Mass and Energy Transfer in Semi-Detached Binary Systems

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1. Introduction Recently, Kitamura and Nakamura[1] have found that the anomalous gravity darkening occurs in semi-detached binary systems. The exponent of gravity darkening for the secondary components, which is defined by $\alpha_c = \frac{d\log F}{d\log g}$ where F is the radiative flux and g is the gravitational accerelation, is significantly greater than the unity as shown in Table 1. We interpret this in terms of the energy transport by the mass outflow from the secondary component filling the Roche-lobe.

Table 1.
Empirical α_c–values and physical quantities of five semi-detached binary systems.

Star	T_c(°K)	$M_c/M_\odot$	M_c/M_1	$\overline{R}_c/R_\odot$	$\log \overline{g}_c$	α_c
u Her	12000	2.9	0.39	4.4	3.61	8.47±0.51
z Vul	8500	2.3	0.43	4.5	3.48	9.73±0.12
LT Her	5200	0.50	0.20	1.6	3.73	5.86±0.08
RZ Cas	5000	0.63	0.36	1.8	3.73	2.25±0.15
VV UMa	5300	0.46	0.22	1.2	3.96	3.78±0.12

From the analysis based on this interpretation, we show that 1) the mass out-flow carries energy if $\nabla < \nabla_{ad}$ and 2) the anomalous gravity darkening requires that the flow must originate from deep interior. The mass loss rate can be estimated from the anomalous gravity darkening. Thus, the anomalous gravity darkening can be used not only to estimate the mass transfer rate but also to probe the interior structure and the evolution of the secondary star of the semi-detached binary. This model links Morton's instability[2] of the secondary with the accretion disk of the primary.

2. Basic Equations We make following assumptions that 1) the mass loss occurs steadily and 2) the flow is polytropic with index n within the secondary. In these assumptions, the energy conservation is given by

$$\nabla \cdot \{\mathbf{F} + (W + \phi + \frac{v^2}{2})\rho\mathbf{v}\} = 0 \quad (1)$$

and the kinetic energy conservation is given by

$$\nabla \cdot \{[\frac{v^2}{2} + \phi + (n+1)\frac{\Re T}{\mu}]\rho\mathbf{v}\} = 0, \quad (2)$$

where F denotes the radiative flux, W the enthalpy and ϕ the gravitational potential.

For the simplicity of analysis, we make 1-dimensional approximation. Then, from these equations, the radiative flux F is given by

$$F(\xi,\zeta) = \frac{A_b}{A(\zeta)}F_b - (n - \frac{1}{\gamma-1})(1 - \frac{T}{T_b})\frac{q(\xi)}{A(\zeta)}\frac{\Re T_b}{\mu}, \quad (3)$$

where ξ and ζ designate the coordinate to specify the flow tube and the coordinate along the tube, respectively, $A(\zeta)$ is the cross section along the ξ-tube, q is the mass flux through the tube $q(\xi) = \rho v A(\zeta)$ which is constant in each flux tube, and the subscript $_b$ means the deep interior where radiative flux F_b is constant. The polytropic index n is to be determined by the radiative transfer, but is tentatively assumed here as $n = 3$ in the radiative region.

Now, we assume an isothermal siphon flow outside the photosphere which passes trough a sonic point designated by subscript s. Then the velocity v_R of the mass flow at the surface of the secondary is determined from

$$\frac{v^2}{c_s^2}\exp\left(-\frac{v^2 - c_s^2}{c_s^2}\right) = \left(\frac{A_s}{A}\right)^2 \exp\left(-\frac{2}{c_s^2}(\phi_s - \phi)\right). \tag{4}$$

The mass flux q is given by $q(\xi) = \rho_R v_R A(\zeta_R)$, where ρ_R is the density at the photosphere which is determined by the usual atmosphere theory with effective gravity modified by the flow as

$$\rho_R = \frac{1}{\kappa \frac{\Re}{\mu} T_e} \frac{\kappa_T + 1 + n(\kappa_\rho + 1)}{n+1}(1+\chi) g \tau_{eff}, \tag{5}$$

using the opacity $\kappa = \kappa_0 \rho^{\kappa_\rho} T^{\kappa_T}$. The modification factor $(1+\chi)$ is given by

$$\chi = \frac{\frac{v^2}{2} - \frac{v_b^2}{2}}{\phi - \phi_b}. \tag{6}$$

Finally, we obtain for the effective temperature at the surface of secondary

$$\sigma T_e^4 = \sigma T_{e,pol}^4 - K \frac{g \tau_{eff}}{\kappa} \frac{T_b}{T_e} v_R \tag{7}$$

where

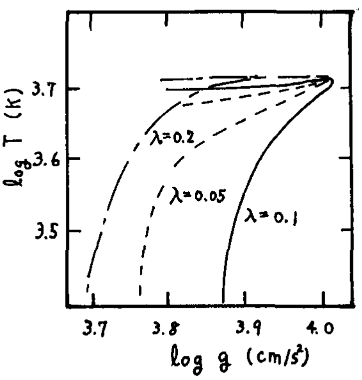
$$K = \left(\frac{1}{\nabla} - \frac{1}{\nabla_{ad}}\right)\left(1 - \frac{T_e}{T_b}\right)\left(1 + \frac{\kappa_T + n\kappa_\rho}{n+1}\right)(1+\chi). \tag{8}$$

3. Result We specify the flow outside the surface of secondary by two parameters λ and l defined by

$$z = \lambda r \sqrt{\tfrac{2}{2l+1}(1 - \cos^{2l+1}\theta)},$$

where z is the distance from the L_1 Lagrange point along the ridge of the potential, r is the radius of the secondary and θ is the angle between lines which direct the L_1 point and a surface point from the center, and we specify the internal model by the temperature T_b at the point where $\nabla = \nabla_{ad}$.

Fig. 1



a) LT Her. We show in Fig. 1 the effective temperature versus surface gravity for various λ. The other parameters are fixed as $T_b = 8 \times 10^5$K and $l = 1$. The $\lambda = 0.1$ model is the best fitted. The mass loss rate is 0.41 for $\lambda = 0.2$, 1.9 for $\lambda = 0.1$ and 1.7 for $\lambda = 0.05$ in units of $10^{-6} M_\odot/y$.

b) Z Vul. As the effective temperature of this star is relatively high, the surface density becomes low due to the large opacity. This reduces the mass flow, so that it becomes necessary to make T_b high ($T_b > 4 \times 10^7$K) in order to give rize to a large gravity darkening.

References

[1] M. Kitamura and Y. Nakamura, Ann. Tokyo Astron. Obs. 2nd Series **21**, 387(1987).
[2] D. C. Morton, Astrophy. J. **132**, 146(1960).