RUDIN-SHAPIRO SEQUENCES FOR ARBITRARY COMPACT GROUPS

For George Szekeres on his sixty-fifth birthday

J. R. McMULLEN and J. F. PRICE

(Received 26 August 1975)

Abstract

Let $G$ be a compact group. A sequence $\{f_n\}_{n=1}^\infty$ of functions in $L^\infty(G)$ is said to be a Rudin-Shapiro sequence (briefly, an RS-sequence) if the following conditions hold:

1. $\inf \|f_n\| > 0$;
2. $\sup \|f_n\| < \infty$;
3. $\lim \|f_n\| = 0$;

The main purpose here is to prove the following theorem:

**Theorem.** Let $G$ be an infinite compact group. Then $G$ has an RS-sequence consisting of trigonometric polynomials.

The proof is carried out in section 1 while in section 2 several applications are given concerning set-theoretic relations between certain function spaces in harmonic analysis. The existence of RS-sequences for infinite LCA groups is well-known.

**Notation.** Let $G$ be a compact group. The Banach space of all continuous complex-valued functions on $G$ we denote by $C(G)$, and the Banach space of all complex Radon measures on $G$ by $M(G)$; $L^1(G)$ will be identified in the usual way with the ideal in $M(G)$ of measures that are absolutely continuous with respect to normalized Haar measure $\lambda_G$.

The symbol $\hat{G}$ will denote a maximal set of pairwise inequivalent continuous irreducible unitary representations of $G$. The representation space of $\gamma \in G$ will be denoted by $H_\gamma$, and its dimension by $d_\gamma$. By $C(\hat{G})$ we mean the linear space of all “sections” over $\hat{G}$, i.e. of all those functions
\( \Phi : \hat{G} \rightarrow \Pi_{\gamma \in \hat{G}} L(H_\gamma) \) such that \( \Phi(\gamma) \in L(H_\gamma) \) for all \( \gamma \in \hat{G} \). Here of course \( L(H_\gamma) \) is the von Neumann algebra of all (bounded) linear operators on \( H_\gamma \).

The Banach spaces \( \mathcal{E}^p(\hat{G}) \) \((1 \leq p \leq \infty)\) and \( \mathcal{E}_0 \), are defined as in (28.34) of Hewitt & Ross (1970). The norms on the \( \mathcal{E}^p(\hat{G}) \) are given by

\[
\| \Phi \|_p = \sup \{ \| \Phi(\gamma) \|_p : \gamma \in \hat{G} \} \quad (\Phi \in \mathcal{E}^\infty(\hat{G}))
\]

\[
\| \Phi \|_{p, r} = \left( \sum_{\gamma \in \hat{G}} d_\gamma \| \Phi(\gamma) \|_{p, r}^r \right)^{1/r} \quad (\Phi \in \mathcal{E}^p(\hat{G}))
\]

where \( \| \|_{p, r} \) denotes the \( p \)th von Neumann-Schatten norm on \( L(H_\gamma) \). In particular \( \| A \|_{p, 2} = [\text{tr}(AA^*)]^{1/2} \), and \( \| A \|_{p, \infty} \) is the operator norm of \( A \).

The Fourier-Stieltjes transform of \( \mu \in M(G) \) we define as an element of \( \mathcal{E}^\infty(\hat{G}) \) by

\[
\hat{\mu}(\gamma) = \int_G \gamma(x^{-1})d\mu(x).
\]

and its Fourier series is the series (suitably interpreted)

\[
\mu \sim \sum_{\gamma \in \hat{G}} d_\gamma \text{tr}(\hat{\mu}(\gamma)\gamma(\cdot)).
\]

The closure of the \( n \)th derived subgroup of \( G \) we denote by \( G^{(n)} \).

1. **Proof of the theorem**

Before commencing the proof we remark that the existence of RS-sequences for infinite LCA groups is well-known; see Gaudry (1970) and (37.19b) of Hewitt & Ross (1970). Also a weaker version is known to exist for infinite compact groups. Specifically, whenever \( t \in [2, \infty] \), a sequence \( \{f_n\}_{n=1}^\infty \) of functions is said to be a \( t \)-RS-sequence if it satisfies conditions (1) and (3) of the definition of an RS-sequence with (2) replaced by

\[
(2') \quad \sup_n \| f_n \|_r < \infty.
\]

In Figà-Talamanca & Price (1972), random Fourier series are used to show that \( t \)-RS-sequences with \( t < \infty \) exist for all infinite compact groups. Also the existence of such sequences with other useful properties is demonstrated in Figà-Talamanca & Price (1972, 1973). We have not been able to generalise these extra properties to RS-sequences.

Since the definition of an RS-sequence involves only three norms, it is easily verified that any RS-sequence may be replaced by an RS-sequence consisting of trigonometric polynomials. In this section we therefore prove merely the existence of an RS-sequence for any infinite compact group.
Whenever the supports of the members of an RS-sequence are contained in some open set \( U \), then we say that this sequence is a U-RS-sequence.

Our proof begins with two special cases, from which we proceed to deduce the general case.

(1.1) Proposition. (Gaudry (1970), Lemma 2.1). Let \( G \) be an infinite compact abelian group and \( U \) a nonvoid open subset of \( G \). Then \( G \) has a U-RS-sequence.

Now, let us say that a compact group \( G \) is **tall** if for every positive integer \( d \) there are at most finitely many elements of \( \hat{G} \) of degree \( d \).

(1.2) Proposition. Let \( G \) be an infinite tall compact group and \( U \) a nonvoid open subset of \( G \). Then \( G \) has a U-RS-sequence.

**Proof.** The following construction depends on repeated applications of the fact that every measurable subset of \( G \) of positive measure has a subset of half its measure.

Let \( V \subseteq U \subseteq G \) be measurable, \( \lambda_G(V) = v > 0 \). Let \( P_1, P_2 \) be disjoint measurable subsets of \( V \) such that \( P_1 \cup P_2 = V \), \( \lambda_G(P_1) = \lambda_G(P_2) \). Let \( \pi_1 = \{P_1, P_2\} \). If \( \pi_{n-1} \) has been defined as a partition of \( U \) into \( 2^{n-1} \) subsets \( P_{n-1,i} \) \( (1 \leq i \leq 2^{n-1}) \) then form \( \pi_n \) by writing \( P_{n-1,i} \) as a disjoint union of two measurable subsets \( P_{n,2i-1}, P_{n,2i} \) of equal measure. Thus \( \pi_n \) is a set \( \{P_{n,i}: 1 \leq i \leq 2^n\} \) of pairwise disjoint measurable subsets of \( V \) of equal measure such that \( V = \bigcup_{i=1}^{2^n} P_{n,i} \).

We now define a sequence of Rademacher functions associated with the sequence \( \pi_n \). Put \( r_1 = \chi_{P_{1,1}} - \chi_{P_{1,2}} \) and more generally let \( r_n = \sum_{i=1}^{2^n} (-1)^{i+1} \chi_{P_{n,i}} \). Then \( r_n \) takes values \( \pm 1 \) and

\[
\int_{P_{n-1,i}} r_n d\lambda_G = \int_{P_{n,2i-1}} r_n d\lambda_G + \int_{P_{n,2i}} r_n d\lambda_G = 0.
\]

(This construction is also used in Figà-Talamanca and Gaudry (1970).) Clearly we have \( \|r_n\|_p = v^{1/p} \) \( (1 \leq p < \infty), \|r_n\|_\infty = 1 \), and \( \int_G r_m r_n d\lambda_G = v\delta_{mn} \) for \( m, n \in \{1, 2, \cdots\} \). Define \( f_n = v^{-\frac{1}{p}} r_n \); then we have

\[
\|f_n\|_p = v^{1/p - \frac{1}{p}}(1 \leq p < \infty),
\]

\[
\|f_n\|_\infty = v^{-\frac{1}{p}}, \quad \text{and}
\]

\[
\int_G f_m f_n d\lambda_G = \delta_{mn} \quad (m, n \in \{1, 2, \cdots\}).
\]

We claim that \( \{f_n\}_{n=1}^\infty \) is an RS-sequence under the assumption that \( G \) is tall.
Indeed, in view of (4) all that is required is to show that \( \|\hat{f}_n\|_\infty \to 0 \) as \( n \to \infty \). Now Parseval's formula for \( G \) is

\[
\|f\|_2^2 = \sum_{\gamma \in G} d_t \text{tr}(\hat{f}(\gamma)\hat{f}(\gamma)^*) \quad (f \in L^2(G)).
\]

Also, by Hewitt and Ross (1963 and 1970), (D.51) we have, for \( A \in \mathcal{L}(H) \),

\[\text{tr}(AA^*) \equiv \|A\|_{\Phi_H}^2\]

and hence

\[
(5) \quad \sum_{\gamma \in \hat{G}} d_\gamma \|\hat{f}_n(\gamma)\|_{\Phi_\gamma}^2 \equiv \|f_n\|_2^2 = 1.
\]

This makes it clear for each \( n \geq 1 \) and each \( \varepsilon > 0 \), the set \( \{\gamma \in \hat{G} : \|\hat{f}_n(\gamma)\|_{\Phi_\gamma} > \varepsilon\} \) is finite (this merely reproves the well known fact that \( \mathcal{E} \subseteq \sigma_\infty \)). Hence we may conclude that

\[
\|\hat{f}_n\|_\infty = \|\hat{f}_n(\gamma)\|_{\Phi_\gamma}
\]

for some \( \gamma_n \in \hat{G} \). Let \( \Delta = \{\gamma_n : n \geq 1\} \). If \( \Delta \) is infinite, then \( d_{\gamma_n} \to \infty \) as \( n \to \infty \), by our assumption about the representations of \( G \). Hence by (5), we have

\[
\|\hat{f}_n\|_\infty = \|\hat{f}_n(\gamma_n)\|_{\Phi_{\gamma_n}} \leq d_{\gamma_n}^{-1/2},
\]

showing that \( \{f_n\}_{n=1}^\infty \) is an RS-sequence as asserted.

In any case, since \( \{f_n\}_{n=1}^\infty \) is orthonormal in \( L^2(G) \), it follows that \( \|\hat{f}_n(\gamma)\|_{\Phi_\gamma} \to 0 \) for each \( \gamma \in \hat{G} \). Thus when \( \Delta \) is finite, we have

\[
\|\hat{f}_n\|_\infty \leq \sup_{\gamma \in \Delta} \|\hat{f}_n(\gamma)\|_{\Phi_\gamma} \to 0 \quad \text{as} \quad n \to \infty
\]

and again \( \{f_n\}_{n=1}^\infty \) is an RS-sequence. This completes the proof.

(1.3) **Lemma.** Let \( \Gamma \) be a closed subgroup of the compact group \( G \). Then there is a quasi-invariant normalised measure \( \lambda \) on the coset space \( G/\Gamma \) with the following property: if \( f \) is a nonnegative extended real-valued \( \lambda_G \)-integrable function on \( G \), then the set of cosets \( x\Gamma \) in \( G/\Gamma \) for which the function \( \xi \to f(x\xi) \) \( (\xi \in \Gamma) \) is not \( \lambda_{\Gamma} \)-integrable is \( \lambda \)-null; the function on \( G/\Gamma \) defined \( \lambda \)-a.e. by

\[
\int f(x) d\lambda_G(x) = \int_{\Gamma} \int_{\Gamma} f(x\xi) d\lambda_{\Gamma}(\xi) d\lambda(x) \Gamma).
\]

The reader is referred to Bourbaki (1963), Chapter VII, section 2, discussion following Théorème 2.

(1.4) **Lemma.** Let \( \phi : G \to G_1 \) be a continuous surjective homomorphism, where \( G \) and \( G_1 \) are compact groups and \( G_1 \) has an RS-sequence. Then \( G \) has an RS-sequence.
This is proved in Edwards & Price (1970), A.3.3.

(1.5) **Lemma.** Let \( \Gamma \) be a closed subgroup of a separable compact group \( G \), and suppose that \( \Gamma \) admits an RS-sequence. Then \( G \) does also.

**Proof.** Since \( G \) is separable, there is a Borel section \( B \) for \( \Gamma \) in \( G \) (Mackey (1951)), i.e., a Borel set \( B \) in \( G \) which meets each coset \( x\Gamma \) in exactly one point (we may assume \( B \cap \Gamma = \{e\} \)). Define a map \( b : G/\Gamma \to B \) by setting \( b(x\Gamma) \) as the unique member of \( B \cap x\Gamma \). Let \( h \in C(\Gamma) \). Put

\[
h^0(x\xi) = h(\xi) \quad (x \in B, \xi \in \Gamma).
\]

The properties of \( B \) ensure that \( h^0 \) is well-defined as an element of \( L^\infty(G) \).

Application of Lemma (1.3) shows immediately that

\[
\|h^0\|_2^2 = \int_{G/\Gamma} \|h\|_2^2 \, d\lambda(x) = \|h\|_2^2.
\]

It is also easy to see that

\[
\|h^0\|_\infty = \|h\|_\infty.
\]

Now, let \( \sigma \in \hat{G} \) be fixed. Then the restriction \( \sigma|_\Gamma \) of \( \sigma \) to \( \Gamma \) admits a decomposition

\[
\sigma|_\Gamma = \bigoplus_{\tau \in \Gamma} n_\sigma(\tau) \cdot \tau
\]

(\( n_\sigma(\tau) \) is the multiplicity of \( \tau \) in \( \sigma|_\Gamma \) and \( n_\sigma(\tau) = 0 \) for all save finitely many \( \tau \)). This decomposition is given via some unitary intertwining transformation from \( \bigoplus_{\tau \in \Gamma} n_\sigma(\tau)H_\tau \) to \( H_\sigma \) which by transport of structure gives rise to a (von Neumann) algebra isomorphism

\[
\alpha_\sigma : \bigoplus_{\tau \in \Gamma} n_\sigma(\tau)L(H_\tau) \to L(H_\sigma).
\]

Then we have by (1.3)

\[
(h_\sigma^0)^*(\sigma) = \int_G h_\sigma(x)x^{-1} \, d\mu(x) = \int_{G/\Gamma} \left( \int_{\Gamma} h_\sigma(x\xi)\sigma((x\xi)^{-1}) d\lambda_\Gamma(\xi) \right) d\lambda(\Gamma)
\]

\[
= \int_{G/\Gamma} \left( \int_{\Gamma} h(\xi)\sigma(\xi^{-1}) d\lambda_\Gamma(\xi) \right) \sigma((b(\Gamma)^{-1})\xi^{-1}) d\lambda(\Gamma)
\]

\[
= \int_{G/\Gamma} \left( \int_{\Gamma} h(\xi)\alpha_\sigma \left( \bigoplus_{\tau \in \Gamma} n_\sigma(\tau) \cdot \tau(\xi^{-1}) \right) d\lambda_\Gamma(\xi) \right) \sigma((b(\Gamma)^{-1}) d\lambda(\Gamma)
\]

\[
= \alpha_\sigma \left( \bigoplus_{\tau \in \Gamma} n_\sigma(\tau)h(\tau) \right) \int_{G/\Gamma} \sigma((b(\Gamma)^{-1})d\lambda(\Gamma).
\]
It follows that we have
\[ \| (h^0)^\prime (\sigma) \|_{\phi_\sigma} \leq \| \alpha_\sigma \left( \bigoplus \n_\tau (\tau) \hat{h} (\tau) \right) \|_{\phi_\sigma} = \| \bigoplus \n_\tau (\tau) \hat{h} (\tau) \|_{\phi_\sigma} \]
\[ = \max \{ \| \hat{h} (\tau) \|_{\phi_\sigma} : n_\tau (\tau) \neq 0 \} \leq \| \hat{h} \|_\infty \]
and hence that \( \| (h^0)^\prime \|_\infty \leq \| \hat{h} \|_\infty \). This combined with equalities (7) and (8) shows that if \{h_n\} is an RS-sequence on \( \Gamma \) (with the \( h_n \) restricted to be continuous—see the opening remarks of this section) then \{h_n^0\} is an RS-sequence on \( G \).

(1.6) LEMMA. [M. F. Hutchinson, private communication]. Let \( G \) be a prosolvable group (i.e. a projective limit of finite solvable groups) in which each derived factor \( G / G^{(n)} \) is finite. Then \( G \) is tall.

PROOF. Since \( G \) is profinite it is totally disconnected. Let \( \gamma \in \hat{G} \). Then \( \gamma(G) \) must be finite since it is a totally disconnected compact Lie group. Furthermore, since \( \gamma(G) \) is also prosolvable, it must be solvable. Let \( d \) be the degree of \( \gamma \).

By Zassenhaus (1938) there is a number \( l > 0 \) depending on \( d \) only such that the solvable length of \( \gamma(G) \) is at most \( l \). Hence, using the fact that \( G / \ker \gamma \) and \( \gamma(G) \) are isomorphic, \( G^{(0)} \subseteq \ker \gamma \).

Now let \( d \) be a fixed positive integer. The members of \( \{ \gamma \in \hat{G} : d_\gamma = d \} \) must all satisfy \( G^{(0)} \subseteq \ker \gamma \) where \( l = l(d) \) and hence this set corresponds under an obvious injective map to a subset of \( (G / G^{(0)})^* \). But the latter set is finite by the hypothesis, and the lemma is proved.

(1.7) CONCLUSION OF THE PROOF. Let \( G \) be an infinite compact group. Then \( G \) has an infinite separable compact quotient group [Hewitt & Ross (1963), Theorem (8.7)]. Lemma (1.4) indicates that it is enough then to prove the theorem under the assumption that \( G \) is separable.

According to McMullen (1974), \( G \) either has an infinite abelian subgroup or an infinite closed topologically-2-generator pro-\( p \) torsion subgroup (\( p \) an odd prime). In either case, let us call the subgroup in question \( \Gamma \). In the first case, \( \Gamma \) has an RS-sequence by proposition (1.1). In the second, the same conclusion follows from lemma (1.6) and proposition (1.2).

Since \( G \) is separable, the theorem now follows from Lemma (1.5).
2. Applications

Techniques for applying RS-sequences to problems in harmonic analysis are well-known. For example, see Hewitt & Ross (1970), (37.19), Gaudry (1970), Edwards & Price (1970) and Figà-Talamanca & Price (1972).

Here we sketch the details of three applications.

APPLICATION A. (2.1) In the case of compact groups, the Hausdorff-Young theorem states that $\hat{f} \in \mathcal{E}^p$ whenever $f \in L^p$, $1 \leq p \leq 2$, and $1/p + 1/p' = 1$. Thus if $f \in C(G)$, then $\hat{f} \in \mathcal{E}^q$ for all $q \in [2, \infty]$. When $G$ is infinite and abelian this is known to be best possible in the sense that there exists $f \in C(G)$ such that $\hat{f}$ belongs to no $\mathcal{E}^q$ for $q \in [1, 2]$ (see Hewitt & Ross (1973), (37.19(c)) where an extension of this result is given for all locally compact abelian groups).

(2.2) Proposition. Whenever $G$ is an infinite compact group there exists $f \in C(G)$ such that $\hat{f}$ belongs to no $\mathcal{E}^q$ for $q \in [1, 2]$.

The proof will use the following lemma, the proof of which follows directly from the definitions of the $\mathcal{E}^p$ and their respective norms.

Lemma. If $\phi \in \mathcal{E}^p$, where $1 \leq p < \infty$, then $\phi \in \mathcal{E}^q$ for all $p \leq q \leq \infty$ and moreover

$$\|\phi\|_q^p \leq \|\phi\|_p \|\phi\|_q^{q-p}$$

holds for $p \leq q < \infty$.

Proof of (2.2). Suppose that the statement of the proposition is not valid, that is, that $f \rightarrow \hat{f}$ defines a map from $C(G)$ into $\bigcup \{E^n : 1 \leq q < 2\}$. Let $\{q_n\}$ be a sequence in $[1, 2]$ which approaches 2 monotonically; then $\bigcup \{E^n : 1 \leq q < 2\} = \bigcup \{E^n : n \geq 1\}$. A direct application of Edwards (1965), Theorem 6.5.5, with $E = \mathcal{E}^2$, $F = C(G)$, $u = \text{Fourier transform}$, $F_n = \mathcal{E}^n$ and $u_n$ the identity map on $F_n$ shows that there exists an integer $k$ such that $C(G)^* \subseteq \mathcal{E}^n$. It now follows from the closed graph theorem that for some $K > 0$ we have

$$\|\hat{f}\|_{q_n} \leq K \|f\|_n$$

for all $f \in C(G)$. Let $f_n$ be an RS-sequence consisting of continuous functions. Then there exist $m, M > 0$ such that

$$m \leq \|f_n\|_2 \leq \|f_n\|_\infty \leq M$$

for all $n \geq 1$. From the preceding lemma, we have

$$\|f_n\|^2 = \|\hat{f}_n\|^2 \leq \|\hat{f}_n\|^2_{q_n} \|\hat{f}_n\|_{2-q_n}^2$$
and so we have
\[ \| \hat{f}_n \|_{q_k} \geq \| f_n \|^2 q_k / \| \hat{f}_n \|^2 q_k \to \infty \quad \text{as} \quad n \to \infty \]
since $1 \leq q_k < 2$. But this contradicts (9), in view of the fact that $\| f_n \|_\infty \leq M$, for $n \geq 1$.

**APPLICATION B.** (2.3) The Fourier transform $f \to \hat{f}$ carries $M$ into $C^\infty$, $L^1$ into $C_0$ and $L^p$ into $C^p$ when $1 < p < 2$. It is known that these maps are surjective if and only if $G$ is finite [Hewitt & Ross (1970), (37.4) and (37.19 (a))]; a direct proof of these facts follows from the existence of an RS-sequence. The surjectivity of the maps is trivial when $G$ is finite.

(2.4) **PROPOSITION.** Let $G$ be an infinite compact group. The images of $M(G)$, $L^1(G)$ and $L^p(G)$ ($1 < p < 2$) under the Fourier transform are properly contained in $C^\infty$, $C_0$, and $C^p$ respectively.

**PROOF.** The proofs are similar in detail to the second part of the proof of the proposition (2.2): one assumes the contrary, establishes an inequality analogous to (9), and obtains a contradiction by substituting therein the members of an RS-sequence.

**APPLICATION C.** (2.5) Given $p, q \in [1, \infty]$, then $\phi \in \mathcal{E}$ is said to be a $(p, q)$-multiplier if
\[ \sum_{\gamma \in \hat{G}} d(\gamma) [\phi(\gamma) \hat{f}(\gamma) \gamma(\cdot)] \]
is the Fourier series of a function in $L^q$ whenever $f \in L^p$ (an equivalent definition is available which makes sense for arbitrary locally compact groups). For example, it is well-known that $\hat{\mu}$ is a $(p, p)$-multiplier for all $p \in [1, \infty]$ whenever $\mu \in M(G)$. On the other hand, when $G$ is an infinite compact abelian group there exist functions in $C$ which are $(p, q)$-multipliers for all $p \in [1, \infty]$ and all $q \in [1, \infty]$, which are not Fourier-Stieltjes transforms; see Brainerd & Edwards (1966), Theorem (4.15). The proof is based upon the existence of an infinite Sidon set. In fact, given the existence of certain lacunary subsets of $\hat{G}$, examples of functions in $C$ with the preceding properties can easily be produced; c.f. (37.22) of Hewitt & Ross (1970). However, there exist compact groups whose duals possess no reasonable lacunary sets. Suppose that a locally compact group has the property of possessing an RS-sequence of functions whose supports are contained in a fixed compact set. Then Theorem 5.7 of Edwards & Price (1970) shows that for such groups there exists a $(p, q)$-multiplier for all pairs $(p, q)$ such that
1 < p ≤ q < ∞, but which is not a Fourier-Stieltjes transform. Since we have seen that every infinite compact group has an RS-sequence we have:

(2.6) PROPOSITION. Let G be an infinite compact group. There exists a function in $\mathcal{E}$ which is a $(p, q)$-multiplier for all pairs $(p, q)$ such that $1 < p ≤ q < ∞$, but is not the Fourier-Stieltjes transform of any measure.

(2.7) REMARKS (i). Proposition (2.6) improves Theorem 4.3 of Figà-Talamanca & Price (1972), the proof of which was based on the existence of $t$-RS-sequences, $t > ∞$, as defined above. As a consequence of the existence of these restricted RS-sequences, roughly all that could be shown was that there exist multipliers of the type under question which are not Fourier transforms of any element in $\mathcal{U}\{L^r : 1 < r ≤ ∞\}$.

(ii) Further cases of proposition (2.6) for infinite compact groups are accounted for by noting that all functions in $\mathcal{E}^*$ are $(p, q)$-multipliers when $1 ≤ q ≤ 2 ≤ p ≤ ∞$ (Table (36.20) of Hewitt & Ross (1970)), whereas by proposition (2.4) there exist elements in $\mathcal{E}^*$ which are not Fourier-Stieltjes transforms.

(iii) In the event of the existence of a $U$-RS-sequence where $U$ is some open subset of $G$ (see propositions (1.1) and (1.2)), application A can be strengthened to show that there exists $f \in C(G)$ with support in $\bar{U}$ such that $\hat{f}$ belong to no $\mathcal{E}^*$ with $q \in [1, 2]$. Applications B and C can be improved in a like manner.

References


University of Sydney,
Sydney, N.S.W. 2006,
Australia

and

University of New South Wales,
Kensington, N.S.W. 2033,
Australia