## Part 9

# Population and Neutron Star Properties 

Section A. Population, Spin-Down and Magnetic Field

# Neutron Star Populations at the Millenium 

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#### Abstract

. Identified neutron star (NS) classes evidently are determined by several intertwined features: kinematics of NS at their formation (spin and translational); magnetic field strength; and binary membership. I discuss the well-known classes of isolated and accreting NS while keeping in mind recent discoveries of magnetars, anomalous X-ray pulsars, and long-period radio pulsars. I summarize the results of several likelihood analyses on radio pulsars, which yield information on the velocity distribution, luminosity function, and birth rate of high-field radio pulsars. I review the evidence for the occurrence of momentum kicks at the time of NS birth. Discerning the relationship of the classic radio pulsars to the more exotic classes probably requires careful comparison of magnetic fields, kinematics and birthrates, a program for the next millenium. Exciting discoveries of classic pulsars will also be made: sub-millisecond pulsars, massive binaries in fast orbits and truly hyper-velocity pulsars that shed light on core-collapse processes in supernovae.


## 1. Neutron Stars: Classic Pulsars \& Generation-X

As we approach the millenium, we know of $\sim 1000$ radio pulsars. One might have expected, with growing numbers of objects, that our certainty about population properties of radio pulsars would converge in some $N^{-1 / 2}$ sense. But, happily, the situation is far more complex than that: high-energy observations as well as massive radio surveys show there are multiple evolutionary branches for a collapsed star to choose from. These now include (as probably overlapping classes) soft-gamma-repeaters (SGRs), magnetars, anomalous X-ray pulsars (AXPs), hypernovae, and perhaps gamma-ray bursts, in addition to classic spin-driven pulsars and X-ray accreting objects. I refer to non-classic objects as the 'Gen-X' pulsars. Also included are the unidentified $\gamma$-ray sources found by CGRO/EGRET, which, like Geminga, may or may not be of the classic pulsar class. We do not know the evolutionary relationships of the various empirical classes. The situation is not unlike that for hominid family trees, which are redrawn nearly each time a new Australopithecus fossil is found. The link between classic radio pulsars [millisecond pulsars $=$ MSPs and moderate-field pulsars (e.g. $10^{12}$ Gauss fields) $=$ MFPs] and their binary progenitors is probably well understood now that RXTE has found fast-spin accreting sources. But the links between EGRET sources, AXPs, SGRs, and magnetars and between them and radio pulsars currently are not known.

In this paper, I address population aspects of the classic pulsars. There is much that we do know, with important implications, but there remain frontiers that promise exciting new discoveries. Also, the birth rate (BR) of classic pulsars is highly uncertain, as I argue below. This BR, if small compared to the supernova rate in the Galaxy, implies that the BR for Gen-X pulsars may be at least as large or larger. The next few years will allow us to determine these BRs more exactly and the relationship between classic and Gen-X pulsars more fully.

## 2. Classic Pulsars: Millisecond Pulsars \& Moderate Field Pulsars

A number of authors have analyzed radio surveys to derive population characteristics of neutron stars. Here I summarize work done at Cornell (with D. Chernoff and Z. Arzoumanian) that uses rigorous likelihood analyses that incorporate all sources of error in pulsar surveys.
MSPs: Our analyis takes into account nondetections in particular beam areas as well as detections of sources. We account for distance uncertainties and the role of interstellar scintillation in surveys. We find that:

- There are $\sim 3 \times 10^{4}$ MSPs in the galactic disk (uncorrected for beaming), implying a birth rate $\mathrm{BR}_{\text {MSP }} \approx 10^{-5.5} \mathrm{yr}^{-1}$.
- MSPs are a low-velocity population, tightly bound to the disk compared to MFPs. They have a galactic $z$ scale height $\sim 0.65 \mathrm{kpc}$, consistent with 3 D space velocities $\sim 84 \mathrm{~km} \mathrm{~s}^{-1}$ (rms); up to about $50 \%$ of this rms is from diffusion of MSPs off gravitational irregularities in the Galactic potential.
- The distribution in pseudoluminosity ( $L_{p} \equiv D^{2} S_{400}, S_{400}=400 \mathrm{MHz}$ period-averaged flux density) scales as $L_{p}^{-2}$.
- The period distribution, if a simple power law, $\propto P^{-2}$, with minimum period $P_{\min } \gtrsim 0.65 \mathrm{~ms}$ ( $99 \%$ confidence).
- Other P distributions also imply significant allowed probability for $\mathrm{P}<1.5$ ms : for a flat distribution from 0.5 ms to a break period, $\mathrm{P}_{b}$, and a power law above $\mathrm{P}_{b}, \mathrm{P}_{b}=5 \mathrm{~ms}$ and the power-law slope is -3.5 . For a linear rise from 0.5 ms to $\mathrm{P}_{b}$ and a power law for $\mathrm{P}>\mathrm{P}_{b}$, we obtain $\mathrm{P}_{b}=4 \mathrm{~ms}$ and slope $=-3.5$.
- The contribution to the MSP space density $<1 \%$ from the galactic halo.
- MSP searches are optimized at moderate galactic latitudes, $|b| \approx 5-30^{\circ}$.

Moderate-Field Pulsars (MFPs; B~10 ${ }^{12}$ Gauss): Our likelihood analysis (Cordes \& Chernoff 1998) exploits information in the $z$-locations combined with proper motions of pulsars, along with distance estimates and their uncertainties. Our analysis uses proper motion measurements on 49 pulsars that had spindown ages $\tau_{s}=P / 2 \dot{P}<10 \mathrm{Myr}$. We find that

- Kinematic solutions exist (for each object in our sample) that have a birthplace within a thin, Population II disk (scale height $\sim 0.13 \mathrm{kpc}$ ); no object needs to have been born well outside the disk.
- 3D velocities of pulsars have a two-component distribution: our best fit is the sum of two Maxwellians with $\sigma_{\text {slow }} \approx 175 \mathrm{~km} \mathrm{~s}^{-1}$ and $\sigma_{\text {fast }} \approx 700 \mathrm{~km}$ $\mathrm{s}^{-1}$, the slow component carrying $86 \%$ of the probability.
- $\gtrsim 20 \%$ of such pulsars will escape the Galaxy; this fraction is probably $\sim 40 \%$ after accounting for velocity selection effects in pulsars surveys.
- MFPs are about $50 \%$ younger, on average, than their spindown ages, $\tau_{s}$. The best fit we find is for torque decay with a $\mathrm{e}^{-1}$ time scale of 3 Myr combined with a braking index $n=2.5$. Alternatively, if there is no torque decay, the best fit yields $n=4.5 \pm 0.5$. Another alternative is that pulsars are born with $P \gg 1 \mathrm{~ms}$ (the "injection" hypothesis of Vivekenand \& Narayan), but the age constraints we get from comparing proper motion velocities with $|z| /$ age are not consistent with this idea.
- There is a paucity of low-velocity pulsars, $V \lesssim 100 \mathrm{~km} \mathrm{~s}^{-1}$, suggesting there should be very few detected old pulsars that emit X-rays by accretion from the ISM.


## 3. Evidence for Momentum Kicks in Supernovae

Substantial evidence exists that a momentum kick is imparted to NS at or near the time of their birth. MFP velocities as high as $1600 \mathrm{~km} \mathrm{~s}^{-1}$ (e.g. previous section; Chatterjee \& Cordes, this volume) are not achievable through mere disruption of compact binaries through symmetric supernova explosions. Population synthesis studies yield too many NS binaries and too many slow pulsars if explosions are symmetric. Geodetic precession of the Hulse-Taylor pulsar B1913+16, inferred from secular changes in pulse shape, and a similar effect possibly seen in B1534+12, requires misalignment between spin and orbital angular momenta that is most likely to have been caused by the supernova explosion producing the companions of the observed pulsars (Weisberg \& Taylor; Stairs, this volume). Classic spin-orbit precession of the pulsar-B-star binary, J0045-7319, also suggests a kick $\gtrsim 100 \mathrm{~km} \mathrm{~s}^{-1}$ (Lai et al. 1995).

## 4. Pulsars as Standard Candles

Most analyses of radio pulsars make use of the pseudoluminosity, as defined above. Instead, we assume that pulsar beams are standard candles in the sense that, for given P and $\dot{\mathrm{P}}$, the physical luminosity of a beam component (core, cone) is determined. Our model for the total beam of a pulsar has a core and multiple conal components, $B(\boldsymbol{\theta})=\operatorname{core}(\boldsymbol{\theta})+\sum_{i} \operatorname{cone}_{\mathrm{i}}(\boldsymbol{\theta})$, where the relative strengths are strong functions of P and $\dot{\mathrm{P}}$, the beam widths scale as $P^{-1 / 2}$, and contributions to the observed flux are strong functions of viewing angle.

As is well known, the pseudoluminosity covers a range of about $10^{5}$ in the classic radio population. We ascribe most of this variation to geometry and the rest to $\mathrm{P}, \dot{\mathrm{P}}$ dependence of the beam luminosities. In a Monte Carlo study (Arzoumanian, Chernoff \& Cordes $1999=\mathrm{ACC}$ ), we initially adopt a core and single conal component beam shape, with a combined luminosity of the form $L=10^{\gamma} \mathrm{P}^{\alpha} \dot{P}^{\beta}$. We split this luminosity in a P-dependent way that is consistent with the observed properties of cores and cones.

A typical beam shape is shown in Figure 1. The figure also shows histograms of pulsar fluxes for the same beam when the orientation angles (spin,
magnetic=beam axis, line-of-sight) are random and when different pulse components are turned on and off.


Figure 1. (left:) A typical core + cone beam shape. (right:) Histograms of flux density for the core + cone model for B1642-03. The different fluxes obtain for different orientation angles between the spin and magnetic axes and the direction to the observer.

Using the standard-candle assumption, we obtain a well-defined fit to the MFP population, as discussed in the next section. Does this result, combined with the plausibility argument for radio pulsars being standard candles, imply that they are standard candles? The short answer is that we have not yet proved the standard-candle case, though we believe it to be true. ${ }^{1}$ To prove the standard candle idea, we must demonstrate that we can fit the pulse shapes of specific pulsars as well as estimate their integrated flux densities. Our work so far suggests that we will be able to fit pulse shapes, but only after we include two conal components in addition to a core.

Why bother with the standard candle hypothesis? The payoff is potentially enormous: with an adequate beam model, we will be able to calculate luminosity distances. Such distances are likely to be much more accurate than those derived from dispersion measures.

These points can be illustrated through a counter example: suppose ordinary stars had beamed emission. Then the magnitude-temperature plot for a cluster like M68 (Figure 2) would be drastically altered and understanding stellar radiation and establishing their distances would have been much more difficult. This is probably the case with radio pulsars and may also be true for gamma-ray burst sources, if they are beamed as seems likely.

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Figure 2. (Left:) Magnitude-temperature diagram for the globular cluster M68 (data from A. Walker, private communication). (Right:) Simulated diagram with stellar beaming into a cone with FWHM $=10$ degrees, oriented randomly. The horizontal line is the magnitude limit for the survey. All stars below the line would be undetected.

## 5. The Full Monte: A Complete Population Analysis of MFPs

In ACC (1999), we solve for population parameters using a forward analysis: (1) creating pulsars on trajectories in the galactic potential according to some velocity distribution; (2) assigning initial P and B and adopting a spindown law; (3) using the beam model outlined above with physical luminosities; (4) allowing for a "death band" with solved-for parameters in which pulsars turn off; (5) creating pulse shapes according to random orientation angles selected by Monte Carlo; (6) testing whether each object would be detected in any of the major $400-\mathrm{MHz}$ surveys; (7) calculating a likelihood function through comparison of simulated objects with known pulsars found in $400-\mathrm{MHz}$ surveys; (8) searching a grid of population parameters that include $\alpha, \beta$ and $\gamma$ in the luminosity model described above; velocity distribution parameters; and death-band parameters; and (9) calculation of the implied birth rate (BR).

Tentative results for the luminosity law are $\alpha=-1.5, \beta=0.63$ and $\gamma=$ 28.8. For the velocity model, we obtain near agreement with the two component model for MFP velocties discussed in $\S 2$ except that the fraction in the highervelocity component is larger, $30 \%$ instead of $16 \%$. This suggests that high velocity pulsars are in fact selected against in surveys. Our results favor a broad death band in the P- $\dot{\mathrm{P}}$ plane, a factor $\sim 10$ wide in P , implying that radio luminosities fade away rather than dropping abruptly. Finally, our results suggest a rather low birth rate, perhaps as small as ( 1000 yr$)^{-1}$. We emphasize that the BR estimate is tentative. It is very sensitive to the particular beaming
model adopted. We infer that this same sensitivity translates to published BRs based on pseudoluminosities, where there is no attempt to model the beam.

## 6. Gen-X Pulsars: Magnetars \& Anomalous X-ray Pulsars

Gen-X pulsars are well discussed in contributions by Gaensler, Gotthelf, Kaspi and others. The small number of known Gen-X objects may signify a large BR if the observable lifetime is quite short, as may be the case. The BR may be comparable to or even larger than that of classic MFPs. To sort this out, the same issues of beaming, galactic volume searched, and lifetime must be addressed for these objects as apply to the classic pulsars. A useful constraint will result from a comparison of the galactic supernova rate and the classic-MFP birth rate. If the supernova rate exceeds the latter, then there is substantial room for the deficit to be accounted for by Gen-X pulsars.

It is also unknown whether Gen-X pulsars represent parallel evolutionary paths that never intersect the rotation-driven-pulsar paths. Evidence so far suggests that this is the case.

## 7. Prospects for New Discoveries

Future pulsar searches will be much more sensitive to pulsars that are fast in three meanings of the word. MSPs faster than 1.5 ms will be found if they exist. These will place important constraints on nuclear equations of state, on evolutionary spin-up processes and on gravitational instabilities. Objects with fast translational motion will be found in deep, high-latitude surveys; the important questions are What is the largest momentum kick achievable? and How does this constrain kick mechanisms that most likely involve the $1-\mathrm{sec}$ of turbulent neutrino propagation in core collapse? Fast, compact binaries (NS-NS and NS-black hole) will be found, perhaps in significant numbers, as suggested by Arzoumanian, Cordes \& Wasserman (1999).

Slow pulsars are also of interest because of the recognition by Matthew Young that a former 2.8 s pulsar was really a misidentified 8.5 s pulsar, well beyond published death lines (but not inconsistent with our death band). Finding more long period pulsars and elucidating their relationship, if any, to the similar-period AXPs and magnetars will explore the nature of radio emission and how it relates to possible thresholds in pair-production processes.

## References

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[^0]:    ${ }^{1}$ My hedging about this point in my oral presentation apparently was misconstrued by some (those of the Princeton pursuasion) to be some kind of oratorial faux pas. They got it wrong! It was a deliberate, perhaps somewhat self-deprecating statement of intellectual honesty.

