PRODUCTION OF PLANETARY NEBULAE

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For various reasons, we have been studying evolution in the pre-white dwarf phase at Rochester. Our attention to the relevance of this work to planetary nebulae came as a result of calculations of the evolution of a one solar-mass iron star carried out at Rochester by S. Vila. Here the neutrino processes drive the peak luminosity to log $L/L_\odot = 4\cdot26$, at an effective temperature log $T_{\mathrm{eff}} = 5\cdot53$. Although these models are brighter than 100 $L_\odot$ for 500 000 years, they are brighter than 1000 $L_\odot$ for 4000 years, and exceed 10 000 $L_\odot$ for 900 years. We are therefore near the luminosity of the planetary-nebula nuclei, but considerably hotter, for a period of the order of the planetary lifetimes. Except for the temperature discrepancy, these models are in rough agreement with the observationally determined evolutionary sequence found by O'Dell (1963) and by Harman and Seaton (1964) and Seaton (1966).

The space density of planetary nebulae and white dwarfs and their Galactic kinematics (to the extent that they are known) are consistent with this evolutionary connection, provided the planetary lifetime is approximately 10 000 years.

Let us examine two observational properties of planetaries for guidance in discovering modes of planetary production. Clearly, the expansion velocities in the range of 30–100 km/sec are well separated from those of the novae and supernovae, which we may call explosive objects. This flow is characteristic of sound speeds of the order of $10^5 \,^\circ$K rather than $10^9 \,^\circ$K. We are led to consider models in which things proceed slowly compared to pulsational periods. This does not preclude Rose's results as being applicable. On the time-scale of 10^3 years, a mass of $0\cdot5$ $M_\odot$ can be accelerated to 30 km/sec by a mean luminosity of 100 $L_\odot$ assuming total efficiency. Removing the material from a deep potential well and possible inefficiencies can easily require the available energy to be nearly a thousand-fold higher.

Secondly, there exist some indications that an object may have only one planetary nebula phase in contrast to the behavior of novae. However, if more than one phase does occur, we suggest a minimum recurrence time for episodes of 10 000 years. This may be compared with the pulses found by Rose (1966) and Weigert (1966) of 10^5 and 4000 years.

We propose to put nuclear fuel into Vila's stars and watch what happens. These differ from earlier models in including neutrino effects; e.g. that the central temperatures remain below a peak temperature which propagates outwards, increasing in

amplitude until it reaches a maximum peak temperature $\log T = 8.92$ at $M/M_* = 0.73$, this peak decreases now in peak temperature as it propagates out to a fractional mass at least as large as 0.99. The time-scale of non-thermal equilibrium is related to the photon diffusion time,

$$t \simeq kM/Rc \simeq 10^4 \text{ years}.$$

Models of pure carbon are reported as the simplest system for which the temperature inversion is expected and for which a large temperature-sensitive energy source is available. This last condition causes us to omit models with helium and hydrogen as the energy sources responsible for the flash. The abundance of elements in white dwarfs is uncertain, although one of us has suggested evidence for a mean charge of 6–8 for one group (Van Horn, 1968).

We estimate that for masses less than $1.08\ M_\odot$, neutrino losses will dominate and hence that no mass loss will occur. In contrast, for M greater than $1.6\ M_\odot$, nuclear reactions begin before the temperature peak is formed, hence the energy-generation peak is in the centre. These models should resemble main-sequence models. Calculations have been carried out for $1.1\ M_\odot$ from initial conditions shown in Table 1,

Table 1

Initial conditions for $1.1\ M_\odot$ C^{12} Star

Polytrope of index 3
$R_*\ = 0.4\ R_\odot$
$\rho_c\ = 10^3\ \text{g/cm}^3$
$T_c\ = 80 \times 10^6\,^\circ\text{K}$
$L\ = 10^3\ L_\odot$
$T_{\text{eff}} = 46000\,^\circ\text{K}$

through the white dwarf stage, assuming hydrostatic equilibrium throughout. The initial model is derived distantly from a polytrope and has no degeneracy, no neutrino emission, etc.

Evolution in the L, T_e plane (see Figure 1) is similar to the $M = 1\ M_\odot$ iron model of Vila (1965) except for the second loop driven by the C^{12} burning. We find that all the nuclear energy is essentially converted into gravitational energy through the work done in expanding the star adiabatically. No real instability (in the sense of Mestel's thermal run-away) occurs.

In Figure 2, note the development of an extended convective shell driven by the extreme peak in the energy generation. The peak rate of surface expansion was only 30 cm/sec, vanishingly small compared to escape velocities exceeding 10^8 cm/sec. Temperature distributions for some of the models listed in Table 2 are shown in Figs. 3a and 3b.

Can these models be consistent with mass loss? We had originally expected an extended model, $R = 10^{12}$ cm with convection throttled in the surface layers, but the
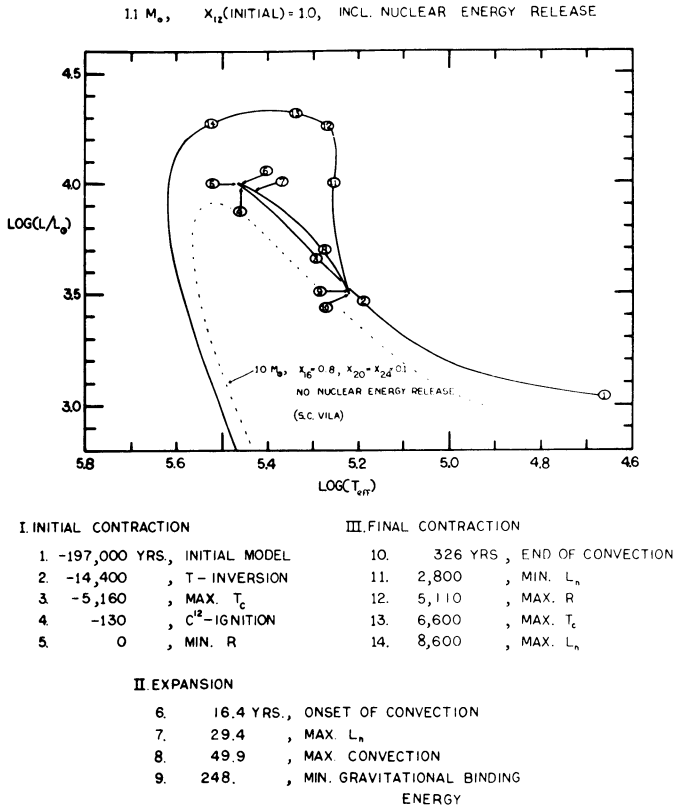
FIG. 1.   L, T_e diagram for $1.10\ M_{\odot}\ C^{12}$ star.

model allows convection only out to $M/M_* = 0.96$. Ledoux has suggested that these models should be vibrationally unstable, with pulsation amplitude increasing until damped by mass loss from the surface. We have been unable to follow this idea into the non-linear regions. Rough estimates show that the amplitude must be comparable to the stellar radius for the velocity to reach the escape velocity.

We had hoped for mass loss through a 'stellar wind' analogous to the Parker 'Solar Corona' Model. This 'corona' could be produced by sonic waves generated in the convective region. Lighthill (1952) and Proudman (1952) has shown that the rate of sonic noise generation by an isotropic turbulent medium is approximately

$$\frac{L_{\text{sonic}}}{4\pi R^2} = 30\rho v_s^3 \left(v/v_s\right)^8.$$

Using conventional mixing length theory we estimate $v = 2$ km/sec and $v_s = 10^3$ km/sec, hence $L = 10^{26 \cdot 7}$ ergs/sec. We fall short of the needed energy by a factor of 10^{10}.

Seaton (1966) has noted that radiation pressure may dominate the stellar models in the region of the planetary nebula nuclei. Setting

$$\frac{GM\rho}{r^2} = \frac{L\kappa\rho}{4\pi r^2 c}$$

we find for electron scattering

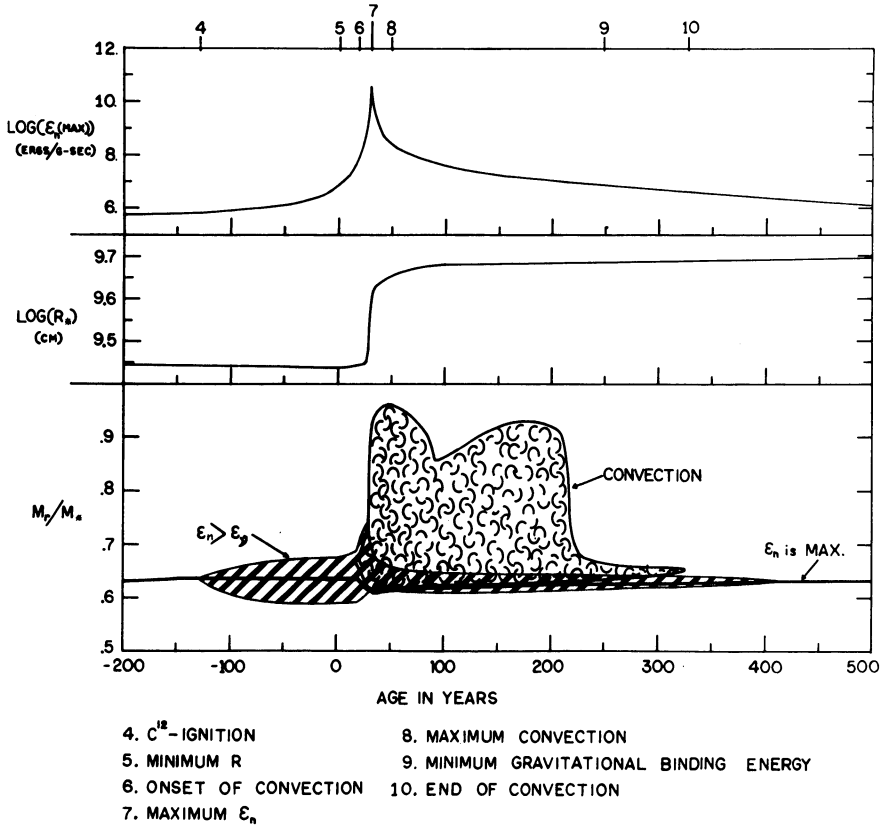$$\frac{L_c}{L_\odot} \sim \frac{10^{4\cdot8} M/M_\odot}{(1 + X)}.$$

FIG. 2.   *Development of* $\log \varepsilon_n$ *(max)*, $\log R_*$, *and extent by mass fraction of nuclear burning* ($\varepsilon_n > \varepsilon_\nu$) *and convection vs. time during the* C^{12} *flash of the* $1\cdot1$ $M_\odot$ *star.*

At the second pass (see Figure 1) peak log luminosity reached $4\cdot33$ at that phase where the temperature peak was at $M/M_* = 0\cdot84$.

Thus an increase of the opacity or the luminosity by a factor of 3 would suffice to free the material gravitationally. We have come close. For $L > L_c$, as a first approxima-

Table 2

Summary of the evolution of the $1 \cdot 1\ M_\odot\ C^{12}$ star

Description	Age (years)	$\log(T_c)$ (°K)	$\log(\rho_c)$ (g/cm³)	$\log(R)$ (cm)	$\log(T_{\text{eff}})$ (°K)	$\log(\frac{L}{L_\odot})$	$\log(\frac{L_\nu}{L_\odot})$	$\log(\frac{L_n}{L_\odot})$	$E_{\text{gravitation}}$ (ergs)
I. Initial Contraction									
1. Initial Model	−197000	7·92	3·00	10·56	4·66	3·04	—	—	−0·155 × 10⁵⁰
2. T-Inversion	−14400	8·61	5·74	9·73	5·19	3·46	3·87	−0·93	−1·16
3. Maximum T_c	−5160	8·64	6·30	9·62	5·29	3·66	4·61	1·90	−1·73
4. C^{12} Ignition	−130	8·51	6·99	9·44	5·46	3·99	5·21	4·71	−3·27
5. Minimum R	0	8·50	7·01	9·44	5·47	4·00	5·24	5·41	−3·29
II. Expansion									
6. Onset of Convection	16·4	8·49	6·99	9·45	5·46	4·00	5·24	5·49	−3·20 × 10⁵⁰
7. Maximum L_n	29·4	8·44	6·88	9·50	5·43	3·96	5·73	8·45	−2·84
8. Maximum Convective Zone	49·9	8·35	6·69	9·66	5·28	3·68	5·16	6·08	−2·16
9. Min. Gravitational Binding Energy	248·	8·33	6·67	9·68	5·22	3·51	4·90	4·88	−2·06
III. Final Contraction									
10. End of Convection	326	8·33	6·67	9·68	5·22	3·50	4·87	4·69	−2·07 × 10⁵⁰
11. Minimum L_n	2800	8·36	6·80	9·89	5·26	4·06	4·71	3·13	−2·46
12. Maximum R	5100	8·37	6·96	9·97	5·26	4·26	4·88	3·46	−2·99
13. Maximum T_c	6600	8·38	7·10	9·85	5·34	4·32	5·00	3·87	−3·50
14. Maximum L_n	8600	8·36	7·30	9·46	5·53	4·27	5·01	4·21	−4·31
15. White-Dwarf Model	856000	7·89	7·71	8·72	4·79	−0·15	0·58	—	−6·27
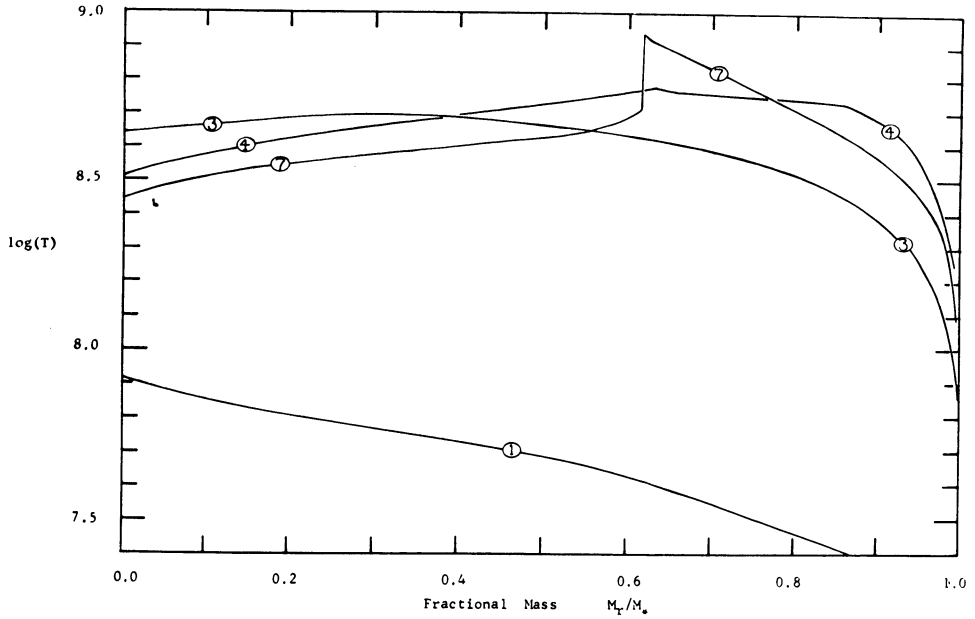
1. -197.000. yrs.. Initial Model   4.   -130. yrs.. C^{12} - Ignition
3. -5.160. yrs.. Maximum T_c 7.   29.4 yrs., Maximum L_n
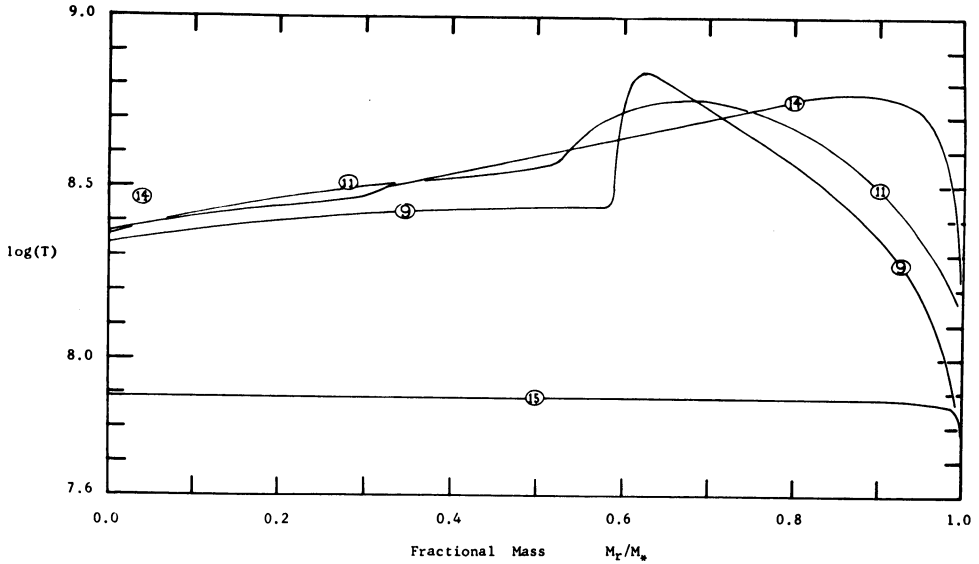
9.   248. yrs., Minimum Gravitational Binding Energy   14.   8,600. yrs., Maximum L_n
11.   2,800. yrs., Minimum L_n 15.   856,000. yrs.. White Dwarf Model

FIG. 3.   *Distribution of log T vs. fractional mass for selected models of the* $1\cdot1\ M_\odot\ C^{12}$ *star.*

tion, the behavior of the gas would resemble free flow into a vacuum until such time as its opacity decreased (recombination) or the luminosity decreased so that the gravitational force becomes dominant again.

The apparent discrepancy in effective temperature and hence radius between our models of the planetary nuclei may be removed by judicious additions of low-molecular weight material to the outer layers of the star. To estimate the effects of this inhomogeneity upon the calculations we require temperature continuity at radius $r_1 = r_2$, that

$$T = \frac{\beta}{4.25} \frac{HGM}{kr_1} \mu_1 \left(1 - \frac{r_1}{R_1}\right) = \frac{\beta}{4.25} \frac{HGM}{kr_1} \mu_2 \left(1 - \frac{r_1}{R_2}\right)$$

be the same both for the homogeneous composition (1) and for the boundary of the outer layer (composition (2)),

$$\frac{R_2}{R_1} = \frac{1}{\frac{\mu_1}{\mu_2} - \left(\frac{\mu_1}{\mu_2} - 1\right)\frac{R_1}{r_1}}.$$

In Table 3 we see that these radii can be appreciably enlarged by the substitution of a 1% helium envelope or a 10^{-5}% hydrogen envelope.

Table 3

Radii of inhomogeneous models based upon 1·1 $M_\odot$ C^{12} star, age 7000 years

$\log T$	R_1/r_1	R_2/R_1	$\log M/M_*$	$\log \rho$	κ
		Helium–Carbon			
8·28	3·6	3·9	·004	3·11	0·14
8·14	3·0	2·3	·002	2·62	·16
		Hydrogen–Carbon			
6·93	1·42	∞	-2×10^{-7}	$-1·37$	·26
6·78	1·38	12·	-5×10^{-8}	$-1·92$	0·30

In summary, we have made another pass through the region of the planetary nuclei; we have come close, but we did not find a nebula. Proposed extensions of this work are being investigated as they seem likely to provide possible planetaries.

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