BOOK REVIEWS

FREMLIN, D. H., *Topological Riesz Spaces and Measure Theory* (Cambridge, 1974), xiv+266 pp., £5·90.

Using the theory of Riesz spaces and methods of functional analysis, the author develops an abstract version of measure theory in which he is able to interpret all the more conventional ideas of the subject. The level of abstraction is justified not only by the generality of the results obtained but also because the development appears natural from a functional analytic point of view; indeed the author states in his preface: " This book is addressed to functional analysts who would like to understand better the application of their subject to the older discipline of measure theory ".

The basic ideas of this subject are developed in Chapters 4, 5 and 6. Starting with a Boolean ring and its Stone representation, the author constructs a linear space S which is effectively a space of " simple functions ". This is completed in the uniform norm to give L^∞ and $L^\#$ is defined as the Riesz space dual $L^{\infty\times}$. Measure rings are then considered and L^1 is represented as the closure of an embedding of a certain subspace of S in $L^\#$. Finally the ordinary measure space appears. It has an associated measure algebra and most of the subsequent development consists of carrying over to the measure space the abstract material established for the measure algebra.

Chapters 1, 2 and 3 discuss Riesz spaces, the types of topologies which are encountered and certain natural classes of linear mappings and functionals on such spaces. These chapters occupy more than a third of the text and, although the material has been selected with a view to its subsequent application to measure theory, they provide a good introduction to the general theory of topological Riesz spaces.

Chapter 7 deals with the representation of sequentially smooth and smooth linear functionals on function spaces. Radon measures and quasi-Radon measures are considered and the Riesz theorem is obtained. Weak compactness is the topic of Chapter 8. This is considered in the Riesz space duals $E^\sim$ and $E^\times$ of a Riesz space E. The results are then applied and extended to give criteria for weak compactness in the various special spaces considered in the previous chapters.

The book has been most carefully written. The proofs of theorems, although often intricate, have been well set out, the individual stages being separated by clear statements of the hypotheses or problems at each stage. Each section also has three useful features: an initial statement of its overall aim, a number of exercises and a concluding subsection of notes and comments. Each chapter ends with a series of examples to illustrate the results and definitions. There are a few minor misprints, none of which should seriously upset the reader's comprehension. The text appears to be mathematically sound.

As well as presenting his material, the author seems to be arguing a case for his approach to measure theory. His book is not for the beginner, but how far he succeeds otherwise must be left to the individual reader to judge. Certainly the reviewer enjoyed reading it and was not unimpressed by his arguments.

I. TWEDDLE

WALKER, P. L., *An Introduction to Complex Analysis* (Adam Hilger, London, 1974), viii+141 pp., £4·50.

This book deals with what the author in his preface calls " the elementary part of complex analysis "; that is, " those concepts which are necessary to comprehend, prove and apply the residue theorem". Although the residue theorem is of great importance to applied mathematicians and engineers, this book is addressed to the embryonic pure mathematician. The reader is assumed to be familiar with the basic results of real analysis and to have a certain expertise in applying such results as the Weierstrass M-test and the Cauchy nth root test. These tools are used to develop the material in the text in a rigorous manner. Many of the problems in a precise treatment of complex analysis are of a topological nature but these have been largely avoided; for instance, in the main part of the text, starred regions are used and simple connectedness is shunted into an appendix. However, some point-set topology is unavoidable.

Chapter 1 deals with various concepts which are used extensively in the sequel. Open and closed sets in the complex plane are defined, the various equivalent formulations of compactness obtained and (path-) connectedness introduced. There is also a collection of results on the calculus of functions of a complex variable. This chapter should probably be referred to as required rather than read straight through.

Chapter 2 contains Cauchy's theorem for closed paths in starred domains, the Cauchy integral formula, Morera's theorem and Liouville's theorem. Chapter 3 deals with Taylor's theorem and Laurent expansions. These are used in Chapter 4 to study the zeros and singularities of functions and this leads naturally to the definition of a residue.

Chapter 5 deals with the residue theorem. The theorem is prefaced by a detailed discussion of the winding number (topological index) of a closed path. The residue theorem is then proved for starred regions. There follow all the usual applications of the theorem to the evaluation of real integrals, trigonometric integrals and the summation of series. The chapter concludes with a section on Rouché's theorem, the ubiquity theorem, the maximum-modulus theorem and results on the inverse of a regular function.

Chapter 6 considers the connection between regular functions and harmonic functions. The Dirichlet problem for Laplace's equation in a disc is solved and the chapter concludes with a proof of the Riesz conjugation theorem concerning the continuity of harmonic conjugation with respect to the pth mean $(1 < p < \infty)$.

Then come three appendices. Appendix A discusses the so-called regulated integral which provides an integration process capable of handling all the functions encountered in the main part of the text. Appendix B deals with some of the topological problems which were side-stepped previously; in particular, simple-connectedness is shown to be the natural condition for Cauchy's theorem in its general form. The Jordan curve theorem is also stated. Finally in Appendix C, there is a discussion of logarithms and fractional powers. Whenever these threaten to appear in the main part of the book, they are prevented from doing so by a change of variable or some other subterfuge. For instance, in Example 5.16 a substitution means that a "keyhole" contour can be avoided. Riemann surfaces and analytic continuation are therefore not mentioned.

Worked examples are sprinkled throughout the text and at the end of each chapter there is an ample supply of exercises for the reader with hints for some of the more complicated investigations.

The suitably qualified reader will find most of the text quite readable, but there are a few places where the going gets tougher. For instance, the last part of the proof of Theorem 5.7 might prove baffling while Theorem B.4 in Appendix B will tax many a student. The latter is one of several instances where a sketch might have been of assistance to the less able reader. There are one or two unfortunate clashes of notation; for example, γ' is used in two different senses within a space of three lines at the top of p. 16. Also, around thirty errors, mainly typographical, were detected; most of these are obvious but those in Definition 5.6 (wrong sign) and Theorem 5.7 (C_2 traversed in the wrong direction) will cause trouble.

The main criticism, however, concerns the cost. Certainly the book is beautifully produced. But when one considers that the main body of the text occupies 106 pages (the rest being appendices, bibliography, indices, etc.), £4·50 seems a tall order even in this day and age. Since books covering essentially the same material are available at under half the price, the high cost may well militate against the recommendation of the book as a text for students.

ADAM C. MCBRIDE

ANDERSON, F. W. AND FULLER, K. R., *Rings and Categories of Modules* (Graduate Texts in Mathematics, vol. 13, Springer-Verlag, 1974), ix+339 pp., DM.36.30, $14.80.

Beginning with the definition, rings are studied from the standpoint of their categories of modules. Care is taken not to become more involved than necessary in general category theory, and categorical ideas are introduced only when they are needed. For example, natural transformations of functors are discussed in § 20 and Morita equivalence in § 21.

Of course one would expect the Morita theorems to be included in such a book, and so they are, but there is much more besides, as the following summary of the contents shows: §§ 1-8, rings, modules and homomorphisms; endomorphism rings; direct sums and products; essential and superfluous submodules; generators and cogenerators; trace and reject: §§ 9-15, semisimple modules; finitely generated, finitely cogenerated (i.e. finitely embedded), artinian and noetherian modules; modules with finite length; indecomposable decompositions; semisimple artinian rings and primitive rings: §§ 16-19, hom and tensor functors; projective, injective and flat modules; projective covers and injective envelopes: §§ 20-24, natural transformations; Morita equivalence and duality: §§ 25-26, direct sums of projective, injective and countably generated modules; characterisations of noetherian rings: §§ 27-29, semiperfect and perfect rings; modules with perfect endomorphism rings.

This is not a book for the expert only, although he will find it a worthwhile addition to his library. The beginner will find this an attractive, well-motivated and informative introduction and account of quite a substantial part of the theory of rings and modules. It is particularly well suited to an M.Sc. course as the presentation is clear and at the end of each section there is a wide selection of exercises to test the understanding.

P. F. SMITH

BERBERIAN, S. K., *Lectures in Functional Analysis and Operator Theory* (Graduate Texts in Mathematics Vol. 15, Springer-Verlag, 1974), ix+345 pp., DM38.50, $15.70.

This is the fifth textbook in analysis that Professor Berberian has written, and it is of the high standard which we have come to expect from the author. In Chapter 1, the basic theory of topological groups is developed up to but excluding the introduction of Haar measure. Topics discussed include neighbourhoods of the identity, subgroups and quotient groups, uniformity in topological groups and metrisability of topological groups. Chapter 2 is devoted to the basic theory of topological vector spaces over the real and complex fields. There are sections on metrisable topological vector spaces, spaces of type (F), normed spaces, Banach spaces, hyperplanes and linear forms, finite-dimensional topological vector spaces and Riesz's theorem. Chapter 3 is entitled " Convexity ". There are sections on convex sets, Kolmogorov's normability criterion, the Hahn-Banach theorem, invariant means, generalised limits, ordered

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