50 km/sec, with the exception

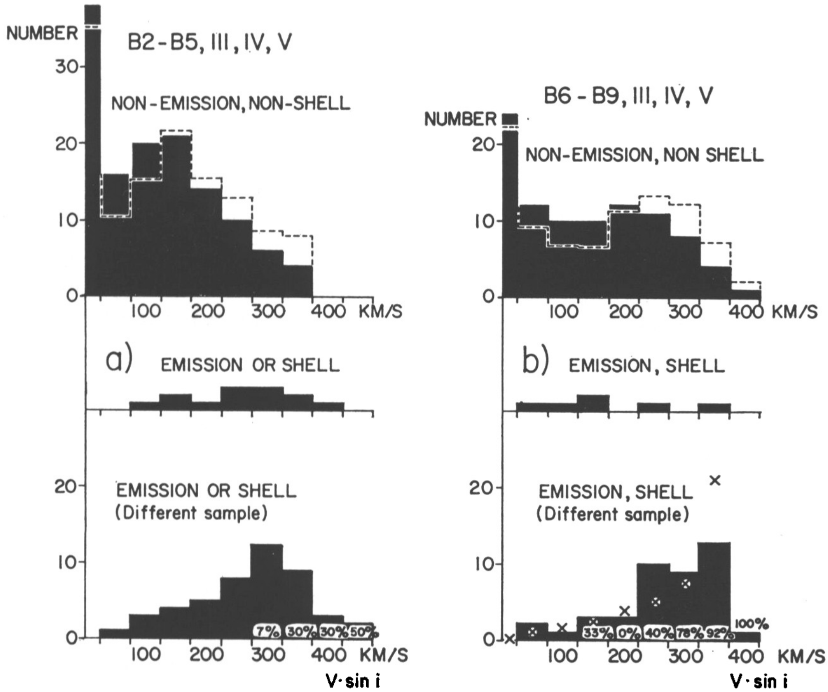

Fig. 1. Distribution of projected rotational velocities of all stars of the indicated spectral types which are brighter than $V=5^{m}.5$ and north of $-20°$ declination (top two lines). The bottom histograms correspond to larger samples of emission or shell stars. Top and bottom graphs clearly exhibit different distributions. Bottom graph b is consistent with the view that all emission stars rotate with 375 km/sec, the distribution of $v \sin i$ being an aspect effect (crosses). The percentages of shell stars are indicated and support this view. The broken lines in the upper graphs show the distribution of true rotational velocities.

of the lowest interval which is only 25 km/sec. The number of stars in this last group has been doubled in order to create equal area for each star in the histogram. In this way the overabundance of slow rotators shows up more clearly.

Since it is difficult to discuss the statistics of 6 stars, we have plotted, at the bottom of the diagram, the distribution of a larger sample of emission-line stars, also taken from the work of Slettebak (1949, 1966). The difference between the distributions for the emission and non-emission stars is striking: the emission-line stars have much higher average velocity. The crosses indicate the distribution to be expected if all emission stars in this sample were rotating with $v=375$ km/sec, and the rotation axes were randomly orientated. We see that the observed data are consistent with the view that all emission-line stars are intrinsically fast rotators, the distribution of $v \sin i$ being an aspect effect.

A further point of evidence is the percentage of shell stars among the Be's: if one looks at the extended disk equator on, the star is hidden behind the disk which causes sharp absorption lines to develop (the shell characteristics). If we interpret the distribution of $v \sin i$ as an aspect effect, we expect a higher percentage of shell stars to show up at higher $v \sin i$; this is indeed observed (see Figure 1b).

The same statements hold for the B2–B5 III, IV, V stars, the distribution of which is displayed in the same manner in Figure 1a. In this case the emission and shell stars comprise about 10% of the sample of 124 stars. Again the distribution of the emission stars, judged from the larger sample shown at the bottom of Figure 1a, can be interpreted as an aspect effect with large intrinsic rotational velocity, although this time we have to assume a mixture of velocities from 325 to 475 km/sec.

Such a mixture is not at all surprising: the breakup-speed decreases during evolution, and our sample certainly has stars of different masses also. What is a bit surprising is the fact that there are a number of fast rotators with *no* emission. Some of them could be at break-up too, because certain stars are known to show only intermittent emission characteristics. At off-times they would be found in the upper sample, but would of course remain breakup-stars. Thus percentages of emission-line stars mentioned above give lower limits only to the number of Be stars at 'break-up' velocity.

All that has been said so far has been mentioned before in the literature, mainly by Slettebak. This evidence is entirely consistent with the hypothesis that Be stars as a class are rotating at critical velocity; this will be assumed throughout the subsequent discussion. In view, however, of its importance in what follows we considered it worthwhile to gather the evidence together here. We now turn to the question of the evolutionary state of the Be stars.

2. Be Stars and Stellar Evolution

Schmidt-Kaler (1964) concluded from the observed location of the Be stars in the HR-diagram (namely $\simeq 1^{\mathrm{m}}$ above the main sequence of non-emission stars) that the Be stars are all in the secondary contraction phase following hydrogen exhaustion in the core (phase 2 to 3 in Iben's tracks as shown in Figure 2). This idea, first mentioned by Crampin and Hoyle (1960), is theoretically attractive: if there is a mechanism that keeps stars in rigid rotation at least during the main-sequence phases (case A), then the rotational velocity should increase rapidly in the secondary contraction phase. The shrinking of the star makes the moment of inertia decrease; the star has to spin faster in order to conserve its total angular momentum. This might bring the rotation to critical speeds, whereupon an extended shell might develop.

Of course this would not work if angular momentum were conserved in independent shells (case B), because then the star would be closest to the critical velocity only at its minimum radius, which is on the zero-age main sequence. We shall therefore simply assume that rigid rotation always holds (with the possible exclusion of the convective

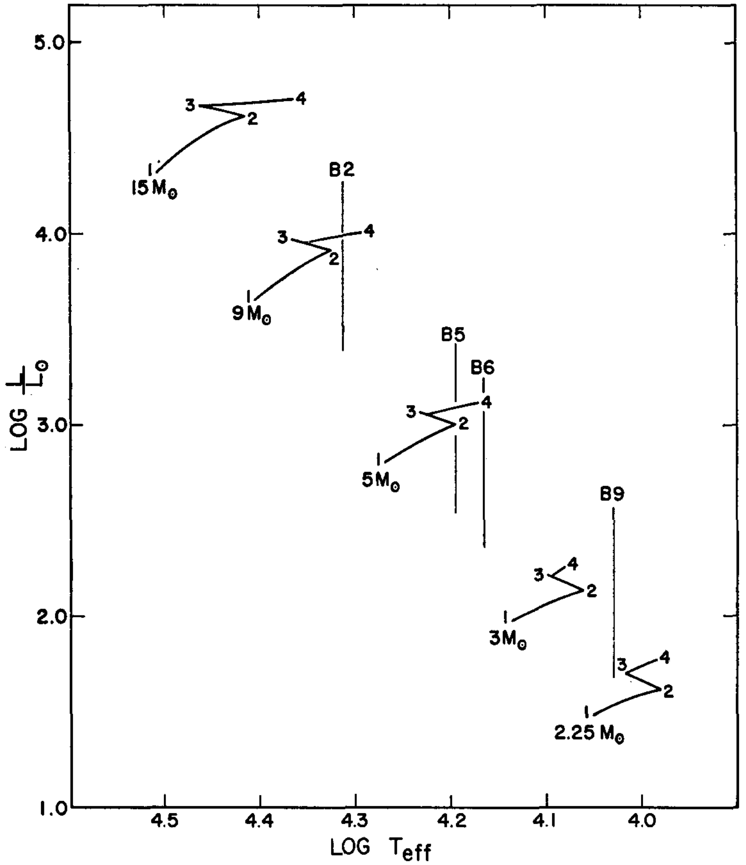

Fig. 2. The main-sequence part of Iben's evolutionary tracks. For numbered points the moment of inertia was evaluated. The spectral types were taken from Morton and Adams (1968).

core), as was tacitly done by Schmidt-Kaler. If we can show that the above idea does not work even under these more favorable assumptions, then it probably can be excluded.

First we consider some data derived from evolutionary calculations for spherical stars by Iben (1967) for various masses, by Kippenhahn *et al.* (1967) for 2 $M_{\odot}$, by Hofmeister (1967) for 5 and 9 $M_{\odot}$, and by Hofmeister *et al.* (1964) for 7 $M_{\odot}$. Listed in Table I, column 3 is the percentage of its main-sequence lifetime that the star spends in what we call the 'evolved phase'. This includes not only the secondary contraction phase but also the establishment of a thick shell-burning zone; it corresponds to stages 2–4 as indicated in Figure 2. The values are obtained from Iben's calculations. Also shown in Table I are the values of $\lambda = \Omega^2 R^3/GM$ (arbitrary units), the ratio of centrifugal force to gravity at the equator computed on the basis of uniform rotation and conservation of angular momentum. The values have been derived from computations for spherical stars and will, therefore be inaccurate near the break-up velocity. The accuracy will, however, be adequate for our purpose. We note a considerable difference between values of λ derived by different investigators; these seem much larger than would be ascribed to composition differences. Fortunately the trend is in each case the same. In Table I we also list $1/R$ and $\sqrt{\lambda}/R$ (arbitrary units), because these are the rotational speeds under assumptions B and A, respectively. The critical velocity v_{cr} of Table I was computed with the value 1.45 for the ratio of critical to non-rotating radius, after Faulkner *et al.* (1968).

A further datum required for this discussion is the distribution of true rotational velocity in each of the above samples. This may be obtained, at least approximately, from the distribution of $v \sin i$, and the results are shown by the dashed lines in Figure 1. Finally for the B2–B5 group we adopt a mean mass $\bar{M} \approx 5\,M_\odot$ while for the B6–9 stars $\bar{M} \approx 3\,M_\odot$ is more appropriate. This choice is justified from the relation between effective temperature and spectral type given by Morton and Adams (1968) as illustrated in Figure 2, and is in fair agreement with Popper's (1967) result. (Although masses as high as 9 $M_\odot$ might occur in the B2–B5 sample, most stars will have the lower value simply because of the rapid decrease in luminosity function with increasing mass.) We are now in a position to examine whether the Be stars occur during the 'evolved phase'.

For the B2–B5 group the answer is clearly negative since 10% of the stars have emission lines whereas stars of this mass spend only ~4.5% of their lifetime in the 'evolved phase'. This conclusion is further strengthened when we consider the distribution of intrinsic velocity. Among B2–B5 stars 65% have intrinsic rotational velocities $v < 225$ km/sec. Adopting a mean critical velocity $v_{cr} \approx 350$ km/sec*, we have $\lambda < 0.42$ for this subgroup. From the changes in λ listed in Table I it therefore follows that none of these stars will ever attain their critical velocity and cannot therefore become Be stars. The ratio of Be stars observed to those expected in the evolved phase is thus increased by at least a factor 3.

Among B6–9 stars, 7% are emission line objects while the stars spend ~8% of their main-sequence lifetime in the evolved phase. However, 68% of these stars have intrinsic velocities $v < 275$ km/sec, while the critical velocity v_{cr} is again in excess of 350 km/sec, therefore $\lambda < 0.62$. Using the evolutionary changes in λ for the appropriate mass range as listed in Table I, it is again clear that none of these stars can reach their critical velocity. This reduces the expected number of Be stars in the evolved phase to at most 2.6% compared to the observed proportion of 7%. Again there are too many Be stars and we therefore conclude that the Be phenomenon cannot be confined to the evolved phase and hence most certainly not to the secondary contraction stage.

We now investigate the possibility that the Be stars became rotationally unstable during main-sequence evolution, that is, between stages 1 and 2. According to Iben's calculations this would require that for the B2–B5 group approximately 10% of the stars were within 1% (for 5 $M_\odot$) of their break-up velocity when they reached the main

* This value of v_c is a lower limit on both observational and theoretical grounds, and thus maximizes λ. (cf. Faulkner *et al.*, 1968; Hardorp and Strittmatter, 1968).

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TABLE I

Evolution of the rotation parameter and of equatorial speeds

Author		Iben						Kippenhahn *et al.*, Hofmeister				Hofmeister			
Composition			$X=0.71$, $Y=0.27$					$X=0.602$, $Y=0.354$				$X=0.739$, $Y=0.240$			
$M/M_\odot$	Phase	Time 2–4	λ	λ^E	$1/R$	$\sqrt{\lambda/R}$	v_{cr} km/sec	λ	λ^E	$1/R$	$\sqrt{\lambda/R}$	λ	λ^E	$1/R$	$\sqrt{\lambda/R}$
1.25	1		1		100	100	389								
	2		1.32		77	101	342								
	3	30%	1.37		78	103	344								
	4		1.61		56	95	291								
2	1							1	1	100	100				
	2							1.13	1.11	67	87				
	3							1.35	1.31	71	98				
	4							1.47	1.42	53	88				
2.25	1		1	1	100	100	449								
	2		1.05	1.03	61	80	352								
	3	10%	1.27	1.23	67	92	366								
	4		1.32	1.28	53	84	326								
3	1		1		100	100	472								
	2		1.08		59	80	364								
	3	8	1.30		64	92	379								
	4		1.34		56	87	354								
5	1		1	1	100	100	520	1	1	100	100	1	1	100	100
	2		1.02	0.97	58	77	394	1.20	1.15	61	86				
	3	4.5	1.35	1.27	62	91	407	1.49	1.43	68	101	1.45	1.37	63	96
	4		1.27	1.18	44	75	344								

(Table I continued)

Author		Iben						Kippenhahn *et al.*, Hofmeister				Hofmeister			
Composition			$X=0.71,\ Y=0.27$					$X=0.602,\ Y=0.354$				$X=0.739,\ Y=0.240$			
$M/M_\odot$	Phase	Time 2–4	λ	λ^E	$1/R$	$\sqrt{\lambda}/R$	v_{cr} km/sec	λ	λ^E	$1/R$	$\sqrt{\lambda}/R$	λ	λ^E	$1/R$	$\sqrt{\lambda}/R$
7	1							1	1			1	1		
	2							1.35	1.22			1.41	1.32		
9	1		1	1	100	100	584	1	1	100	100	1	1	100	100
	2		1.22	1.11	51	79	418	1.41	1.30	56	89				
	3	3.2	1.74	1.56	57	100	443	1.18	1.65	62	106	1.64	1.49	59	98
	4		1.36	1.19	39	73	365								
15	1		1	1	100	100	649								
	2		1.50	1.27	47	84	442								
	3	2.4	2.08	1.71	54	106	476								
	4		1.75	1.39	33	76	373								

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sequence. For B6–B9 stars a similar argument requires 7% of the stars to have initial velocities within 4% of the critical value. The precise numbers here clearly depend on details of rotational distortion of the surface, the actual distribution of masses within the spectral group, etc. However, from Iben's calculations it would appear that a considerable peak is required at the break-up velocity in the initial rotational velocity distribution function. This in itself would not be surprising since those stars with excess angular momentum would merely have shed matter during the later stages of pre-main sequence contraction allowing them all to arrive at the ZAMS rotating at their critical speed. Subsequent evolution would, in case A, keep them at break-up velocity and cause further shedding of material to replenish the emission region. (This would not, of course, occur in case B.) More disconcerting, however, is the fact that results derived from similar calculations by Hofmeister present a rather different picture. On the basis of her computations 10% of B2–B5 stars would have to have initial rotational velocities within 10% of the break-up value, if main-sequence evolution is to produce the emission phenomenon; this clearly requires no sharply-peaked initial distribution function. We are unable to comment on the origin of these discrepancies between the various computations but feel that they merit further investigation. We can, however, conclude that evolution under case A assumptions will maintain the Be star phenomenon throughout the main sequence and evolved phases. The question of whether this involves an excess of stars rotating at their break-up velocity on the ZAMS cannot be settled until discrepancies in the evolutionary models are resolved.

The conclusions drawn here do not apply to types earlier than B2. Table I shows that the increase of λ during the main-sequence phase is substantially larger as one goes to 9 or 15 solar masses. Anand and Sackmann (1970) computed the evolution of a 10 solar mass star, including the effects of rotation explicitly. They found that it is hard to prevent such a star from reaching the critical velocity (again under the assumption of rigid rotation all the way through): 250 km/sec at phase 1 leads to breakup as early as phase 2. For the validity of our conclusions it is therefore important to assign the right values of masses to the stars in question. It would be interesting to see Anand and Sackmann's method applied to the case of 5 $M_{\odot}$, in order to have a check on our rougher method.

One possible objection has still to be discussed: If we are right that Be stars are more or less in the same stage of evolution as non-emission B stars, why is it that they appear to lie *above* the main sequence for non-emission stars by approximately one magnitude? Meisel (1967) found that 10 Be stars in visual binaries lie an average $0\overset{m}{.}7$ above the mean MK-main sequence. This, however, is just the shift one expects from the effect of gravity darkening for stars at critical velocity (see, for example, Collins 1966). This need not therefore be an evolutionary effect, or even cannot be an evolutionary effect unless the amount is considerably larger than 1^{m}.

Finally we note that evolution under case B seems rather unlikely. Certainly the rotational parameter would decrease below its critical value as soon as the star left the ZAMS and the star would never become unstable again. In view of the fact that many Be stars lose their emission characteristics for long periods it seems to us that continuous replenishment of gaseous material in the disc is required to maintain the emission phenomenon. Since this is precluded under case B we feel that detailed conservation of angular momentum can probably be excluded. This is in agreement with results of stability analyses by Goldreich and Schubert (1967) and by Fricke (1967).

3. Angular Momentum Transport

We have assumed throughout this paper that there is sufficient transport of angular momentum to maintain rigid rotation during the main-sequence phase. For later type stars this has been shown to be likely by Faber and Danziger (1970). We did not dare to subdivide our small sample to do the same analysis for the earlier types, but in principle this could be done. In this appendix we merely want to summarize what mechanisms can be made responsible for maintaining rigid rotation.

TABLE II

Mechanisms for transport of angular momentum

Mode	Time scale	Years
1. Viscous stresses	R^2/ν	10^{12}
2. Dynamical instability	$(R/h)^{3/2}/\Omega$	10^2
3. Spin down	$(R^2/(\nu\Omega))^{1/2}$	10^4
4. Magnetic stresses	$R(4\pi\varrho)^{1/2}/H$	10^4/H

Table II lists four processes with their repective time scales (here h = scale height, ν = microscopic viscosity, ϱ = density, H = magnetic field).

Clearly viscous stresses are too slow, whereas each of the remaining processes could act on a time scale short compared to the main-sequence lifetime. The dynamical instability is the one discussed by Goldreich and Schubert (1967) and by Fricke (1967); its time scale is rather uncertain. The spin-down process has been discussed for an analogous case by Howard *et al.* (1967). Processes 2 and 3 involve transport of material from the inner regions towards the surface and would therefore make for well mixed stars whose evolution is not in line with observations. However, process 2 could nevertheless be at work: Goldreich and Schubert have shown that dynamical instability is inhibited by a stable molecular weight gradient of the type established in the core region due to the shrinking of the convective core during evolution. If this type of dynamical instability is responsible for the outward transfer of angular momentum then we should expect the convective core not to participate in this transport. How would this fact alter the conclusions drawn in the preceding paragraph?

We recomputed the moment of inertia of the models, omitting those regions which were convective on the zero-age main sequence. The corresponding numbers for the rotation parameter λ^E of these exterior regions are given in Table I. The results are very similar to those calculated previously, because the core contributes too little to the moment of inertia. For this reason no observational test of whether the core is rotationally decoupled from the rest of the star seems possible at the present.

There could, however, be another reason why the convective core has to be excluded from angular momentum considerations: It has been suggested by Gough and Lynden-Bell (1968) that vorticity is expelled from convective regions. If this were the case, the convective core would not rotate at all underneath a rotating exterior. In cases like the sun with a convective exterior it would be the other way round: the exterior would expel its angular momentum to the inner regions, which might explain Dicke's oblateness measurements.

We cannot go into theoretical or experimental details of this process here but merely wish to list two more points of astronomical evidence for it. The first is the observed break in the main-sequence rotational velocity distribution at spectral type $\simeq$F2. This coincides with the development of a strong surface convective zone. The second is Kraft's (1968) observation of rotational velocities of the four Hyades giants, as compared with what one expects from the velocities on the main-sequence: He observed $\leqq 8$ km/sec, whereas velocities of 40 km/sec would be expected on the rigid rotation assumption, and 20 km/sec with detailed conservation of angular momentum. The explanation could be that angular momentum is transferred to the interior radiative regions as soon as the star moves to the right in the HR diagram and develops a deep outer convection zone (we do not claim that this is the only possible explanation). Once these stars evolve further and eventually move to the left again, onto the horizontal branch, they cease to have a convective envelope. Then the angular momentum locked up in the interior could speed up the outer parts again on a time scale for dynamical instabilities. This might explain the high rotational velocities of horizontal branch stars in M67 even though these are presumably evolved G stars (Deutsch, 1967). Clearly, the mechanism of vorticity expulsion could be of considerable astrophysical importance. Further evidence, both observational and from laboratory experiments, is required before its true significance can be assessed.

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Discussion

Collins: (1) I feel, without further comment, that one should be careful of small number statistics, particularly when one of the cases agrees with the theory to be disproven.

(2) It is not at all clear that Be stars are rotating at the critical velocity. In the event they are not, the argument used to eliminate the several cases of small statistics disappears. This is a result of the fact that 'gravity-darkening' corrections to $v \sin i$ do not apply.

(3) Care must be exercised when one talks about 'gravity-darkening' corrections in the spectral type-luminosity plane. Indeed, the correction is in the other direction (i.e., to earlier types).

(4) If the Be stars lie less than 1.5 magnitude above the main sequence you cannot have the Crampin-Hoyle (or Schmidt-Kaler) result as they require the existence of rapid rotation and the rotation effects of about 1 magnitude added to the evolutionary effects of about 1 magnitude to imply a height of 2 magnitudes above the main sequence.

Hardorp: In answer to your second comment, if Be stars were not rotating at critical velocity, our conclusions would not be at all changed: we could not apply the gravity-darkening correction to $v \sin i$, which simply means that not $\frac{2}{3}$ but about $\frac{1}{2}$ of the B6–B9 stars are now too slow to ever rotate as fast as the Be stars. In the case of the B2–B5 stars, our point would even be strengthened since we are then justified in using the spherical models right up to the point where emission sets in.

Jaschek: I think you underestimated the proportion of Be stars because the figures are higher if you count as Be stars not only those which at the time of the survey show emission lines, but also all those which showed emission at any other time.

Hardorp: The quoted proportions are certainly underestimates, which strengthens our conclusions.

Roxburgh: We do not know what limits the angular velocity of a star. It may not be equatorial shedding, but an instability that sets in earlier, such as a pulsational instability. Could your arguments be reversed to calculate the maximum rotational velocity such that enough Be stars are produced by evolution?

Hardorp: No, because the maximum rotational velocity is taken from the observations and is therefore fixed.

Van den Heuvel: Could Be stars not be stars contracting toward the main sequence? Such stars are also expected to rotate with the break-up velocity.

Hardorp: Only a very small percentage of B stars can be expected to be contracting towards the main sequence – therefore, the Be stars cannot be identified with them. However, since all B stars pass through the contraction phase, the extended rings could be remnants of that phase.

Roxburgh: In the calculation of the increase in rotational velocity it is important to remember that the moment of inertia does not have to decrease by a large factor in order to increase the rotational velocity by a similar factor, as the equatorial radius increases very rapidly for a small change in moment of inertia if the star is near maximum rotational velocity. Have you included this effect in your calculations?

Hardorp: Yes, insofar as we showed that not enough stars ever come near the critical velocity through evolutionary effects, so that your argument does not apply. In fact, near critical velocity the equatorial radius blows up by a factor 1.45 which makes for an extra λ-increase by a factor $1.45^3 = 3$ near critical velocity. This just means that near this velocity our way of reasoning is not applicable in any case, because it relies on spherical models.

Dicke: This is a question addressed both to the speaker and the other participants. The problem of rotation on the horizontal branch is very important. What is the status of the observations concerning such a rotation?

Deutsch: The so-called 'blue stragglers', like those in M67, are rotating two or three times too slowly for A stars, but 40 or 50 times too fast for G stars. At least one of these objects is probably a close-binary remnant, in the sense of Van den Heuvel, and I think others are likely to be so. For this reason and others, I now disbelieve my earlier conjecture that these stars are metamorphs of red giants that have lost their outer layers but have retained their initially high angular momentum in the interior.

Relative to the misunderstanding between Dr. Ostriker and Dr. Hardorp, one should note that if a star rotates rigidly while its density profile changes as the result of evolution, then some process must indeed occur to transfer angular momentum within the star.

When stars are red giants, they have chromospheres and, probably, stellar winds. These can transport angular momentum outwards very effectively, as the work of O. C. Wilson and R. P. Kraft has shown for solar-type dwarfs. The angular momentum therefore need not be expelled into the interior to account for the rotation seen in any metamorphs of those stars that are found near the main sequence – if there are any such stars!

Hardorp: I agree. The expulsion of angular momentum from convective layers to the interior was only proposed as an alternative mechanism.

Ruben: Evolutionary time scales depend on chemical composition. How much are your theoretical values of 5% for B2–B5 and 8% for B6–B9 influenced by the composition of the models?

Hardorp: The higher the helium abundance, the harder it is to disprove Schmidt-Kaler's hypothesis. Not only does the relative duration of the secondary contraction phase rise with helium abundance, but there is also a larger increase of the rotation parameter during evolution.

Ostriker: Have there been any masses determined for Be stars?

Cowley: Sure!