RESEARCH ARTICLE

Ratemaking in a changing environment

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Abstract

In pricing insurance contracts based on the individual policyholder's aggregate losses for non-life insurers, the literature has mainly focused on using detailed information from policies and closed claims. However, the information on open claims can reflect shifts in the distribution of the expected claim payments better than closed claims. Such shifts may be needed to be reflected in the ratemaking process earlier rather than later, especially when insurers are experiencing environmental changes. In practice, actuaries use ad hoc techniques to adjust data to current levels to determine premiums. This paper presents an intuitive ratemaking model, employing a marked Poisson process framework, which ensures that the multivariate risk analysis is done more routinely using all reported claims and makes an adjustment for Incurred But Not Reported claims. Utilizing data from the Wisconsin Local Government Property Insurance Fund, we find that by determining rates based on current data, the proposed ratemaking model leads to better alignment of premiums and provides insurers with a more financially sound portfolio.

1. Introduction

In non-life insurance, premiums are set to cover the expected future cost and also allow for the earmarked underwriting profit through a process known as ratemaking. The pure premium method and the loss ratio are two popular traditional methods used for ratemaking. These methods ensure that the total premium will cover the total costs while allowing for the targeted underwriting profit. To obtain rates, historical claims are developed to ultimate using macro-level reserving techniques and adjusted to a level applicable to the future effective period using trending techniques. See Werner and Modlin (2010) and Brown and Lennox (2015) for details on these methods.

Accurate risk pricing is expected to provide stronger incentives for more caution, resulting in lower claim frequencies and reductions in insurance loss costs (Cummins, 2002). As a result, in addition to determining the overall average rate change from traditional methods, actuaries employ univariate risk classification techniques to identify a base rate and risk classification variables. The base rate is assigned relativity of one, and different risk classification variables are assigned other relativities. Thus, the rates are individual rates, reflecting the individual policyholder's characteristics.

The main shortcoming of the univariate risk classification techniques is that they do not consider all rating variables simultaneously. In contrast, multivariate risk classification techniques allow for the consideration of all rating variables simultaneously and automatically adjust for exposure correlations between rating variables. Hence, by employing multivariate risk classification techniques, actuaries develop rates to align premiums with expected costs better. The generalized linear models (GLMs) and machine learning algorithms are two popular multivariate risk classification techniques (Taylor and McGuire, 2016; Wüthrich and Merz, 2023).

An observation from the ratemaking literature is that the multivariate analysis data are often based on closed claims, where the ultimate amount paid for all claims is known. This observation is not surprising because practitioners want to use the most recent data but are limited by the requirement that the data

be complete, thus leading to a natural friction between using only closed claims from older policy years and using the information on all reported claims, including information on open claims. For example, last year's data are most representative of current trends but are certainly incomplete (censored).

With closed claims, all uncertainties in information on open claims are eliminated. However, using only closed claims may not reflect shifts in the expected claim payments' distribution. Practicing actuaries are well aware of these biases and have developed ad hoc on-leveling methods to adjust the data to the current level. For example, premium trending techniques are used to adjust the historical premium to the level expected during the future time period, and the actuary needs to determine how to measure and incorporate the shifts that have occurred. In contrast, the information on open claims can reflect shifts in the expected claim payments' distribution automatically. Hence, to better align premiums with expected future costs, actuaries need to extract all the information they can from open claims, especially in these pandemic times.

This paper presents an intuitive framework for ratemaking that ensures that the multivariate risk analysis is done more routinely using the information on claims that have been closed and payments on open claims. To model the complete development of claims for ratemaking purposes, we employ the marked Poisson process (MPP) framework with three hierarchical building blocks. The first building block concerns the number of claims per policy by modeling the number of claims reported and accounts for the future cost relating to Incurred But Not Reported (IBNR) claims through a unique feature of the MPP framework by analyzing the reporting delay distribution of claims. The second and third building blocks model the conditional number of payment transactions for a claim and the conditional payment sizes for each transaction, respectively. For Reported But Not Settled (RBNS) claims, the number of transactions is censored at the ratemaking date, which is duly addressed. One advantage of the MPP is that the likelihood of the claims process can be decomposed into independent blocks, which allows each block to be maximized in isolation (Larsen, 2007). As a result, the parameters of each block are estimated with the appropriate GLMs. We use policy covariates that are readily available for new and existing policyholders for an observation period.

The MPP framework, which was introduced by Arjas (1989), Jewell (1989), Norberg (1993), and (1999), has been widely used for individual-level loss reserving. For example, Antonio and Plat (2014) and Verrall and Wüthrich (2016) apply the marked Poisson process for non-life insurance loss reserving where claims occurrences are assumed to follow a non-homogeneous Poisson process, and the stochastic characteristics about the claims are treated as marks. The MPP framework's hierarchical makeup also provides flexibility in modeling different events and their features in the ratemaking process. In insurance pricing, hierarchical models are not new. For example, the frequency-severity model, a popular multivariate approach to model the claim frequency and payments arising from closed claims, forms a two-level hierarchical pricing model. See Frees (2014) for details and the application of the frequency-severity models. Frees and Valdez (2008) and Frees *et al.* (2009) extended the frequency-severity model to a hierarchical model with three building blocks relating to the frequency, type, and severity of claims. Shi *et al.* (2016) provide a hierarchical framework for modeling insurance loss cost with a complex structure and propose a copula regression to accommodate various sources of dependence, and Karoui *et al.* (2017) discuss a robust optimization approach to promptly identify shifts in the frequency of insurance claims by detecting any disruptions in the intensity of the counting process, even when changes occur at unpredictable and unobservable times.

We contribute to the literature in two main ways: First, the proposed framework ensures that the multivariate risk analysis is done using the information on both open and closed claims in a more efficient way leading to better alignment of premiums. Second, by automatically accounting for the expected cost relating to open claims without using a separate claim reserving model, the proposed framework bridges the gap between ratemaking and reserving and makes the ratemaking process complete and balanced for individual risks.

The rest of the paper is organized as follows. Section 2 presents the MPP model and its application to ratemaking. Section 3 discusses parameter estimation and the loss cost prediction for the MPP model. Section 4 evaluates the ratemaking performance of the MPP model using simulation studies.
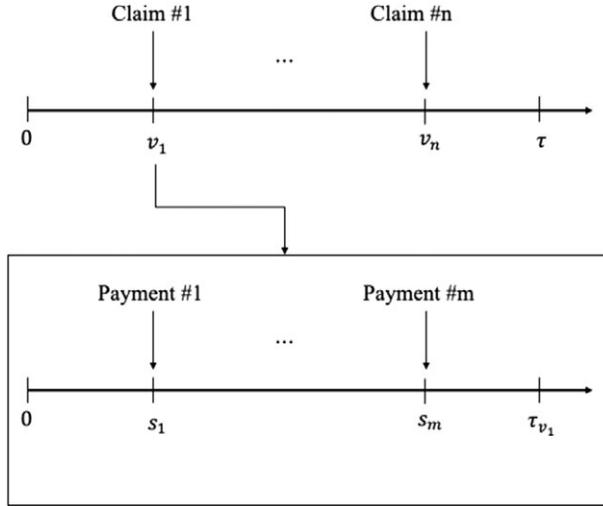
Figure 1. *Claim occurrence and payment development process.*

Section 5 provides the model fitting results using a training dataset and evaluates the MPP model's quality of prediction using out-of-sample data. Section 6 concludes.

2. Claim modeling

2.1. Data structure

Let $j = 1, \ldots, J$ represent the index for policies in the portfolio, and $t = 1, \ldots, T_j$ represent the policy years observed for each policy, then the observable responses at the ratemaking date are:

- N_{jt}, the number of claims reported within a policy year t for each policy j.
- $M_{jt,i}$, the number of transactions for each claim, where $i = 1, \ldots, N_{jt}$ is the claim index. For open claims, $M_{jt,i}$ is censored; then denote $\delta_{jt,i} = 1$ when claim is closed or $\delta_{jt,i} = 0$ otherwise.
- $P_{jt,ik}$, the payment amount per transaction. Where the payment transaction index is $k = 1, \ldots, M_{jt,i}$.

The exposure e_{jt} is measured as a fraction of years, which provides the length of time in the policy year as of the ratemaking date. For the explanatory covariates, policy-level characteristics represented by $\boldsymbol{x}_{jt}$ are used. Additionally, $U_{jt,i}$ is the reporting delay variable which is the difference between claim occurrence and reporting times for claim i reported in the $\{jt\}$ observation period. Then, the data available can be summarized as:

$$\{e_{jt}, N_{jt}, U_{jt,i}, (M_{jt,i}, \delta_{jt,i}), P_{jt,ik}, \boldsymbol{x}_{jt}; t = 1, \ldots, T_j, j = 1, \ldots, J\}. \tag{2.1}$$

For the ratemaking exercise in this paper, we assume $\{N_{jt}, M_{jt,i}\}$ are recorded for each policy year, but the analysis done in this paper can be easily extended to other periods. Further, insurers usually record the reporting and claim occurrence dates, and we will assume the reporting delay variable $U_{jt,i}$ is in days.

2.2. Marked Poisson process

Figure 1 elaborates on the timeline for claim occurrence at times $V_1 = v_1, V_2 = v_2, \ldots, V_n = v_n$ and transaction occurrence at times $S_1 = s_1, S_2 = s_2, \ldots, S_m = s_m$ in a fixed period $[0, \tau]$. In this paper, τ represents the ratemaking date. From the figure, it is clear there are two counting processes, one relating to claims occurrence and the other the transaction occurrence after reporting.

The associated counting process $\{N(v), 0 \leq v\}$ of the claim occurrence process in Figure 1 is Poisson and records the cumulative number of claims that the process generates. We denote $H(v) = \{N(u) : 0 \leq u < v\}$ to be the history of the claims occurrence process at time v. Then, the intensity function, determined only by v, for the claim occurrence process, is given by:

$$\rho(v|H(v)) = \lim_{\Delta v \downarrow 0} \frac{\Pr\{\Delta N(v) = 1|H(v)\}}{\Delta v} = \lim_{\Delta v \downarrow 0} \frac{\Pr\{\Delta N(v) = 1\}}{\Delta v} = \rho(v), \qquad (2.2)$$

where $\rho(v)$ is a non-negative integrable function and $\Delta N(v)$ represents the number of claims in the short interval $[v, v + \Delta v]$. Further, observable covariates $x_j(v)$ for policyholders $j = 1, \ldots J$ that affects claim occurrence may be incorporated in the model by including the covariate information in the process history. Thus, the heterogeneities among the policyholders can be accounted for by specifying the intensity function of the form:

$$\rho(v|x_j^v) = \rho_0(v) \exp(x_j'(v)\boldsymbol{\beta}), \qquad (2.3)$$

where $x_j^v = \{x_j(u) : 0 \leq u \leq v\}$ is the covariate history. $\rho_0(v)$ is the baseline function that relates to policyholders for whom $x_j(v) = 0$ for all v, and $\boldsymbol{\beta}$ is a vector of regression coefficients for the covariates.

The marked Poisson process (MPP) framework is employed for the claims modeling for insurance pricing. For a marked Poisson process in $[0, \tau]$, the likelihood that n claims occur at times $V_1 = v_1$, $V_2 = v_2, \ldots, V_n = v_n$, with marks $Z_1 = z_1, Z_2 = z_2, \ldots, Z_n = z_n$ is given by:

$$\Pr[N = n, (V_i, Z_i) = (v_i, z_i), i = 1, 2, \ldots, n] = \left(\prod_{i=1}^{n} \rho(v_i) P_{Z|v_i}(z_i)\right) \exp\left(-\int_0^\tau \rho(v)dv\right). \qquad (2.4)$$

Here, the claim occurrence counting process $N(v)$ is a Poisson process with intensity function $\rho(v)$. The distribution of the marks $P_{Z|v}$ is conditional on $\Delta N(v) = 1$.

The MPP framework allows for the modeling of the entire claim process, including occurrence, reporting, and development after reporting. Let $\boldsymbol{Z}_i = (U_i, \boldsymbol{W}_i)$, with U_i and $\boldsymbol{W}_i$ denoting the reporting delay and the claim development process after reporting, respectively. As seen in Figure 1, $\boldsymbol{W}_i$ includes payment transaction occurrence times S_{ik} and the severity of each transaction P_{ik}, where $k = 1, \ldots, m_i$ index payment transactions for the ith claim. Then, the distribution of the marks $P_{Z|v}$ is specified as $P_{Z|v} = P_{U|v} \times P_{W|v,u}$. This paper assumes that the claim occurrence process and the marks, such as the reporting delay and transaction payments, are independent. It is also assumed the marks are independent of each other.

The reporting delay distribution U given occurrence time v, $P_{U|v} = P_U$, can be modeled using various distributions from survival analysis, but we specify a mixed distribution comprising of a discrete distribution for a reporting delay below or equal to r days, and a Weibull distribution for reporting delays above r days with density function f_U. The likelihood for the reporting delay is given by:

$$\sum_{r=0}^{d} q_r 1(U = r) + \left(1 - \sum_{r=0}^{d} q_r\right) f_{U|U>r}(u), \qquad (2.5)$$

where the probability mass for a reporting delay of r days is given by q_r. Specifically, we use $d = 0$, that is, a probability mass for a reporting delay of zero days (reporting in the same day of occurrence, q_0). To incorporate the policyholder characteristics x_j that may impact the reporting delay distribution of claim i, we specify a Weibull distribution with the scale parameter θ_i that depends on the policyholder characteristics and a constant shape parameter κ given by:

$$f_{U|U>r}(u_i; \kappa, \theta_i) \sim \frac{\kappa u_i^{\kappa-1}}{\theta_i^\kappa} \exp[-(u_i/\theta_i)^\kappa], \quad \theta_i = \exp(x_j'\gamma). \qquad (2.6)$$

The other component of $P_{Z|v}$ is the distribution of the claim development process after reporting, $P_{W|v,u}$. We assume the occurrence of transactions for claim i also follow a non-homogeneous Poisson process in $[0, \tau]$, and transaction payment amounts are treated as marks. Then, the likelihood that m_i transactions occur at times $S_{i1} = s_{i1}, S_{i2} = s_{i2}, \ldots, S_{im_i} = s_{im_i}$, with marks $P_{i1} = p_{i1}, P_{i2} = p_{i2}, \ldots, P_{im_i} = p_{im_i}$ is given by:

$$\Pr[M_i = m_i, (S_{ik}, P_{ik}) = (s_{ik}, p_{ik}), k = 1, 2, \ldots, m_i] = \left(\prod_{k=1}^{m_i} \lambda_i(s_{ik}) f_P(p_{ik}) \right) \\ \times \exp\left(-\int_0^\tau \lambda_i(s) ds \right). \quad (2.7)$$

Here, the transaction occurrence counting process $M_i(s)$ is a Poisson process with intensity function $\lambda_i(s)$, k applies to all payments in $[0, \tau]$, and the distribution of the transaction payment amounts is conditional on $\Delta M_i(s) = 1$. The density function for the payment severity is denoted by $f_P(p_{ik})$.

3. Statistical inference

3.1. Estimating parameters

At the ratemaking time τ, there are reported claims whose full or partial development process is observed, that is, $\mathcal{C}^{rep} = ((v, u, w) \in \mathcal{C} | v + u \leq \tau)$, and IBNR claims whose development process is totally unobserved, that is, $\mathcal{C}^{ibnr} = ((v, u, w) \in \mathcal{C} | v \leq \tau, v + u > \tau)$. Then, the occurrence of reported claims follows an independent Poisson process with intensity function $\rho(v)F_{U|v}(\tau - v)$ and that of IBNR claims also follows an independent Poisson process with intensity function $\rho(v)(1 - F_{U|v}(\tau - v))$ (Wüthrich and Merz, 2008).

It follows that the observed likelihood of the claims process is given by:

$$L = \left(\prod_{i:v_i+u_i \leq \tau} \rho(v_i)F_{U|v}(\tau - v_i) \right) \exp\left(-\int_0^\tau \rho(v)F_{U|v}(\tau - v)dv \right) \times f_{W|v,u}^{\tau - v_i - u_i}(w_i), \quad (3.1)$$

where $f(\cdot)$ and $F(\cdot)$ denote a pdf and a cdf, respectively. The superscript in the claim development term (last term in (3.1)) represents that a claim that occurred at v_i and with reporting delay u_i is censored at $\tau - v_i - u_i$ time units after reporting. As discussed earlier, given the occurrence time v and the reporting delay u, the claim development process W can be decomposed into the payment transactions occurrence times S and the transaction payment amounts P. Then, the observed likelihood in (3.1) becomes:

$$L = \left(\prod_{i:v_i+u_i \leq \tau} \rho(v_i)F_{U|v}(\tau - v_i) \right) \exp\left(-\int_0^\tau \rho(v)F_{U|v}(\tau - v)dv \right) \\ \times \prod_{i:v_i+u_i \leq \tau} \left(\prod_k \lambda_i(s_{ik}) \exp\left(-\int_0^{\tau_i} \lambda_i(s) ds \right) \right) \\ \times \prod_{i:v_i+u_i \leq \tau} \prod_k f_P(p_{ik}). \quad (3.2)$$

Here, k applies to all payments in $[0, \tau_i]$, where $\tau_i = \min(\tau - v_i - u_i, S_i)$ and S_i is the total waiting time from reporting to settlement of claim i. We emphasize that in addition to the claim occurrence counting process $N(v)$ with intensity function $\rho(v)$, the transaction occurrence counting process $M_i(s)$ is also a Poisson process with intensity function $\lambda_i(s)$.

Considering the ratemaking data are organized by the $\{jt\}$ observation period, where $j = 1, \ldots, J$ index policyholders, and $t = 1, \ldots, T_j$ index the policy years for each claim; let the intensity function

for the counting process of claim occurrence $N_{jt}(v)$ be $\rho_{jt}(v)$ and that of counting process of transaction occurrence $M_{jt,i}(s)$ be $\lambda_{jt,i}(s)$. Then, the likelihood for the observed claims process in (3.2) becomes:

$$
\begin{aligned}
L &= \prod_{j=1}^{J} \prod_{t=1}^{T_j} \left(\prod_{i=1}^{n_{jt}} \rho_{jt}(v_{jt,i}) F_{U|v}(\tau - v_{jt,i}) \exp\left(-\int_{t-1}^{t} w_{jt}(v) \rho_{jt}(v) F_{U|v}(\tau - v) dv \right) \right) \\
&\times \prod_{j=1}^{J} \prod_{t=1}^{T_j} \prod_{i=1}^{n_{jt}} \left(\prod_{k=1}^{m_{jt,i}} \lambda_{jt,i}(s_{jt,ik}) \exp\left(-\int_{0}^{\tau_{jt,i}} \lambda_{jt,i}(s) ds \right) \right) \\
&\times \prod_{j=1}^{J} \prod_{t=1}^{T_j} \prod_{i=1}^{n_{jt}} \prod_{k=1}^{m_{jt,i}} f_P(p_{jt,ik}),
\end{aligned}
\tag{3.3}
$$

where $\tau \in [T_j - 1, T_j]$, the exposure at time v is given by $w_{jt}(v)$, n_{jt} denote the number of reported claims that occur in the $\{jt\}$ observation period, and $m_{jt,i}$ is the number of transactions for claim i reported in the $\{jt\}$ observation period. With regard to the ratemaking application, the number of transactions to settlement is of interest. But for RBNS claims, the number of transactions is censored at the ratemaking date τ. Therefore, we denote, $\delta_{jt,i} = I(S_{jt,i} \leq \tau_{jt,i})$ to indicate whether the claim has been closed by the ratemaking time. Note that, $S_{jt,i}$ is the total waiting time from reporting to settlement of claim i reported in the $\{jt\}$ observation period. Thus, $\delta_{jt,i} = 1$ for closed claims, and $\delta_{jt,i} = 0$ for RBNS claims at τ. Additionally, given that $S_{jt,i} > \tau_{jt,i}$ for RBNS claims, it means that $M_{jt,i}(S_{jt,i}) \geq M_{jt,i}(\tau_{jt,i}) = m_{jt,i}$.

The likelihood in (3.3) for reported claims can be broken down into three building blocks: the number of claims per policy in a policy year, the conditional number of payment transactions for a claim, and the conditional payment sizes for each transaction. The likelihood is decomposed into independent blocks, which can be maximized in isolation. But, the MPP is a continuous-time model, and the data on the claims occurrence and transaction occurrence recorded and available for statistical inference are discrete. Thus, we assume a piece-wise constant specification for the intensity functions that allow the use of the recorded number of claims and the number of transactions per claim for estimation. Each block is discussed below.

3.1.1. Poisson process for claim frequency

The first line in the likelihood in (3.3) relates to the occurrence of reported claims. A multiplicative form of the intensity function is assumed where $\rho_{jt}(v) = \rho_0(v; \boldsymbol{\alpha}) \exp(\boldsymbol{x}'_{jt} \boldsymbol{\beta})$. Here, $\boldsymbol{x}_{jt}$ are the rating variables, and $\{\boldsymbol{\alpha}, \boldsymbol{\beta}\}$ are parameters to be estimated. To estimate $\rho_{jt}(v)$, we assume the claim occurrence follows a Poisson process with a non-homogeneous piece-wise constant intensity ρ_{jt}, such that the baseline rate function is given by:

$$
\rho_0(v; \boldsymbol{\alpha}) = \alpha_t \quad a_{t-1} < v \leq a_t,
\tag{3.4}
$$

where $t = 1, \ldots T$, and T is the most recent policy effective year. The parameters of the baseline rate function are denoted by $\boldsymbol{\alpha} = (\alpha_1, \ldots, \alpha_T)$, and $a_0 < a_1 < \cdots, a_T$ are the cut-points of the intervals for the baseline function where $a_0 = 0$ and $a_T = T$. Then, the occurrence of reported claims follows an independent Poisson process with intensity function $\rho_{jt} F_{U|v}(\tau - v)$. The corresponding likelihood for the occurrence of reported claims is given by:

$$
L = \prod_{j=1}^{J} \prod_{t=1}^{T_j} \left(\rho_{jt}^{n_{jt}} \times \prod_{i=1}^{n_{jt}} F_{U|v}(\tau - v_{jt,i}) \exp\left(-e_{jt} \rho_{jt} \int_{t-1}^{t} F_{U|v}(\tau - v) dv \right) \right),
\tag{3.5}
$$

where $\rho_{jt} = \alpha_t \exp(\boldsymbol{x}'_{jt} \boldsymbol{\beta})$, and $e_{jt} = \int_{t-1}^{t} w_{jt}(u) du$ is the exposure time in $(a_{t-1}, a_t]$ for policyholder j. For ratemaking purposes, the likelihood in (3.5) can be maximized via a Poisson regression with a log link where $\rho_{jt} = \exp(\ln \alpha_t + \boldsymbol{x}'_{jt} \boldsymbol{\beta})$ is the mean of the response N_{jt}. Here, $\ln(e_{jt})$ and $\ln\left(\int_{t-1}^{t} F_{U|v}(\tau - v) dv \right)$ are specified as offset variables to account for the exposure as at the ratemaking date and to adjust parameters to account for IBNR claims, respectively. An estimate of the reporting delay distribution is obtained by

fitting the distribution in (2.5). Hence, the estimation for the claim frequency is a two-stage approach. First, the reporting delay model is estimated, and second, the estimated parameters from the reporting delaying model are plugged in to estimate claim frequency model.

3.1.2. Poisson process for transaction frequency

The second line in the likelihood in (3.3) relates to transaction occurrence conditional on having at least a claim. The transaction counting process $M_{jt,i}(s)$ is also Poisson with intensity measure $\lambda_{jt,i}(s) = \lambda_0(s; \boldsymbol{b}) \exp(\boldsymbol{x}'_{jt}\boldsymbol{\pi})$. Again, a piece-wise constant intensity $\lambda_{jt,i}$ is assumed such that the baseline rate function is given by:

$$\lambda_0(s; \boldsymbol{b}) = b_t \quad a_{t-1} < s \leq a_t, \tag{3.6}$$

where $\boldsymbol{b} = (b_1, \ldots, b_T)$ are parameters of the baseline rate function. Here, the likelihood for the transaction occurrence can be specified as proportional to the product of Poisson likelihoods shown as:

$$L = \prod_{j=1}^{J} \prod_{t=1}^{T_j} \left(\prod_{i=1}^{n_{jt}} (b_t \exp(\boldsymbol{x}'_{jt}\boldsymbol{\pi}))^{m_{jt,i}} \exp\left(-b_t \exp(\boldsymbol{x}'_{jt}\boldsymbol{\pi})\right) \right). \tag{3.7}$$

For RBNS claims, $M_{jt,i}(S_{jt,i}) \geq m_{jt,i}$, then the likelihood in (3.7) can be maximized using censored Poisson regression with a log link where $\lambda_{jt,i} = \exp(\ln b_t + \boldsymbol{x}'_{jt}\boldsymbol{\pi})$ is the mean of the response $M_{jt,i}$. The likelihood for the censored Poisson is given by:

$$\Pr(m_{jt,i}, \delta_{jt,i}) = [f(m_{jt,i})]^{\delta_{jt,i}} \left[1 - \sum_{k=0}^{m_{jt,i}-1} f(k) \right]^{1-\delta_{jt,i}}, \tag{3.8}$$

where $f(\cdot)$ is a Poisson density function and $\delta_{jt,i} = 1$ if the claims are closed or $\delta_{jt,i} = 0$ if open.

3.1.3. Transaction severity modeling

The conditional severity block describes the claim payment size per transaction. Different distributions can be used to model the transaction payments $P_{jt,ik}$ with conditional mean $\mu_{jt,ik} = \exp(\boldsymbol{x}'_{jt}\boldsymbol{\phi})$. The gamma GLM is frequently used in insurance pricing to model payment sizes (Henckaerts *et al.*, 2018), but the goodness-of-fit test needs to be performed to select the proper distribution for the data. Also, certain characteristics of the data may inform model choices. For example, Antonio and Plat (2014) built different models for the first transaction payments and the later transaction payments.

3.2. Loss cost prediction using the MPP Model

A rating formula based on the MPP ratemaking framework will be achieved by the product of exponentiated estimates from the claim frequency, transaction frequency, and severity models. The following rating formula calculates the predicted loss cost:

$$\begin{aligned}
\text{Loss Cost} &= e_j \exp(\ln \hat{\alpha}_T + \boldsymbol{x}'_j \hat{\boldsymbol{\beta}}) \times \exp(\ln \hat{b}_T + \boldsymbol{x}'_j \hat{\boldsymbol{\pi}}) \times \exp(\boldsymbol{x}'_j \hat{\boldsymbol{\phi}}) \\
&= \text{Exposure} \\
&\quad \times \text{Expected number of claims} \\
&\quad \times \text{Expected number of transaction per claim} \\
&\quad \times \text{Expected payment per transaction.}
\end{aligned} \tag{3.9}$$

where e_j is the exposure variable and $\boldsymbol{x}_j$ are rating factors for the new contract. $\{\hat{\alpha}_T, \hat{b}_T\}$ are the fitted trend parameters from the most recent policy year for the reported claim and transaction frequency models. Also, $\{\hat{\boldsymbol{\beta}}, \hat{\boldsymbol{\pi}}, \hat{\boldsymbol{\phi}}\}$ are the fitted parameters for rating variables from the claim frequency model, transaction frequency model, and the severity model building blocks.

In this approach, including information on open claims, such as the number of reported open claims, the claim transaction frequency on open claims, and the payments on open claims, automatically accounts for the reserves on reported claims because the regression parameters are accurately adjusted. Also, the claim frequency regression parameters are adjusted for IBNR claims to account for IBNR reserves. Further, trends in time are captured by the trend parameter estimates $\{\hat{\alpha}_T, \hat{b}_T\}$ and will help to adjust the experience to a cost level applicable to a future effective period. In addition, the trends in time, other rating variables, and open claims data would help promptly capture environmental changes. Finally, regulatory requirements are an important challenge in implementing advanced ratemaking models. Especially, the need to explain and quantify the effect of each policyholder characteristic on the total premium. By multiplying the fitted parameters found in the three layers, the MPP helps to satisfy this requirement.

We note that the predicted loss cost in (3.9) is based on reported claims on which the insurer has made payments. However, if the number of payment transactions, $M_{jt,i}$, in the transaction frequency model is allowed to take on the value of zero, where $M_{jt,i}$ can be equal to zero when claims are reported, but the insurer has not made any payment transactions at the ratemaking date; then, the expected payment per transaction is given by $\hat{\mu}_{jt,ik} = q_j \times \exp(x'_{jt}\hat{\phi})$, where q_j is the probability of positive payment transactions from the transaction-level data.

4. Ratemaking performance evaluation using simulated data

This section highlights the importance of open claims in the ratemaking process using simulated data from the MPP model. We show that using only closed claims for ratemaking leads to biased estimates and inaccurate premiums. We also underscore the advantages of the MPP model over the frequency-severity model that incorporates information on open claims.

4.1. Simulation design

In this simulation, for simplicity, the number of claims for policyholder j within a policy year t is assumed to be a homogeneous Poisson regression with the conditional mean specified as:

$$\rho_{jt} = \exp(\ln \alpha_t + x'_{jt}\boldsymbol{\beta}) = \exp(\ln \alpha + x'_{j1}\beta_{11} + x'_{j2}\beta_{12}), \tag{4.1}$$

where $x_{jt} = \{x_{j1}, x_{j2}\}$ are policy-level covariates. We assume $x_1 \sim Bernoulli(0.3)$, representing a discrete rating variable and $x_2 \sim Normal(0, 1)$, corresponding to a continuous rating variable. The reporting delay distribution for claims is assumed to be from a Weibull distribution with a constant shape parameter κ and scale parameter θ_i given by:

$$\theta_i = \exp(x'_{jt}\gamma) = \exp(\gamma_{10} + x'_{j1}\gamma_{11} + x'_{j2}\gamma_{12}). \tag{4.2}$$

For this simulation, we use $\kappa = 0.2$ and $\gamma = \{1.5, 0.3, 0.1\}$. In addition, claim occurence times, v_i, are assumed to follow Uniform(0, 5). In this specific setting, we set $\tau = 5$. Given that a claim is reported, $M_{jt,i}$, the number of transactions to the settlement of claim i reported in the observation period $\{jt\}$ is also assumed to be a homogeneous Poisson regression with the conditional mean specified as:

$$\lambda_{jt,i} = \exp(\ln b_t + x'_{jt}\boldsymbol{\pi}) = \exp(\ln b + x'_{j1}\pi_{11} + x'_{j2}\pi_{12}). \tag{4.3}$$

Further, given that there are $k = 1, \ldots, M_{jt,i}$ transactions, the payments are assumed to be from a gamma regression with logarithmic link function and dispersion parameter $1/\sigma$. The conditional mean is specified as:

$$\mu_{jt,ik} = \exp(x'_{jt}\boldsymbol{\phi}) = \exp(\phi_{10} + x'_{j1}\phi_{11} + x'_{j2}\phi_{12}). \tag{4.4}$$

Finally, we let $\delta_{jt,i} \sim Bernoulli(\eta)$, that is, whether a claim is closed or censored is simulated using a Bernoulli distribution with probability η. We vary η to examine the effect of the proportion of closed

claims in the portfolio on the estimation and prediction results. The parameters used for data generation under the claim frequency, transaction frequency, and transaction severity models are shown in Table 1. We employ Algorithm 1 in the Online Appendix to construct the simulation data.

4.2. Parameter estimates

The estimation results using the simulated data at the ratemaking date are given in Table 1 and based on $S = 100$ replications. Following the likelihood-based method described in Section 3.1, we obtain the parameter estimates and the associated standard error of each simulated sample based on data at the ratemaking date. We present results for different proportions of closed claims and different numbers of policies. Specifically, we report estimations results when the proportion of closed claims are 30%, 80%, and 100% and when the number of policies are 500, 1000, and 1500.

In Table 1, we present the average bias (Bias), the nominal standard deviation of the point estimates (SD), and the average standard error (SE) of the estimates. It can be seen that both the average bias and uncertainty of the average bias for parameters in the claim frequency, transaction frequency, and transaction severity models decrease as the number of policies increases. Further, for the transaction frequency model and transaction severity model, the average bias and uncertainty of the average bias decrease as the proportion of closed claims in the portfolio increases. This observation shows that ignoring open claims will only lead to more biased parameter estimates. The results also show that the average standard error is comparable to the nominal standard deviation, indicating the accuracy of variance estimates.

4.3. Loss cost prediction

This section focuses on the proposed MPP model's prediction performance under different proportions of closed claims and number of policies in the portfolio. The results reported are based on $S = 100$ replication. The loss cost prediction for each policy is obtained using (3.9).

We compare the actual loss cost from the simulated data to that of the loss cost predictions from the MPP model and that of a frequency-severity model which uses only closed claims to estimate parameters named FS_Closed. For the frequency-severity model, the Poisson model is used to model the claim frequency, and a gamma GLM with a logarithmic link is used to model the loss amounts from claims (Frees, 2014). See the Online Appendix for the rating formula based on the frequency-severity model. Further, we compare the loss cost prediction from the MPP model to a frequency-severity model that incorporates information on all reported claims named FS_All. For the open claims in the FS_All model, to eliminate the uncertainty in reserve predictions, we use the actual ultimate amount in the model of loss amounts. This implies that the loss cost predictions from the FS_All are the best-case scenario from the frequency-severity model.

The Gini index measure, which was motivated by the economics of insurance and developed in Frees *et al.* (2011), is employed to aid in the comparison of loss cost predictions between the different models and the actual loss cost. The Gini index is a measure of profit and thus insurers that adopt a rating structure with a larger Gini index are more likely to enjoy a profitable portfolio. The Gini index can be interpreted as the covariance between the profit (loss minus premium) and the rank of relativities (score divided by premium). In addition, the Gini index can be described as twice the area between the ordered Lorenz curve and the 45-degree line (line of equality), where the ordered Lorenz curve is a graph of the ordered premium versus ordered loss distribution based on the relativity measure. The Gini index may range over $[-1, 1]$. The results in Table 2 assume that the insurer has adopted the FS_Closed as a base premium for rating purposes, and the insurer wishes to investigate alternative scoring methods (in this case, the MPP model) to understand the potential vulnerabilities of this base premium. Similarly, the results in Table 3 assume that the insurer has adopted the FS_All as a base premium.

Table 2 summarizes several comparisons using the Gini index when the number of policies are 500, 1000, and 1500 with different proportions of closed claims. We report the average Gini index and the

Table 1. *Estimation results for MPP for different number of policies and different proportion of closed claims.*

$S=100$		Bias			SD			SE	
Parameter	$J=500$	1000	1500	500	1000	1500	500	1000	1500
Proportion of closed claims $=30\%$									
			Poisson claim frequency model						
$\ln(\alpha)=-0.105$	-0.003	0.003	-0.004	0.067	0.041	0.035	0.071	0.050	0.041
$\beta_{11}=0.25$	0.009	0.001	0.001	0.081	0.059	0.041	0.093	0.065	0.053
$\beta_{12}=1$	-0.001	-0.002	0.005	0.037	0.027	0.024	0.045	0.032	0.026
			Censored Poisson transaction frequency model						
$\ln(b)=0.406$	0.072	0.078	0.078	0.075	0.056	0.046	0.062	0.043	0.035
$\pi_{11}=0.5$	0.063	0.070	0.071	0.094	0.057	0.049	0.072	0.050	0.041
$\pi_{12}=0.2$	0.029	0.025	0.025	0.044	0.032	0.028	0.037	0.025	0.021
			Gamma transaction severity model						
$\phi_{10}=5.522$	-0.011	-0.007	-0.006	0.075	0.055	0.045	0.080	0.056	0.046
$\phi_{11}=1$	0.004	0.010	0.007	0.089	0.063	0.048	0.087	0.061	0.050
$\phi_{12}=0.75$	0.010	0.002	-0.001	0.044	0.027	0.024	0.045	0.031	0.025
$\sigma=0.7$	0.011	0.007	-0.002	0.028	0.025	0.017	0.031	0.022	0.018
Proportion of closed claims $=80\%$									
			Poisson claim frequency model						
$\ln(\alpha)=-0.105$	-0.003	0.003	-0.004	0.067	0.041	0.035	0.071	0.50	0.041
$\beta_{11}=0.25$	0.009	0.001	0.001	0.081	0.059	0.041	0.093	0.065	0.053
$\beta_{12}=1$	-0.001	-0.002	0.005	0.037	0.027	0.024	0.045	0.032	0.026
			Censored Poisson transaction frequency model						
$\ln(b)=0.406$	0.011	0.022	0.018	0.060	0.040	0.036	0.056	0.039	0.032
$\pi_{11}=0.5$	0.009	0.007	0.009	0.073	0.042	0.037	0.062	0.043	0.035
$\pi_{12}=0.2$	0.005	0.001	0.003	0.032	0.023	0.022	0.032	0.022	0.018
			Gamma transaction severity model						
$\phi_{10}=5.522$	-0.008	-0.006	-0.005	0.068	0.042	0.039	0.068	0.048	0.039
$\phi_{11}=1$	0.004	0.003	0.006	0.074	0.047	0.041	0.074	0.052	0.043
$\phi_{12}=0.75$	0.008	0.003	0.000	0.039	0.021	0.021	0.038	0.027	0.022
$\sigma=0.7$	0.006	0.004	-0.002	0.026	0.020	0.013	0.026	0.018	0.015
Proportion of closed claims $=100\%$									
			Poisson claim frequency model						
$\ln(\alpha)=-0.105$	-0.003	0.003	-0.004	0.067	0.041	0.035	0.071	0.050	0.041
$\beta_{11}=0.25$	0.009	0.001	0.001	0.081	0.059	0.041	0.093	0.065	0.053
$\beta_{12}=1$	-0.001	-0.002	0.005	0.037	0.027	0.024	0.045	0.032	0.026
			Censored Poisson transaction frequency model						
$\ln(b)=0.406$	-0.003	0.007	0.002	0.052	0.037	0.033	0.054	0.038	0.031
$\pi_{11}=0.5$	0.002	-0.003	0.000	0.066	0.039	0.033	0.059	0.042	0.034
$\pi_{12}=0.2$	0.001	-0.004	-0.001	0.030	0.022	0.020	0.031	0.021	0.017
			Gamma transaction severity model						
$\phi_{10}=5.522$	-0.008	-0.008	-0.005	0.062	0.043	0.036	0.064	0.045	0.037
$\phi_{11}=1$	0.000	0.005	0.007	0.070	0.047	0.039	0.070	0.049	0.040
$\phi_{12}=0.75$	0.007	0.003	-0.001	0.037	0.021	0.019	0.036	0.025	0.020
$\sigma=0.7$	0.007	0.004	-0.002	0.025	0.018	0.012	0.025	0.018	0.014

Table 2. *Gini indices of MPP predictive claim scores with frequency-severity model based on only closed claims (FS_Closed) as base premium.*

% of closed claims	S = 100	J = 500 (%)	J = 1000 (%)	J = 1500 (%)
30%	Mean	11.50	11.17	11.85
	SE	3.38	2.60	2.25
80%	Mean	4.08	5.41	5.26
	SE	3.41	2.64	2.25
100%	Mean	3.32	4.22	4.37
	SE	3.40	2.67	2.23

Table 3. *Gini indices of MPP predictive claim scores with frequency-severity model based on all reported claims (FS_All) as base premium.*

% of closed claims	S = 100	J = 500 (%)	J = 1000 (%)	J = 1500 (%)
30%	Mean	3.81	4.43	4.69
	SE	3.36	2.67	2.24
80%	Mean	3.24	4.34	4.41
	SE	3.42	2.65	2.23
100%	Mean	3.30	4.19	4.34
	SE	3.41	2.67	2.22

average standard error (SE) of the Gini index estimates. The standard errors were derived in Frees *et al.* (2011) where the asymptotic normality of the Gini index was proved. From the results, with 500 policies and a 30% proportion of closed claims, the Gini index is 11.50%, which indicates that insurers using FS_Closed for premiums could look to the MPP model to detect discrepancies between their loss and premium distributions. The standard error implies that the discrepancy is statistically significant. Similar observations are made when the policy numbers increase to 1000 and 1500 with just 30% proportion of closed claims. However, the discrepancies may not be significant when the proportion of closed claims increased to 80% and 100%, meaning the Frequency-Severity model based on closed claims and the MPP model produce similar predictions only when the proportion of closed claims is high.

Further, the Gini index results from Table 3, where the FS_All model is the base premium, show that the MPP model produces loss costs that are slightly better than that from the best-case scenario of the frequency-severity framework. But, the standard error implies that the discrepancies may not be statistically significant. Even with similar loss cost predictions from the MPP and the FS_All models, the MPP framework provides the advantage of performing the ratemaking exercise without needing a reserving model to develop claims to ultimate.

To show that the MPP can account for claim reserves, Table 4 presents the actual versus expected analysis of the total loss cost values (A/E). The A/E is a common actuarial model projection tool and provides the actual total loss cost as a percentage of predicted loss cost from the models. We report the average A/E scores and the nominal standard deviation of the A/E scores (SD). For accurate model projections, we expect the average A/E to be close to 100%. The results show that the MPP performs better than the best-case scenario of the frequency-severity model (FS_All) because the FS_All model does not account for IBNR claims. Hence, the FS_All model will only be competitive when the claim occurrence frequency model is adjusted to account for IBNR claims. Also, the frequency-severity models based on only closed claims (FS_Closed) performed worse because, in addition to not accounting for IBNR claims, it does not use the information of reported claims that are open. This further shows that the MPP framework provides the advantage of performing the ratemaking exercise without the need for a reserving model to develop claims to ultimate and also accounts for IBNR claims.

Table 4. *Loss cost prediction results.*

% of closed claims	$S = 100$	A/E % ($J = 500$)			A/E % ($J = 1000$)			A/E % ($J = 1500$)		
		MPP	FS_All	FS_Closed	MPP	FS_All	FS_Closed	MPP	FS_All	FS_Closed
30%	Mean	94.06	141.01	438.39	93.96	143.08	444.75	94.81	143.92	452.91
	SD	8.98	11.04	65.80	5.66	8.49	42.64	5.76	8.53	39.76
80%	Mean	98.21	146.54	182.98	99.19	148.28	185.35	99.22	148.91	185.97
	SD	6.86	10.48	14.83	5.16	8.59	13.75	5.31	8.46	12.19
100%	Mean	98.60	148.18	148.26	99.52	149.81	149.95	99.78	150.63	150.78
	SD	6.88	10.79	10.82	5.06	8.58	8.59	5.16	8.40	8.45

Table 5. *Summary statistics at the policy and claim level for building and contents coverage.*

	Policy level				Claim level	
Effective year	Average frequency	Average severity	Average coverage (Million)	Number of policies	Average No. of transaction	Average payment per transaction
2006	0.734	10,083	32.363	1159	1.276	9554
2007	0.925	7095	35.143	1143	1.291	8946
2008	0.746	6730	37.150	1130	1.245	7991
2009	0.924	4864	40.275	1114	1.206	9864
2010	1.088	20,827	41.123	1114	1.123	17,152
2011	0.948	8367	42.426	1096	1.173	17,156

5. Empirical analysis using the MPP

5.1. Data

The data we use for the ratemaking exercise in this paper are from the Wisconsin Local Government Property Insurance Fund (LGPIF). The Fund insures local government entities, including counties, cities, towns, villages, school districts, and library boards. The primary coverage of the Fund, which is the focus of this paper, is the building and contents coverage. But the Fund also provides coverage for inland marine (construction equipment) and motor vehicles. The dataset has already been used in other ratemaking papers; for example, see Frees and Lee (2015).

Though the LGPIF data spans from January 1, 2006, to December 31, 2013, we focus on the dataset from effective years 2006-2011, where all claims are marked as closed as of December 31, 2013. Here, we use data from the policy, claim, and transaction databases. Table 5 shows the summary statistics, at the policy and claim level, from effective years 2006-2011. High variability across years in the average claim frequencies and severity is observed at the policy level, highlighting the importance of using current information in claim modeling for ratemaking purposes. Further, from the summary statistics at the claim level, the average number of payments transactions to settlement per claim is gradually reducing, and the average payment per transaction is increasing. This observation suggests a change in claims processing of the LGPIF. Such environmental changes affect the distribution of future losses, and using current information on open claims allows capturing such changes promptly.

Table 6 describes the rating variables considered in this paper. Tables 1 and 2 in the Online Appendix show that the rating variables are correlated with the claim frequency and severity at the policy level, and with the transaction severity at the claim level, which indicates that they will be significant predictors of claims in the ratemaking model. For the ratemaking exercise, the data from effective years 2006-2009 are used as the training sample to calibrate the MPP model. Here, we assume that by December 31, 2009, policyholders' rates have to be updated for the policy year 2010. The rating factors from the calibrated

Table 6. *Description of rating variables.*

Variable	Description
EntityType	Categorical variable that is one of six types: Village, City, County, Misc, School, or Town
Region	Categorical variable which identifies region of the county of an entity belongs to: Northern, Northeastern, Southeastern, Southern, or Western
LnPolicyDed	Deductible for the policy in logarithmic of dollars
LnPolicyCov	Total building and contents coverage for the policy in logarithmic millions of dollars
AlarmCredit	Categorical variable that is one of five types:(0%, 5%, 10%, 15%, or a combination of credits), for automatic smoke alarms in main rooms
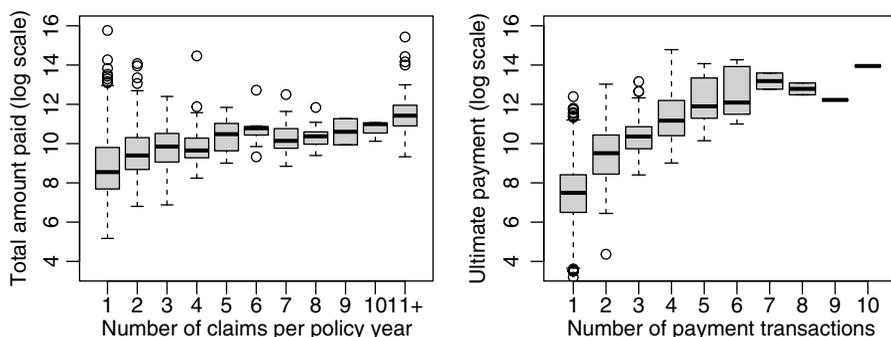
Figure 2. *Left panel: shows the distribution of total amount of payment per policy by number of claims per policy year. Right panel: shows the ultimate amount paid by number of transactions to settlement.*
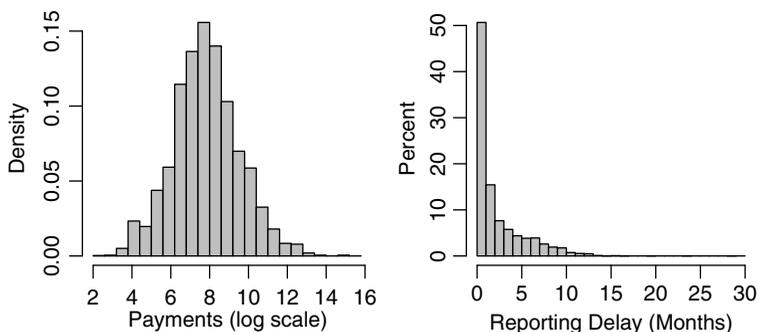
Figure 3. *Distribution of marks. The left panel shows the histogram of log-transformed transaction payment amounts. The right panel shows the observed reporting delay distribution.*

MPP model are then applied to the 2010 rating variables to predict 2010 loss costs (claims scores).

The left panel of Figure 2 presents the relationship between the total amount of payment and the number of claims per policy based on the training sample. We note that the policies are all one-year policies. The plot suggests an increase in the number of reported claims in a policy year increases the total amounts paid by the insurer. A similar positive relationship is observed in the right panel between the ultimate amount paid for claims and the number of transactions to settlement. The small circles identify outliers. Table 1 in the Online Appendix shows that the transaction payments are heavily right-skewed. The left panel of Figure 3 shows the histogram of the transaction payment amounts based on the training sample. Here, the log scale is used to handle the skewness. The reporting delay is a key

Table 7. *Summary statistics for closed, RBNS, and IBNR claims as of December 31, 2009.*

Effective year	Number of claims				Average payments		
	Closed	RBNS	IBNR	Total	Closed	RBNS	Total
2006	785	1	0	786	16,501	5,398,051	23,348
2007	987	6	4	997	13,281	376,109	15,473
2008	747	20	10	777	13,509	51,165	14,491
2009	478	136	184	798	5631	8904	6356
Total	3230	163	198	3591	14,133	60,668	16,368

driver of IBNR claims, and the right panel of Figure 3 shows the distribution of the reporting delays in months. The distribution appears to be right-skewed.

Table 7 provides a summary of the number of closed, RBNS, and IBNR claims as of December 31, 2009. As expected, the very recent effective year 2009 is associated with the highest RBNS and IBNR claims. Therefore, ratemaking models that rely on only closed claims will lose current information needed to produce accurate premiums. The table also summarizes the average payments for reported claims (closed and RBNS claims) as of December 31, 2009. It is seen that the average payments across years for RBNS claims are more significant than closed claims, and RBNS claims tend to decrease as the effective year increases indicating that bigger claims take a longer time to settle. This is because, as discussed in Okine *et al.* (2022), large and complicated claims naturally involve multiple interested parties, demand special expertise, and are more likely to be litigated. Hence, the payments on the RBNS claims can reflect the change in the Fund's risk profile and capture environmental changes in a timely manner compared to closed claims.

For robustness checks, we also present an analysis of the MPP framework using the effective years 2007–2010 as the training sample. We compare the loss cost predictions from this new model to the out-of-sample data from 2011.

5.2. Estimation results

This section presents the estimation results from the three building blocks in the MPP framework fitted using maximum likelihood. The training data contain observations from effective years 2006–2009, but we also show the parameter estimates using only data from the recent effective year 2009 to examine the effect on estimation when the proportion of closed claims in the portfolio is reduced.

Figure 4 explores different distributions to fit the transaction payment amounts. The figure plots the empirical distribution functions of the transaction payment amounts to the distribution functions from the fitted Gamma, Pareto, and Log-normal distributions. The Pareto and the Log-normal distributions appear to provide a reasonable fit to the data. But, the Akaike Information Criterion (AIC) for the Log-normal fit is 69,749.6, and that of the Pareto fit is 69,758.0. Hence, we select the Log-normal distribution for modeling the transactions payment block.

Figure 5 compares the reporting delay distribution with the fitted mixed distribution with a probability mass for a reporting delay of zero and a Weibull distribution for reporting delays above zero. The fitted mixed distribution seems to fit the observed reporting delay data reasonably well from the plots. The reporting delay distribution may vary depending on the policyholder characteristics; therefore, we expand the Weibull distribution for reporting delay to include the entity types. Table A.1 in the Appendix shows the parameter estimates of the fitted Weibull model in (2.6).

Table A.2. in the Appendix presents the estimation results for the MPP model using the information on reported claims at the ratemaking date and following the likelihood-based method described in Section 3.1. The rating variables used are described in Table 6. The results from the claim frequency
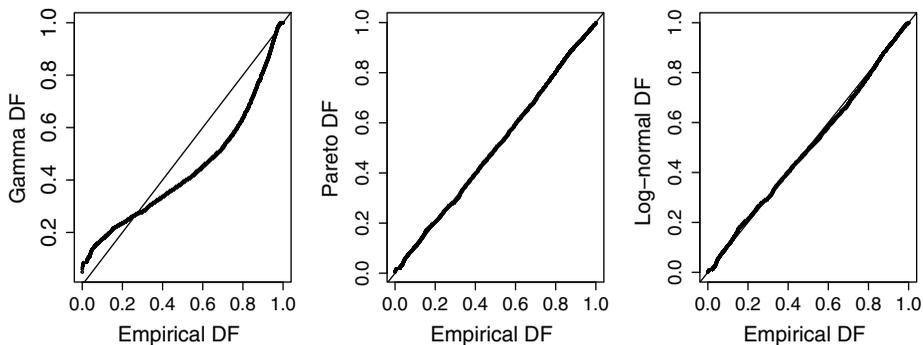
Figure 4. *Comparing the empirical distribution function (DF) to the distribution function of the Gamma fit (left panel), Pareto fit (middle panel), and Log-normal fit (right panel).*
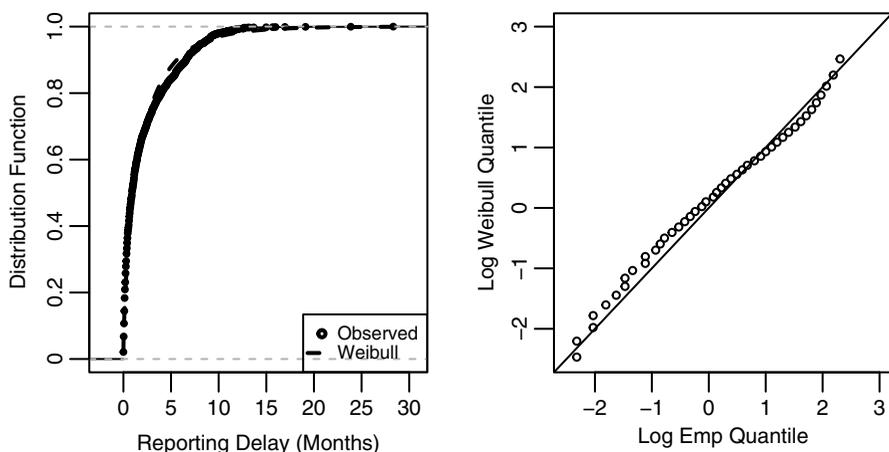
Figure 5. *Comparing the observed reporting delay distribution to the fitted Weibull. Left panel overlays fitted Weibull distributiion function (DF) over the empirical DF. The right panel provides the Quantile-Quantile Plot.*

model capture the trend in reported claims across years when using all the training data. For the loss cost prediction, the baseline parameter from the most recent year α_{2009} is used. Further, the censored Poisson transaction frequency model and the Log-normal severity model results are provided in the middle and bottom layer of Table A.2. The effect of a rating variable can be obtained by adding up the effects from all three building blocks. For example, using all the training data, the parameter estimate for LnPolicyCov is positive in the claim frequency model (1.107), negative in the transaction frequency model (-0.022), and negative in the transaction payment model (-0.349). In this case, the overall effect can be interpreted as positive (0.737). In addition, the last two columns in Table A.2 provide results using all the training data with policy coverage (LnPolicyCov) and time (policy years) as interaction variables. The results suggest differences in the effect of the policy coverage values on claim frequency in different years.

5.3. Out-of-sample performance

This section provides the loss cost prediction based on the MPP model fitted in Section 5.2. Thus, we do not consider expenses associated with the loss amounts. The predictions were generated based on the 2010 out-of-sample rating variables, and we compare the predictions to the 2010 out-of-sample claims. Also, loss cost prediction results are provided for the MPP model with policy coverage and
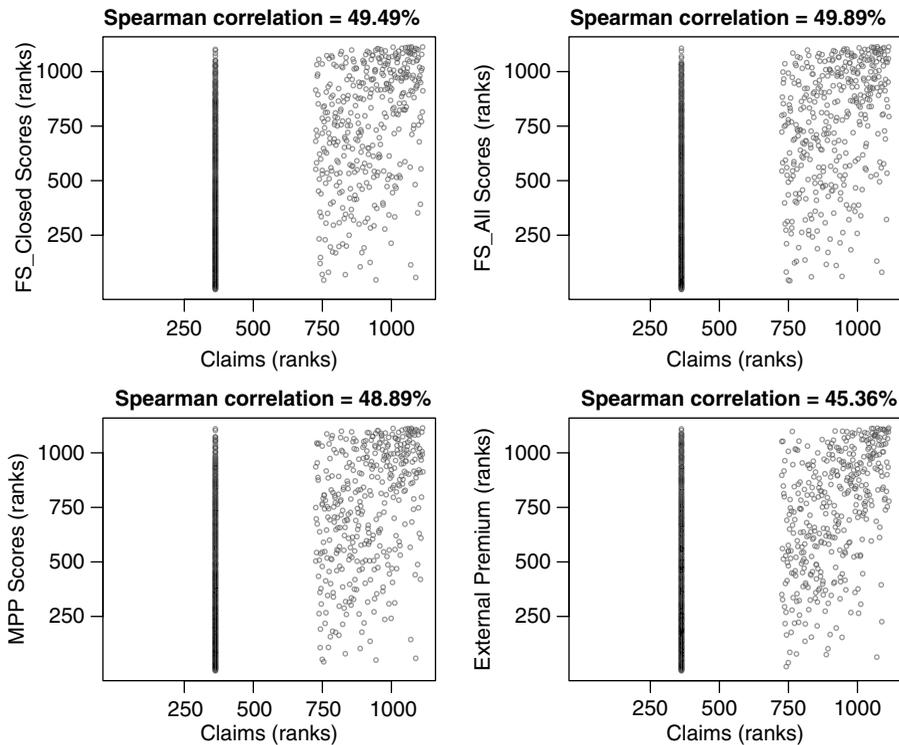
Figure 6. *Comparing the 2010 out-of-sample claims to loss cost predictions from different models (based on data from effective years 2006-2009), and the 2010 external out-of-sample premiums.*

time as interaction variables, named MPP_Int. In addition, we compare the out-of-sample results to that of a frequency-severity model that uses only closed claims in the estimation of parameters named FS_Closed and another that uses all reported claims in the model building, named FS_All. For the open claims in the FS_All model, we use the incurred payment (amount paid plus loss reserve) as an estimate for the ultimate payment amount. Further, we compare the 2010 out-of-sample premiums in the dataset, which an external agency generated, to the loss cost predictions from the models. These external agency premiums are the actual premiums paid by the policyholder for the 2010 effective year, and results based on these premiums are named EA_Premiums.

Figure 6 shows the scatterplot of the ranks for the 2010 out-of-sample claims and the predicted loss costs from the models, where each point relates to a policyholder. It also shows a scatterplot of the ranks between the out-of-sample claims and the external out-of-sample premiums. The vertical line formed by the points relates to policyholders without losses in the 2010 effective year, and rank ties are replaced by their mean. The correlation of the rank of the scores from the MPP and the 2010 out-of-sample claims is 48.89%, and that of the scores from the external 2010 out-of-sample premiums is 45.36%. The higher correlation from the MPP model shows that the MPP framework offers a promising alternative to the existing rating methodology.

The Gini index measure is employed to aid in the comparison of loss cost predictions between the different models, the external out-of-sample premiums, and the out-of-sample claims. As discussed in Section 4.3, insurers that adopt a rating structure with a larger Gini index are more likely to enjoy a profitable portfolio. We show that the MPP framework helps align premiums with the underlying risk better than the frequency-severity approach and is highly competitive against the existing rating methodology used to generate the external out-of-sample premiums. As seen from Table 5, for the out-of-sample year 2010, the average severity is higher than in other years due to two unusually large claims as a result of the Great Milwaukee Flood. These two large claims were paid to two different policyholders,

Table 8. *Gini indices of projected loss cost based on 2010 hold-out-sample with FS_All as base premium.*

Base premium	Effective year 2009			Effective years 2006–2009			
	FS_Closed (%)	MPP (%)	EA_Premiums (%)	FS_Closed (%)	MPP (%)	MPP_Int (%)	EA_Premiums (%)
	With outliers						
FS_All	−51.91	−8.52	48.22	38.90	49.24	58.70	43.70
Standard errors	15.91	16.85	9.83	25.85	20.40	14.91	9.96
	Without outliers						
FS_All	−24.99	−2.30	42.47	12.57	22.17	28.25	44.88
Standard errors	10.12	10.11	6.87	12.95	10.15	7.95	6.12

and the total amounts paid to the policyholders are $12, 922, 218 and $4, 776, 206. Hence, we provide the Gini index results with and without the policies with the unusual claims in the 2010 out-of-sample data.

Table 8 presents the Gini index results, where we use FS_All as a base premium. The columns summarize results using the FS_Closed, MPP scoring methods in addition to the external premium (EA_Premiums). The table shows that when the ratemaking model was built using data from the effective year 2009, the frequency-severity model based on only closed claims did not fare well compared with the base model. Also, the MPP model discrepancies were not statistically significant compared to the base model, and the external agency premiums fared well compared to the base premium (with or without the unusual claims). However, when the data from the effective years 2006-2009 are used in the model building and the unusual claims are used in the 2010 out-of-sample data, the Gini index for the MPP model and the external agency premiums are 49.24% and 43.70%, respectively. Also, the MPP model with interaction variables has a Gini index of 58.70%. The standard error implies that the differences are statistically significant. The results suggest that the MPP framework promotes equity in pricing compared to the base premium (FS_All) and indicates loss cost predictions from the MPP model will lead to more equitable rates. Similar results are seen when the unusual claims are removed from the 2010 out-of-sample data. It is worth noting that, when all the training data are used in the model building, with the proportion of closed claims increased, the difference between the base model FS_All and the FS_Closed is not statistically significant. The ordered Lorenz plots relating to the Gini indexes with FS_ALL as a base premium are shown in Figure 7. The figure demonstrates that the area between the Lorenz curve and the 45-degree line generally decreased when unusual claims were removed, visually explaining the changes in the Gini index.

Table 9 further investigates the FS_All model loss cost projection using reserve predictions instead of the incurred payment in the dataset. Here, the reserves for reported but not settled claims are estimated using two individual-level reserving methods, the joint model and the Marked Poisson Process. The former jointly model the longitudinal model for cumulative payments and the survival model for time-to-settlement data, incorporating the payment-settlement association in the reserving process. The entire claim process is modeled in the latter, including the claim occurrence reporting and development after reporting. Both methods were applied in Okine *et al.* (2022) on the LGPIF dataset for reserving purposes. FS_All_JM and FS_All_MPP are the names for the Frequency-Severity model that uses the joint model and Marked Poisson Process models to obtain the ultimate amount of reported but not settled claims, respectively. The models are based on data from effective years 2006-2009. Not surprisingly, the results are similar to that reported in Table 8.

With the external agency premiums being the actual premiums policyholder paid, these premiums provide an interesting state-of-the-art baseline for model comparison. Hence, we provide the Gini index results with the external agency premiums as a base premium in Table 10 and the corresponding ordered

Table 9. *Gini indices of projected loss cost from different frequency-severity models based on* 2010 *hold-out-sample.*

Base premium	FS_Closed (%)	MPP (%)	MPP_Int (%)	EA_Premiums (%)
	With outliers			
FS_All_JM	38.69	48.72	58.61	43.82
Standard errors	25.82	20.52	14.84	9.52
FS_All_MPP	38.86	47.67	58.38	40.82
Standard errors	26.77	20.36	14.25	9.17
	Without outliers			
FS_All_JM	11.99	21.08	27.62	44.53
Standard errors	12.96	10.39	7.90	6.13
FS_All_MPP	10.04	20.92	27.68	44.12
Standard errors	12.78	9.34	7.31	6.28
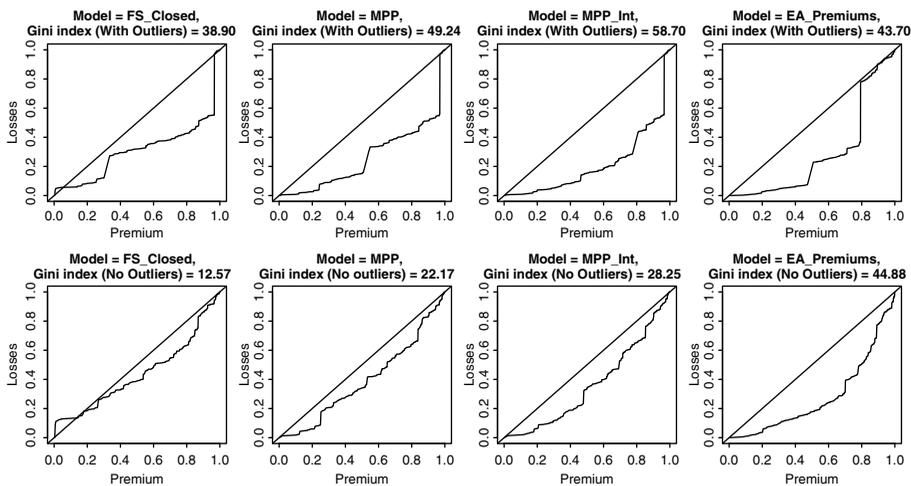
Figure 7. *Ordered Lorenz curve of projected loss cost based on 2010 hold-out-sample with FS_All as base premium.*
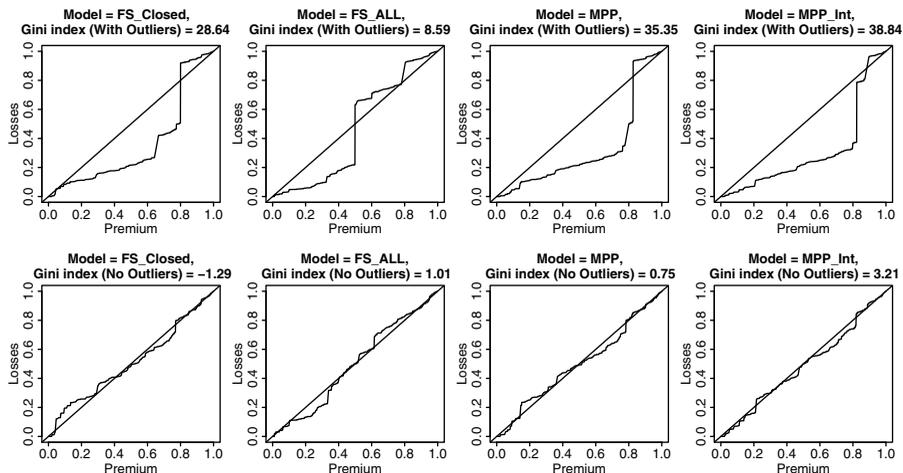
Figure 8. *Ordered Lorenz curve of projected loss cost based on 2010 hold-out-sample with EA_Premiums as base premium.*

Table 10. *Gini indices of projected loss cost based on 2010 hold-out-sample with external agency premium (EA_Premium) as base premium.*

Base premium	Effective year 2009			Effective years 2006–2009			
	FS_Closed (%)	FS_All (%)	MPP (%)	FS_Closed (%)	FS_All (%)	MPP (%)	MPP_Int (%)
	With outliers						
EA_Premium	−16.34	−0.44	0.11	28.64	8.59	35.35	38.84
Standard errors	14.04	8.76	11.00	14.14	7.89	13.76	12.19
	Without outliers						
EA_Premium	−5.72	−1.09	0.39	−1.29	1.01	0.75	3.21
Standard errors	8.77	7.32	6.98	10.25	6.16	9.16	8.70

Table 11. *Gini indices of projected loss cost for robustness check based on* 2011 *hold-out-sample with FS_All as base premium.*

Base premium	Effective year 2010			Effective years 2007–2010			
	FS_Closed (%)	MPP (%)	EA_Premiums (%)	FS_Closed (%)	MPP (%)	MPP_Int (%)	EA_Premiums (%)
FS_All	1.69	16.26	7.73	0.97	2.85	0.5	23.35
Standard errors	9.22	10.98	6.82	10.89	11.06	10.99	7.87

Table 12. *Gini indices of projected loss cost for robustness check based on* 2011 *hold-out-sample with external agency premium (EA_Premium) as base premium.*

Base premium	Effective year 2009			Effective years 2006–2009			
	FS_Closed (%)	FS_All (%)	MPP (%)	FS_Closed (%)	FS_All (%)	MPP (%)	MPP_Int (%)
EA_Premium	22.25	19.75	23.01	16.07	24.06	16.61	14.69
Standard errors	5.97	6.06	7.27	6.80	6.34	6.68	6.41

Lorenz plots in Figure 8, and the results show that the MPP framework offers a promising alternative to the existing rating methodology.

Tables 11 and 12 provide a robustness check for the results in Tables 8 and 10. Here, we use data from the effective years 2007-2010 as the training dataset and observations from the effective year 2011 as the hold-out-sample. Again, the performance of the MPP models is higher than the base models, which emphasizes that the MPP model promotes equity in rates and will lead to a more profitable portfolio.

5.4. Limitations of current framework

Based on simulations and real-world data analysis via the Marked Poisson Process, this paper has shown that extracting information from open claims leads to more accurate insurance pricing. However, the proposed framework has some limitations which will be the focus of future research and discussed below:

1. Independence assumption between claim occurrence and marks and between marks: In this paper, for simplicity, it is assumed that the claim occurrence process and the marks, such as

the reporting delay process and transaction payments, are independent. We also assume the marks are independent of each other. But, we do not need to require independence. We could allow dependence by incorporating unobservable policy-specific random effects or by using copula modeling. For example, Shi *et al*. (2016, 2022) and Frees *et al*. (2021) accommodate the dependence of the multilevel structure of claims using copula modeling, and Okine *et al*. (2022) use random effects to capture the association between the size of claims and time to settlement. However, we note that Frees *et al.* (2016) verify that dependence modeling has little influence on the claim scores under the frequency-severity model approach. The analysis of incorporating dependence modeling under the MPP framework is left to explore in the future.

2. Modeling claim and transaction frequencies with distributions other than Poisson: The Marked Poisson Framework employed in this paper means that we focus on using the Poisson distribution to model the claim and transaction frequencies. The Poisson process possesses attractive mathematical properties for insurance applications and thus is the most popular claim number process. However, insurance claim counts often experience over-dispersion. For this reason, other distributions, such as the negative binomial distribution or zero-inflated versions of distributions, are used to model the claim frequencies. One approach to modeling over-dispersion is using random effects in the intensity function to capture process dependencies and unobservable characteristics (Cook and Lawless 2007). Hence, future work will focus on incorporating over-dispersions in the proposed framework.

6. Conclusion

Through the ratemaking process, insurance rates are set to cover the total future expected cost, which includes liabilities from both RBNS and IBNR claims. Actuaries develop rates by employing multivariate risk classification techniques based on information from the policy and the claim history to promote better alignment of premiums with claims experience. But the observation from the literature is that the data used in the multivariate analysis are usually based on closed claims, where the ultimate amount paid for all claims is known, leaving out open claims. Ignoring the information from open claims could lead to inaccurate rates because the ratemaking data lacks the current information that may capture shifts in the insurer's book risk profile. Practicing actuaries are well aware of these biases and have developed cumbersome on-leveling methods to adjust the data to the current level and develop claims to ultimate during the ratemaking exercise.

This paper employs the marked Poisson process (MPP) framework, which has primarily been used for micro-level reserving, to provide formal procedures to make adjustments and improve on insurance pricing. The MPP framework specified ensures that the multivariate risk analysis is done using the information on claims that have been closed by the ratemaking date, payments on claims not yet closed, and reporting times to adjust rates to account for IBNR claims. The MPP framework specified for ratemaking purposes uses three hierarchical building blocks. The first building block drives the number of claims per policy, the second building block models the conditional number of payment transactions for a claim, and the third building block concerns the conditional payment sizes for each transaction. Each block is modeled with the appropriate GLMs. The results using data from a property insurance provider show that the proposed framework promotes equity in the ratemaking algorithm and leads to a more profitable portfolio.

In these COVID-19 pandemic times and in addition to the insurance industry experiencing a rapid pace of product innovation and intense competition, by using the information on all reported claims, the proposed approach will provide actuaries and regulators a more disciplined method of ascertaining promptly if rate increases are necessary.

Supplementary material. To view supplementary material for this article, please visit https://doi.org/10.1017/asb.2023.23.

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Appendix

Appendix A: Estimation results for the MPP model

Table A.2. presents the estimation results for the MPP model based on reported claims at the ratemaking date and following the likelihood-based method described in Section 3.1 of the paper. The upper layer of the table presents the estimation results for the claim frequency Poisson model. The baseline trend parameters and the rating variables with parameter estimates that are not significant are not shown. When using all the training data (without interaction), as expected, the coefficient for LnPolicyDed is negative, meaning higher deductible is associated with lower claim frequency, but the coefficient switches to positive and significant when using only observations from the effective year 2009. Also, the coefficients for the LnPolicyCov are positive and significant in both results. Compared to the reference category "Village," all entity types experience lower claims frequency except "Town" in both results. In addition, based on the Region rating factor, there are significant differences in claim frequency driven by the geographical location. Further, when using all the training data, the difference in the baseline parameters

Table A.1. *Weibull model parameter estimates for reporting delay (using all observations in the training data).*

	Estimate	Std. error
(Intercept)	3.461	0.075
TypeCity	0.581	0.086
TypeCounty	0.930	0.087
TypeSchool	0.292	0.085
TypeTown	0.302	0.188
TypeMisc	0.832	0.192
$\ln(\kappa)$	−0.283	0.014

Table A.2. *Parameter estimates for Poisson reported claim frequency model, censored Poisson transaction frequency model, and Log-normal transaction severity model.*

	Effective year 2009		Effective years 2006–2009			
			Without interaction		With interaction	
	Estimate	Std. error	Estimate	Std. error	Estimate	Std. error
Poisson claim frequency model						
(Intercept)	−4.983	0.272	−2.600	0.133	−2.646	0.161
α_{2007}	–	–	0.159	0.048	0.423	0.150
α_{2008}	–	–	−0.136	0.051	0.148	0.160
α_{2009}	–	–	0.114	0.055	−0.372	0.180
LnPolicyDed	0.214	0.035	−0.161	0.014	−0.164	0.014
LnPolicyCov	1.148	0.049	1.107	0.021	1.122	0.029
TypeCity	−0.595	0.195	−0.522	0.077	−0.520	0.077
TypeCounty	−0.729	0.206	−0.425	0.084	−.0422	0.085
TypeSchool	−1.412	0.197	−1.005	0.076	−1.001	0.076
TypeTown	0.089	0.474	0.421	0.149	0.409	0.149
TypeMisc	−2.209	0.364	−1.780	0.150	−1.758	0.150
RegionNorthern	−0.280	0.180	0.153	0.084	0.152	0.084
RegionSoutheastern	−1.335	0.126	0.261	0.055	0.256	0.055
RegionSouthern	0.142	0.107	0.714	0.055	0.713	0.055
RegionWestern	−0.386	0.142	0.232	0.065	0.233	0.065
AlarmCredit(0%)	–	–	−0.110	0.063	−0.115	0.063
AlarmCredit(5%)	–	–	−0.387	0.169	−0.374	0.169
AlarmCredit(10%)	–	–	−0.324	0.132	−0.321	0.132
AlarmCredit(15%)	–	–	−0.011	0.046	−0.001	0.047
$\alpha_{2007} \times$ LnPolicyCov	–	–	–	–	−0.057	0.031
$\alpha_{2008} \times$ LnPolicyCov	–	–	–	–	−0.061	0.033
$\alpha_{2009} \times$ LnPolicyCov	–	–	–	–	0.097	0.035
−2 Log L	2186		9959		9932	

α_t compared to α_{2006} captures the trend in reported claims across years. The last two columns in Table A.2 provide results using all the training data with LnPolicyCov and policy years as interaction variables. The results suggest differences in the effect of the policy coverage values on claim frequency in different years. The estimation results for the Log-normal severity model with dispersion parameter σ^2 are given in third layer of Table A.2. The dependent variable is the observed transaction payments $P_{jt,ik}$, and the results show a significant difference in claim transaction payments based on geographical location. When using all the training data, there are also differences in the transaction payments by entity types.

Table A.2. *continued*

| | Effective year 2009 | | Effective years 2006–2009 | | | |
| | Estimate | Std. error | Without interaction | | With interaction | |
			Estimate	Std. error	Estimate	Std. error
	Censored Poisson transaction frequency model					
(Intercept)	0.097	0.042	0.307	0.052	0.261	0.087
b_{2007}	–	–	−0.012	0.043	0.034	0.112
b_{2008}	–	–	−0.029	0.046	0.042	0.126
b_{2009}	–	–	−0.097	0.053	0.015	0.163
LnPolicyCov	–	–	−0.022	0.009	−0.012	0.017
$b_{2007} \times$ LnPolicyCov	–	–	–	–	−0.010	0.023
$b_{2008} \times$ LnPolicyCov	–	–	–	–	−0.016	0.026
$b_{2009} \times$ LnPolicyCov	–	–	–	–	−0.023	0.032
−2 Log L	1040		6968		6967	
	Log-normal transaction severity model					
(Intercept)	10.921	0.417	9.125	0.159	9.125	0.159
LnPolicyDed	−0.574	0.043	−0.076	0.022	−0.076	0.022
LnPolicyCov	–	–	−0.349	0.0293	−0.349	0.0293
TypeCity	–	–	0.292	0.112	0.292	0.112
TypeCounty	–	–	0.959	0.130	0.959	0.130
TypeSchool	–	–	0.523	0.118	0.523	0.118
TypeTown	–	–	−0.630	0.224	−0.630	0.224
TypeMisc	–	–	0.080	0.231	0.080	0.231
RegionNorthern	1.318	0.304	0.573	0.128	0.573	0.128
RegionSoutheastern	2.055	0.179	0.766	0.085	0.766	0.085
RegionSouthern	1.250	0.186	0.221	0.085	0.221	0.085
RegionWestern	1.315	0.240	0.340	0.098	0.340	0.098
$\ln \sigma$	0.445	0.030	0.484	0.012	0.484	0.012
−2 Log L	9361		69,295		69,295	