Inside the Stars, IAU Colloquium 137
ASP Conference Series, Vol. 40, 1993
Werner W. Weiss and Annie Baglin (eds.)

IS IT POSSIBLE TO DETERMINE WHETHER A STAR IS ROTATING ABOUT A UNIQUE AXIS?

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INTRODUCTION

Rotating stars are normally presumed to rotate about a unique axis. Would it be possible to determine whether or not that presumption is correct? This is a natural question to raise, particularly after the suggestion by T. Bai & P. Sturrock that the core of the sun rotates about an axis that is inclined to the axis of rotation of the envelope.

A variation with radius of the direction of the rotation axis would modify the form of rotational splitting of oscillation eigenfrequencies. But so too does a variation with depth and latitude in the magnitude of the angular velocity. One type of variation can mimic the other, and so frequency information alone cannot differentiate between them. What is different, however, is the structure of the eigenfunctions. Therefore, in principle, one might hope to untangle the two phenomena using information about both the frequencies and the amplitudes of the oscillations.

We consider a simple model of a star which is divided into two regions, each of which is rotating about a different fixed axis. We enquire whether there are any circumstances under which it might be possible to determine seismologically the separate orientations of the axes.

OSCILLATIONS OF A STAR WITH OBLIQUELY ROTATING CORE

We consider the stellar model illustrated in Figure I*a* whose core rotates with angular velocity Ω_c about a unique axis inclined at an angle β to the angular velocity Ω_e of the envelope. We ask whether β can be determined seismologically. Assuming Ω_c and Ω_e to be independent of appropriate colatitude, the rotational splitting of high-frequency acoustic modes is a seismic average $\overline{\Omega}$ given by the triangle rule (Gough & Kosovichev, 1992) of Figure I*b*.

The amplitude spectrum of intensity measurements of a quadrupole multiplet is plotted in Figure II*a*, assuming the singlets have equal energy. It depends only on the magnitude of $\overline{\Omega}$ and the angle α, which is defined in Figure I*b*. From the relative amplitudes of the rotationally split singlets, and with enough data to average fluctuations, α might be determined. However, that provides no information about β.

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https://doi.org/10.1017/S0252921100018467 Published online by Cambridge University Press

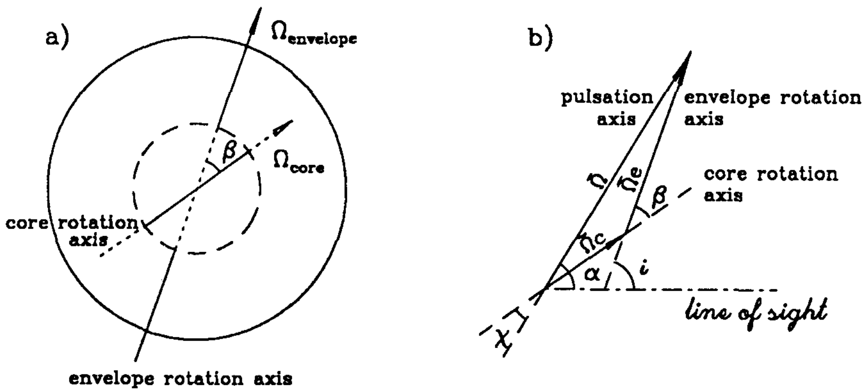

FIGURE I a) A stellar model with obliquely rotating core; b) Triangle rule for seismic averages of angular velocities

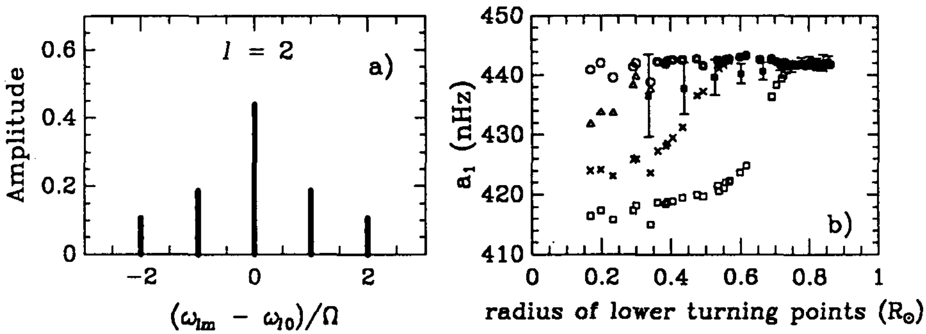

FIGURE II a) Rotational multiplets for whole-disk observations (α = 30°); b) Averaged rotational splitting coefficients a_1 [as defined by Duvall *et al.* (1986)] with $(r_c/R_\odot, \beta)$ = 0, 0 (∘); 0.3, 40° (△); 0.5, 40° (×); 0.7, 40° (□) together with BBSO data (with error bars)

To demonstrate that β cannot be determined from frequency information, even if $v \sin i$ and the magnitude of the angular velocity of the surface were measured, we consider the case of the sun, in which an obliquely rotating core has been suggested by Bai & Sturrock (1992). In Figure II*b* we compare a variety of theoretical frequency splittings of models with cores of various radii r_c uniformly rotating with angular velocity Ω_c determined by assuming zero torque with the envelope (and assuming stress is proportional to shear). The results are at variance with the data unless either r_c or β is small. However, if the core were to be assumed to be rotating more rapidly, as in Figure III, to counter the geometrical reduction of the splitting caused by β being nonzero, the splitting data (if viewed from an inertial frame) can be reproduced. Actually, there is fine structure introduced by the earth's rotation about the sun, as illustrated in Figure IV (cf Goode & Thompson, 1992), but with a resolution of 0.1 μHz, which is better that has been achieved to date, a power spectrum is indistinguishable

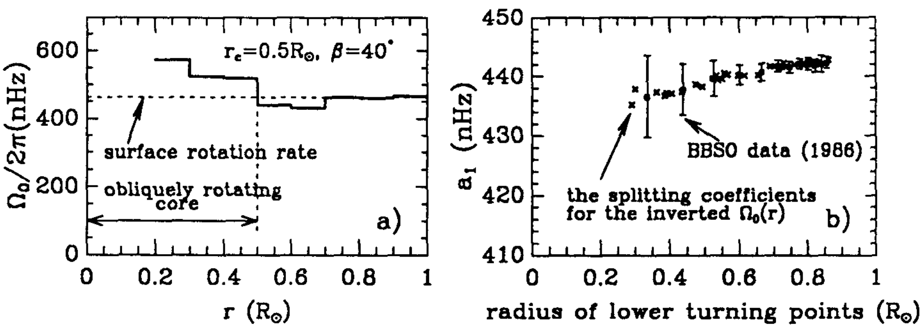

FIGURE III a) Equatorial angular velocity in the sun inferred from our inversions of BBSO data (Libbrecht & Zirin, 1986) assuming an obliquely rotating core; b) the coefficients of the rotational frequency splittings for the inverted $\Omega_0(r)$

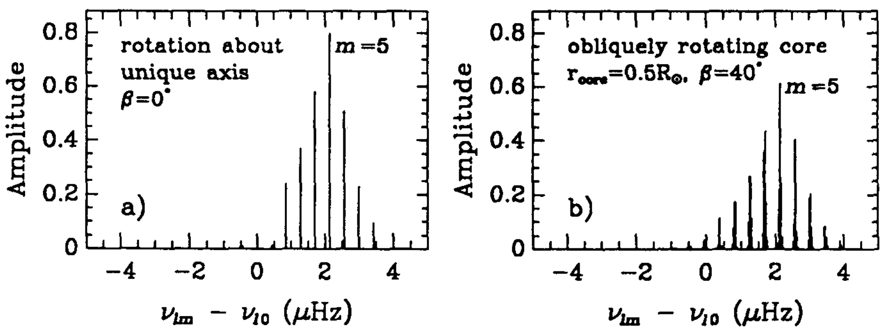

FIGURE IV Spatial filtering of a p mode multiplet of degree $l = 10$ by the Libbrecht & Zirin (1986) mapping for $l = 10, m = 5$ of artificial high-resolution Doppler observations

from that with $\beta = 0$. Therefore, we cannot yet determine whether the solar core is oblique. Since the solar system does not rotate measurably about any other star, this fine structure is not present in stellar spectra. Therefore, we cannot tell seismologically whether stellar cores rotates obliquely. However, if one is prepared to assume that a star rotates about a unique axis, from observations of both singlet frequencies and amplitudes one could in principle determine i seismologically, whatever the functional form of $\Omega(r, \theta)$.

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https://doi.org/10.1017/S0252921100018467 Published online by Cambridge University Press