ONTARIO MATHEMATICAL MEETINGS

The third Ontario Mathematical Meeting was held Saturday, January 28, 1967 at Sidney Smith Building, University of Toronto. Research papers were presented in the morning, followed by lunch at Sir Daniel Wilson Residence, followed by a one-hour address by Professor Richard Kadison (University of Pennsylvania) entitled: A Survey of von Neumann Algebras. Abstracts of research papers which were presented are as follows:

67.1 J. L. B. Cooper (University of Toronto)
Functional Equations for Linear Transformations

Let A and B be spaces of functions on intervals of the real line R . A group $W(\alpha)$ of transformations of A is called appropriate if it is a representation of the additive group of R and $[W(\alpha)f](x) = Q(x, \alpha)f(V(\alpha)x)$ where Q is a multiplier, $V(\alpha)$ a group of transformations of the interval. T obeys an appropriate functional equation if there is an appropriate group on B , $W_1(\alpha)$, so that $TW(\alpha) = W_1(\alpha)T$. It is shown that all cases can be reduced by decomposition, changes of variable, and multiplication by constant functions to four types: -

I. $[f(x+\alpha)](u) = [Tf(x)](u+\alpha)$,

II. $[Tf(x+\alpha)](u) = e^{\alpha q(u)}\,[Tf(x)](u)$,

III. $[Te^{\alpha p(x)}f(x)](u) = [Tf(x)](u+\alpha)$,

IV. $[Te^{\alpha p(x)}f(x)](u) = e^{\alpha q(u)}\,[Tf(x)](u)$.

Solutions of I, under certain assumptions, are convolution transforms; of II and III, exponential transforms; of IV, multiplications and changes of variable.

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67.2 M.A. McKiernan (University of Waterloo)

Vanishing n-th Ordered Differences and Hamel Bases

We study the general solutions of the equation

$$(*) \qquad \Delta_\nu^{n+1} f(x) \equiv 0$$

and also $\Delta_\nu^n f(z) = g(\nu)$ when no regularity assumptions are imposed. It is known that if f satisfies $(*)$, is continuous at one point, or bounded on a set of positive measure, then $f(x)$ is continuous at all points and is therefore a polynomial of degree n.

Let A_p denote a symmetric multi-additive function on R^p to R, the reals: $A_p(x_1, x_2, \ldots, x_p)$ satisfies Cauchy's functional equation in each argument, and $A_p(x_1, \ldots x_p) = A_p(x_{i_1}, \ldots, x_{i_p})$ for permutations $(i_1, \ldots, i_p)$ of $(1, \ldots, p)$.

By A^p is meant the function on R to R obtained by diagonalizing A_p, that is

$$A^p(x) = A_p(x, x, \ldots, x) .$$

The function A^p will play a role analogous to the power function $x \to xp$.

THEOREM. *If $\Delta_\nu^{n+1} f(x) \equiv 0$ for all x and ν real, then there exists symmetric multi-additive functions A_p for $p = 1, 2, \ldots, n$ such that*

$$f(x) = A^0 + A^1(x) + \ldots + A^n(x) \quad \text{for all } x ,$$

where A^0 is constant. Conversely, any such $f(x)$ satisfies $\Delta_\nu^{n+1} f(x) \equiv 0$.

COROLLARY 1. *A necessary and sufficient condition for the equation*

$$(**) \qquad \Delta_\nu^n f(x) = g(\nu)$$

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to have a solution is that $g(\nu) = n!A^n(\nu)$ where A^n is the diagonalization of a multi-additive function A_n of n arguments. The general solution of (**) is then $f(x) = A^n(x) + h(x)$ where $h(x)$ is the general solution of $\Delta_\nu^n h(x) \equiv 0$.

COROLLARY 2. A necessary and sufficient condition for $f(x)$ to be of the form $A^n(x)$ is that $\Delta_\nu^n f(x) = n!f(\nu)$.

67.3 P.B. Chapman (University of Toronto)
Method of Steepest Descent

The extension is given of the method of Steepest Descent to cope with cases in which the integrand of

$$I = \int_{-\infty}^{\infty} e^{-tz^2/2} g(z)\, dz$$

contains singularities whose location depends on independent variables. The technique permits the construction of a uniform Poincaré asymptotic expansion of I , $t \to \infty$, for singularities in $g(z)$ arbitrarily close to $\mathrm{Im}(z) = 0$.

67.4 R.G. Lintz (McMaster University)
Non-determinist Mathematics

By non-determinist mathematics it is understood a general program of research in which the concepts of function, continuity and derivative are generalized and given by non-determinist definition. In the paper to be published soon it is studied the concept of g-derivative. The main results are:

THEOREM 1. Every paracompact space is a Gauss space.

THEOREM 2. The chain-rule for usual derivative is true also for g-derivatives.

THEOREM 3. Given a real function $\varphi(x)$ of real

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variable having derivative $\varphi'(x)$ everywhere, there is a g-function f, generating φ and such that $D_x f = |\varphi'(x)|$, where $D_x f$ is the g-derivative of f at x.

67.5 H. Langer (University of Toronto)
<u>Spectral Functions for a Class of J-Operators</u>

Let H denote a J-space, that is Hilbert-space, where besides the usual positive definite scalar product $(x, y)(x, y \varepsilon H)$ an indefinite scalar product $[x, y]$ is defined by the equation $[x, y] = (Jx, y)(x, y \varepsilon H)$; here $J = P_1 - P_2$ with $P_1^2 = P_1^* = P_1$, $P_1 + P_2 = I$. A bounded linear operator A in H is called J-selfadjoint if $[ax, y] = [x, Ay]$ $(x, y \varepsilon H)$. M.G. Krein and the author proved, that in case $\dim P_2 H < \infty$, the J-selfadjoint operator A has a certain resolution of the identity E_λ, defined for all real λ with exception of a finite number of "critical points". It is shown here, that a corresponding result remains true for a J-selfadjoint operator A in an arbitrary J-space, provided the operator P_1AP_2 is compact and the non-decreasingly-ordered eigenvalues μ_j of $P_2AP_2A^*P_1$, each repeated according to its multiplicity, satisfy the condition $\sum_j \frac{\mu_j}{2j+1} < \infty$. In this case the set of "critical points" is $(\sigma_1 \cap \sigma_2) \cup \sigma$, where σ_j denotes the set of all points of the spectrum of the operator P_jAP_j in H_j, which are not isolated eigenvalues of finite multiplicity $(j=1, 2)$ and σ is an at most countable set with no accumulation points outside $\sigma_1 \cap \sigma_2$.

67.6 B.J. Müller (McMaster University)
<u>Classification of Algebras by Dominant Dimension</u>

The following homological dimensions for finite dimensional algebras R over a field have been introduced by Nakayama and Tachikawa: right-dominant-dimension

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$R \geqq n$ if there exists an exact sequence $0 \to R \to X_1 \to \ldots \to X_n$ of projective-injective right-modules X_i ; left-dominant-dimension, Nakayama-dimension analogously using left- respectively bi-modules instead of right-modules. The following theorems hold:

1. r-dom.dim. R = 1-dom.dim. R = N-dim. R for any algebra R .
2. dom.dim. $R \geqq 1$ if and only if R is QF-3 .
3. dom.dim. R. $\geqq 2$ if and only if R is the endomorphism ring of a fully faithful module ${}_A X$.
4. If dom.dim. $R \geqq 1$ then dom.dim $R \geqq n+1$ if and only if $\mathrm{Ext}_R^k(S, R) = 0$ for all $1 \leqq k \leqq n$ and all simple modules B non-isomorphic to ideals.
5. If dom.dim. $R \geqq 2$ then dom.dim. $R \geqq n+2$ if and only if $\mathrm{Ext}_A^k(X, X) = 0$ for all $1 \leqq k \leqq n$, where ${}_A X$ is fully faithful and R = Endo $({}_A X)$.