SECOND ORDER PERTURBATIONS OF ELLIPTIC ELEMENTS WITH RESPECT TO THE INITIAL ONES

F.J. MARCO CASTILLO
Depto de Matemáticas, Universitat Jaume I, Castellón, SPAIN.
M.J. MARTÍNEZ USÓ
Depto de Matemática Aplicada, ETSII, Universidad Politécnica de Valencia, SPAIN.
and
J.A. LÓPEZ ORTÍ
Depto de Matemáticas, Universitat Jaume I, Castellón, SPAIN.

Abstract.

The following paper is devoted to the theoretical exposition of the obtention of second order perturbations of elliptic elements and is a follow-up of previous papers (Marco *et al.*, 1996; Marco *et al.*, 1997) where the hypothesis was made that the matrix of the partial derivatives of the orbital elements with respect to the initial ones is the identity matrix at the initial instant only. So, we must compute them through the integration of Lagrange planetary equations and their partial derivatives.

Such developments have been applied to the individual corrections of orbits together with the correction of the reference system through the minimization of a quadratic form obtained from the linearized residual. In this state two new targets emerged:

1. To be sure that the most suitable quadratic form was to be considered.
2. To provide a wider vision of the behavior of the different orbital parameters in time.

Both aims may be accomplished through the consideration of the second order partial derivatives of the elliptic orbital elements with respect to the initial ones.

1. First and second order derivatives

Let

$$\dot{\sigma_i} = f_i(\overrightarrow{\sigma}, t) \tag{1}$$

be the i^{th} Lagrange planetary equation, where σ_i $1 \le i \le 6$ denotes the orbital elements at a given time, say t and let $\sigma_{ij}(t) = \partial\sigma_i/\partial\sigma_j^0\ (t)$ be its first partial derivative. As it was shown in(Marco *et al.*, 1997) these variables verify the differential equations

$$\frac{d\sigma_{ij}}{dt} = \sum_{m=1}^{6} \left.\frac{\partial f_i}{\partial \sigma_m}\right|_t \sigma_{mj}(t) \tag{2}$$

with the initial conditions $\sigma_{ij}(t_0) = \delta_{ij}$. Also, we denote the second partial derivatives as $\sigma_{ij}^k(t) = \left.\frac{\partial^2\sigma_k}{\partial\sigma_i^0\partial\sigma_j^0}\right|_t$ and its temporal derivatives are given by

$$\frac{d\sigma_{ij}^k}{dt} = \sum_{m=1}^{6} \sigma_{mi}(t) \left.\frac{\partial^2 f_k}{\partial\sigma_m\partial\sigma_r}\right|_t \sigma_{rj}(t) + \sum_{r=1}^{6} \left.\frac{\partial f_k}{\partial\sigma_r}\right|_t \sigma_{ij}^r(t) \tag{3}$$

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with the initial conditions $\sigma_{ij}^{k}(t_0) = 0$. Finally, the equations (1), (2) and (3) must be integrated all together in order to obtain the desired values $\sigma_{ij}^{k}(t), \sigma_{ij}(t)$ and $\sigma_i(t)$.

2. Third order derivatives of the perturbation function

In the present section we devote our efforts in presenting a formulation to obtain the derivatives of the perturbation function which are necessary to evaluate the derivatives of the Lagrange equations. To compute the first and second derivatives of the perturbation function with respect to the orbital elements we follow (Simon, 1988)

$$\frac{\partial^2 \Re}{\partial \sigma_i \partial \sigma_m} = \overrightarrow{V_i}.(\partial^2 \Re).\overrightarrow{V_m} + \overrightarrow{V_{i,m}}.\overrightarrow{\partial \Re} \tag{4}$$

where $\overrightarrow{V}$ is the rectangular ecliptic vector of position, $\overrightarrow{V_i}$ the partial derivative with respect to the σ_i element and $\partial \Re$ and $\partial^2 \Re$ are, respectively, the gradient and the matrix of the derivatives in second order of the perturbation function in ecliptic rectangular coordinates. Considering the derivatives of (4) we obtain $\Re_{kjl} \equiv \frac{\partial^2 \Re_k}{\partial \sigma_j \partial \sigma_l}$

$$\Re_{kjl} = \overrightarrow{V_{kjl}}.\overrightarrow{\partial \Re} + \overrightarrow{V_{kj}}(\partial^2 \Re)\overrightarrow{V_l} + \overrightarrow{V_{kl}}(\partial^2 \Re)\overrightarrow{V_j} + \overrightarrow{V_k}(\partial^3 \Re \otimes \overrightarrow{V_l})\overrightarrow{V_j} + \overrightarrow{V_k}(\partial^2 \Re)\overrightarrow{V_{jl}} \tag{5}$$

where the following notation is employed

$$\partial_i^2 \Re = \frac{\partial \Re}{\partial x_i} \text{ i=1,2,3 } ; \partial^3 \Re = \left[\partial_1^2 \Re, \partial_2^2 \Re, \partial_3^2 \Re\right] ; \overrightarrow{V_{kj}} = \frac{\partial \overrightarrow{V_k}}{\partial \sigma_j} ; \overrightarrow{V_{kjl}} = \frac{\partial \overrightarrow{V_{kj}}}{\partial \sigma_l} \tag{6}$$

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