

# FULL AMPLITUDE MODELS OF 15 DAY CEPHEIDS

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## ABSTRACT

Numerical models of Cepheids have been computed with a range of effective temperatures and compositions. The amplitudes increase if the helium abundance increases or if the effective temperature decreases. The latter effect is contrary to observational data. The models also exhibit velocity amplitudes which are much lower than those observed.

The purpose of the calculations to be discussed here is to see how successful non-linear hydrodynamic models are in reproducing the observed trends in the amplitudes of Cepheid variables. To narrow this down, we have computed a series of models that have periods close (within 1%) to 15 days, and masses derived from evolutionary calculations. Initially we wanted to look at two trends: (1) the variation of amplitude as the effective temperature is changed, and (2) the effect upon the amplitude of changing the helium abundance in the envelope.

There exists sufficient data to be able to draw conclusions about the observed variation of amplitude with color (Cogan, 1980). If one looks at Cepheids with periods greater than 12 days, there is a systematic trend of decreasing bolometric amplitude from the blue to the red side of the instability strip. In addition there are a few low amplitude blue stars. This is based upon 26 stars all of which have well-determined intrinsic colors. The bolometric amplitudes are derived from the Johnson (B,V) color and light curves and Flower's (1977) bolometric correction scale.

Our models were computed with the DYNSTAR code incorporating Stellingwerf's method for obtaining strictly periodic non-linear solutions (cf. Cox, *et al.*, 1976). The results of these calculations

are displayed in Fig. 1, in which bolometric amplitudes are plotted against effective temperature, for both our models and the observational data. The observations of Cepheids with periods between 12.5 and 20.0 days are represented by open and filled circles. The open circles represent poorly determined data or values that may be subject to substantial systematic errors. The effective temperatures have been determined by applying Flower's scale for supergiants to the intensity mean,  $\langle B \rangle - \langle V \rangle$ .

Two principal series of models were calculated with chemical compositions (X,Y,Z) of (0.70, 0.28, 0.02) and (0.60, 0.38, 0.02). These are represented by the two lines in Fig. 1, the line to the left being for the Y = 0.38 models. A single model with (0.80, 0.18, 0.02) was computed and is indicated by a cross. In addition, a model with a 15 day first-overtone period and a composition of (0.70, 0.28, 0.02) is represented by an asterisk. However the stability analysis of this model indicates that it is unstable to switching over to the fundamental mode.

Since these models include no convective energy transport, we did not extend the calculations to lower effective temperatures where convection is important. The cut-off was determined by examining equilibrium models which included convection, calculated with a temperature-dependent local mixing length similar to that described by Deupree and Varner (1980). Non-linear models were not computed if convection carried more than about 50% of the flux at any point in the envelope of the corresponding convective equilibrium model.

The velocity amplitudes of the models can also be compared with observed radial velocities, as shown in Fig. 2. The observed velocity amplitudes have been multiplied by 1.3 to convert them to physical pulsation velocities. Due to the scarcity of good data no clear trend can be seen, except that the computed amplitudes are systematically smaller than the observed ones.

Of the two trends we were interested in initially, we can consider the second quite briefly. Our models indicate that at a given period and effective temperature, the amplitude is fairly sensitive to helium abundance: an increase of 0.1 in Y increases the bolometric amplitude by about 0.25 mag and the velocity amplitude by about 15 km/sec. The trend of the amplitudes of the models with temperature is the opposite of what is observed--the amplitudes increase as the effective temperature decreases.

Although the computed bolometric amplitudes are about the same size as those observed, the velocity amplitudes are much smaller. This can be seen clearly in Fig. 3, where the two amplitudes are plotted against each other. All the models from Fig. 1 lie in a band well below all of the higher-quality data points.

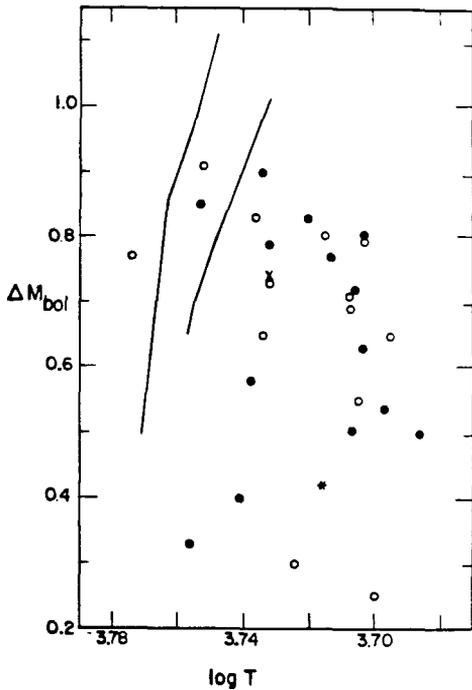


Fig. 1

Bolometric amplitudes of models and observations as a function of effective temperature. Symbols defined in the text.

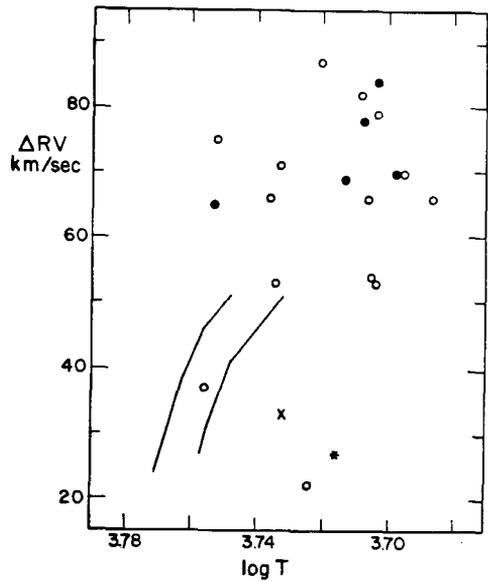


Fig. 2

Velocity amplitudes of models and observations as a function of effective temperature. Symbols as in Fig. 1.

To try to explain this result, we computed some additional models that were modifications of the  $Y = 0.28$ ,  $\log T = 3.748$  model. These are indicated by letters in Fig. 3. The A and B models have the threshold in the artificial viscosity, as defined by Stellingwerf (1975), increased from 0.02 to 0.10 and 0.05, respectively. (The A and B models in the lower right part of the figure represent the same modification to the overtone model.) Model C includes a simple time-dependent convection calculation, and lies in the diagram very close to the original model. Model D has a modified surface boundary condition and E has the mass reduced to 2/3 of the evolutionary value. However in both D and E it was necessary to reduce the artificial viscosity threshold to zero, and this is probably the main cause of the change in the amplitudes. The net result of these numerical experiments is that none of the changes is of much help in getting the model amplitudes to match the data.

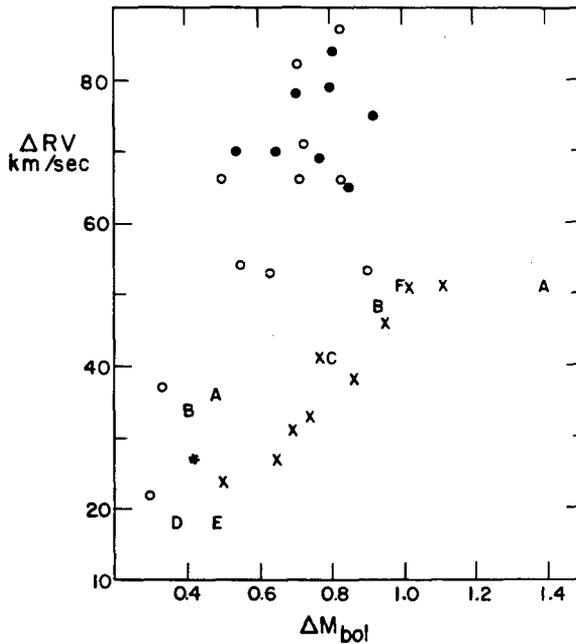


Fig. 3

Velocity amplitude as a function of bolometric amplitude. Open and filled circles: observed Cepheids. Crosses and asterisk: fundamental and overtone models from Fig. 1 and 2. Letters represent models described in the text.

In summary, there are two important aspects in which our models fail to match observed amplitudes: the amplitudes increase as the effective temperatures decrease, and for a given bolometric amplitude the velocity amplitude is much too small. The first discrepancy also can be seen in the models computed by Stobie (1968) some years ago. If the data have been interpreted correctly, this cannot be ascribed to a neglect of convection since the trend extends to the blue edge where the convective flux is very small. The problem of the velocity amplitudes may be solved by a more detailed modeling of the optically thin layers. In any case we seem to be a long way from understanding the amplitude behavior of these stars.

## REFERENCES

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## DISCUSSION

SIMON: I have a number of comments. First of all, the theoretical result is not surprising. It is just what everybody has found. According to Bob Stobie, as you decrease the temperature you increase the amplitude. This seems to be true of everything that has been calculated. Second, I wonder about the truth of that observational trend. I am inclined to wonder because if you plot amplitude versus period, which is much better defined, then above ten days you start to see an increase in amplitude. In the Ledoux and Walraven article you see that. I have done it for a big selection of Cepheids and I also see it. There is a dip at ten days and then you start seeing a slow climb.

COGAN: My point is though, that within the longer period Cepheids you do get this decrease to the red across the strip. Essentially, I took the data and did a least squares fit and you get this trend. All of the very large amplitude Cepheids are sitting near the blue edge of the strip.

FITCH: Could it be that the colors are bad?

COGAN: For all of these stars the colors are either from Pel's work or from Dean, Warren, and Cousins work. There are a couple from the older work of Parsons and Bell.

A. COX: Perhaps you happen to know the answer to this. In Christy's work he got the amplitude to fall off toward the red. Is he way over to the red and maybe beyond the red edge?

COGAN: I don't know. I do know that Bob Stobie's calculations show this same trend and in the amplitude diagram I plotted they fall in the midst of our results.

STOBIE: My models were extremely crude. I don't necessarily trust my amplitude systematics.

COGAN: In fact, this is one reason we did this. We thought that maybe after fifteen years we might have learned something and could fit the data a little better.

STOBIE: As Norm Simon pointed out, I think plotting amplitudes at

constant period may confuse things because you are getting two effects. One is that the luminosity over mass ratio changes as you go up the strip and the other is the effective temperature variation at constant luminosity. What we are trying to detect here, at least I think, is the effective temperature variation.

COGAN: Essentially.

LUB: How do you correct your radial velocities for the depths of formation of the lines in the extended atmosphere?

COGAN: We interpolate for the velocity of the photosphere. For the models that I have looked at, it appears that in the outer few layers the velocity amplitudes do not change from one zone to the next by any large amount.

DEUPREE: Your experiment with convection is very interesting. If I read your graph correctly, what you did was increase the luminosity variations without changing the radial velocity variation.

COGAN: By a very small amount. I had to move the points a little bit to get them on the graph.

DEUPREE: The point I wish to bring out is that when we calculated both RR Lyrae and Cepheid models with the 2D code, precisely the opposite happened, starting from a radiative model and letting convection rapidly build itself up. It kept the same radial velocity, but the luminosity variation went down. It went down by a noticeable amount. That would shift you to the left in the diagram. That would help but it doesn't necessarily say that the full amplitude result would be different, but the trend is more or less in the right direction as opposed to what you got.

KING: I wonder if the point that Lub made might not be worth pursuing. We do look only at the photosphere and if we look at a given shell outside of this it looks about the same, but in the real star you may be looking at quite different geometric depths over the cycle. Maybe this is why the observed velocity amplitudes are considerably larger than the ones we get in our models.

COGAN: It is certainly true that we don't treat the outer layers of the star in the kind of detail that we would like to, either in the number of zones or perhaps even the way radiative transfer is handled. That is certainly a place that one could look in trying to resolve this problem.

J. COX: Were the velocity curves relatively smooth?

COGAN: Yes. They were fairly smooth.