# FORUM

# A Simple Graphical Solution for Satellite Orbits

## *from* Frank George

THE first amazement at hearing of the successful launching of the Russian Earth-satellite in October may have been followed in the reader's mind by some bewilderment at its rapid and seemingly erratic progress over all the world's capitals, large cities and outlying island groups. One supposed that a mathematical prediction would be possible but hesitated to attempt anything so complex. Yet, with some simplifying assumptions which must in any case be rather closely approximated by any near satellite that can continue to rotate in orbit, the problem is not difficult and lends itself to graphical treatment. The construction here proposed will give a good insight into the behaviour of such satellites and provide reasonably accurate predictions for a day or two ahead.

In addition to treating the Earth as a sphere the following assumptions are made:

(i) That the satellite is in a free orbit which lies in a plane, containing the Earth's centre, and that this plane remains fixed while the Earth rotates on its axis;

(ii) That the orbit is a circle (concentric with the Earth) and that the velocity of the satellite is constant.

Deviations from the first of these assumptions will necessarily arise from perturbations by the Sun and Moon and also from irregularities in the Earth's gravitational field. Over any short period, however, the departure from motion in a plane will be negligible. A further assumption could be made that the plane itself rotates slowly in some appropriate way; but with this we are not concerned for relatively short-term predictions. The second assumption is also unlikely to be satisfied exactly since the laws of motion require only that the orbit should be an ellipse with one focus at the centre of attraction. This would lead to an inconstant velocity in the orbit and further variations will arise from perturbations and the irregularities of the gravitational field. Still, if the orbit is everywhere within a few hundreds of miles of the Earth's surface, it cannot differ very much from a circle. It is the irregularity of angular velocity which concerns us here and gives rise to a situation analogous to the equation of time in navigational astronomy. One may imagine a 'true' satellite and a 'mean' satellite but it would not seem that they are likely to be separated by more than one or two degrees of the orbit, a distance which the satellite would cover in less than a minute of time.

We may therefore adopt these two simplifying assumptions, while recognizing that it is the departure from them that will chiefly interest the astronomers and the geophysicists.

The method now proposed serves to trace the successive points at which the satellite is overhead; it does not deal with its height above the Earth. The



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method will indicate the time of a transit or near transit, the quarter of the sky in which it is to be found and whether it will be high or low in the sky; it will not provide a prediction of the satellite's azimuths and angular elevations from a given station. The point where the satellite is overhead lies on the intersection of its orbital plane with the Earth's surface, and therefore on a terrestrial great circle. But, because the Earth rotates on its axis, while this orbital plane does not, the great circle will itself move continuously westwards over the Earth's surface. The method consists in locating this circle on a world map after any given interval has elapsed and then fixing the position (of the 'mean satellite') on this circle. The most convenient base is a polar stereographic projection, in which the meridians are straight lines and the parallels of latitude concentric circles. All other great circles on the sphere are circles on the projection.

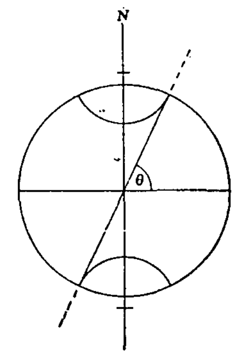


*Fig. 1*

Two elements are sufficient to define the motion of the satellite; the inclination of its orbital plane to the plane of the equator and its period of rotation. These elements are assumed to remain constant and for the first Russian satellite they were said to be 64° 20′ and $96^m\ 10^s$ respectively. It is also necessary to know which way the satellite goes round in its orbit; in this instance it was, and usually will be, anticlockwise as seen from the north pole, since this requires a lower launching velocity relative to the Earth.

Turning now to Fig. 1, which is an equatorial stereographic projection, it will be seen that the most northerly and southerly points on the great circle of intersection with the orbital plane are in $\theta$ N. or S., where $\theta$ is the inclination of the orbit. This is analogous to the tropics of Cancer and Capricorn in relation to the Sun's motion and we may call these parallels the 'tropical latitudes' for a given satellite. Fig. 2, which is a polar projection, shows how the orbit circle touches the two tropical latitudes. It may be noted that the radius of a parallel of latitude $\phi°$ on a polar stereographic projection is $r \tan \frac{1}{2} (90° - \phi)$, where $r$ is the radius of the equator. If the orbit circle is drawn on tracing paper and pivoted at the pole of the projection it can evidently be made to occupy the successive positions of the satellite's orbit.

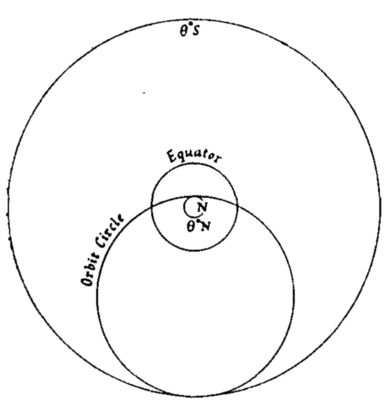


*Fig. 2*

It remains to subdivide the orbit circle and sixteen equal intervals of the orbit will be convenient. For the first satellite with a period of $96^m\ 10^s$ these intervals will be almost exactly $6^m$, or $0.1^h$. Unfortunately the divisions of the orbit are not equally spaced on the orbit circle of the projection. As those familiar with map projections will know, it is necessary to find the pole of the orbit circle and a set of equally spaced orthogonal circles through this point. The construction is not difficult and proceeds as follows (Fig. 3):

Let N once more be the north pole; draw RS perpendicular to a diameter

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through N; draw tangents at R and S to cut the diameter in Z; draw ZX perpendicular to the diameter; let an arc, centre Z radius ZS, cut the diameter in P, and ZX in $C_2$ and $C_2'$; draw the angles Z $PC_1 = 22\frac{1}{2}°$ and Z $PC_3 = 67\frac{1}{2}°$; similarly find the points $C_1'$ and $C_3'$; draw arcs through P with centres $C_1$, $C_2$, $C_3$ . . ., &c. to cut the orbit circle.

It can be shown that these arcs, together with the extremities of the diameter and the points R and S, provide the required sixteen divisions of the orbit. P is the pole of the orbit and evidently lies in latitude $(90° - \theta)$ N. or S. It will be convenient to number the divisions in minutes, 6, 12, 18 . . . 96, starting and ending at R, the Ascending Node, the point where the orbit cuts the equator (satellite moving into NE. quartant).

This completes the graphical solution of the problem and we can now follow a satellite round its orbit from any initial point of observation, remembering to rotate the orbit-circle itself, westwards, by an appropriate amount. It will be convenient to divide the circumference of the map into intervals of longitude equal to the periodic time of the satellite and to mark angles equal to the sixteen sub-divisions at the edge of the tracing paper (see Fig. 3).

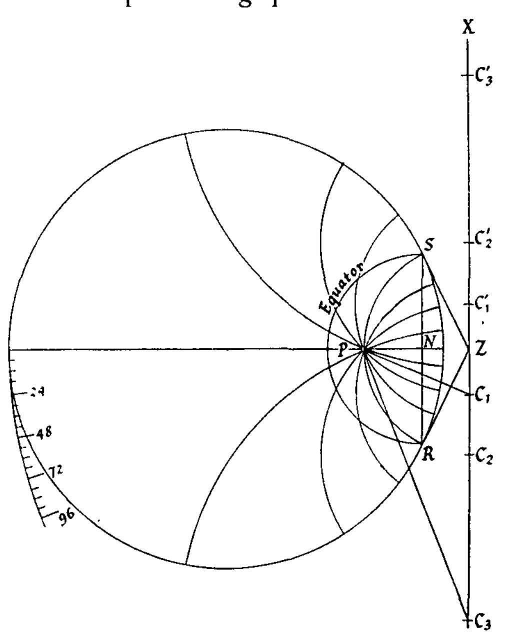


*Fig. 3. Subdivision of the orbit-circle for $\theta = 64° 20'$.*

It is an interesting point, though of no significance for this construction, that the rotation of the Earth about its axis should be taken as about $23^h\ 56^m$ and not $24^h$, as for the conversion of mean and siderial time. We are concerned with the transit of a plane which we have assumed to remain fixed with respect to the first point of Aries, whereas mean time is related to the transit of the Sun. In one rotation of the satellite, however, the difference in longitude will only be a few minutes of arc and we have probably already neglected larger irregularities in the satellite's motion. That the period of the Russian satellite ($96^m\ 10^s$) was so nearly a sub-multiple of 24 hours was presumably fortuitous, but the close correspondence of transits at the same places on successive days can be seen from Table I, extracted from the predictions broadcast by Moscow Radio, as reported in the *Daily Telegraph*. This correspondence suggests that the simplifying assumptions we have made are valid over periods of several days at least.

A practical disadvantage of the method of construction described above is that the scale of the northern hemisphere will be very small if the figure is to extend to the southern 'tropical latitude'. If, however, one is satisfied with predictions for a single hemisphere, it would be well to use a published hemi-

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sphere map on the stereographic projection, and only that part of the orbit circle which falls in the chosen hemisphere. To transfer from one hemisphere to the other would be rather awkward and the method, though perhaps more practical, is less instructive. Other projections could also be used for the base map, but with no obvious advantages. For the Mercator chart, the orbit will plot as an exponential curve not unlike a sine-curve; it is to be moved westwards parallel to the equator. It seems probable that future artificial satellites will have orbits and motions not very different from the first. For each new satellite it will be necessary to draw a new orbit circle for the appropriate angle of inclination ($\theta$) and to graduate the circumference of the map in terms of the new period.

Finally, this graphical treatment of the problem will show how, at least in theory, the characteristics of a new satellite (inclination and period) can be inferred from two well spaced observations of transit within a single rotation. The angular distance between the two stations and the elapsed time between the observations will give a first approximation to the period. The position of the first station is then to be carried westwards (as if it were a position line) to the time of the second observation. The orbit circle is now drawn through these two points on the map and through their antipodes. This will determine the 'tropical latitudes' and hence the inclination of the orbit. Of course, a considerable number of such observed transits would be needed for a reliable result.

TABLE I. PREDICTED TRANSITS (G.M.T.)

| Station | Oct. 9 | Oct. 10 | Oct. 11 |
|---|---|---|---|
| Fiji | 08h 11m | 08h 08m | h m |
| Coral Harbour | — | 08 36 | 08 35 |
| Fairbanks | 11 48 | 11 47 | 11 45 |
| Detroit | — | 11 57 | 11 55 |
| Vladivostok | 13 15 | 13 14 | — |
| Denver | — | 13 35 | 13 33 |
| San Francisco | 15 14 | 15 12 | — |
| Karachi | — | 16 22 | 16 19 |
| Rio de Janiero | 22 34 | 22 32 | — |
| Archangel | 23 01 | 23 00 | 22 56 |

*Mr. D. H. Sadler comments:*

The basic principles of the above graphical solution are sound, although the simplifying assumptions limit severely both the range of applicability and the accuracy of prediction obtainable. It will, however, suffice to give a general picture. The method has something in common with the admittedly rather crude predictions currently being issued by H.M. Nautical Almanac Office. On a basic conical orthomorphic projection a mean track (of approximately the correct inclination and eccentricity) is constructed, marked with heights above the surface of the Earth and times from apex (i.e. greatest northern latitude) passage; actual predictions take the form, for each evolution of the satellite, of the time of apex passage at that time. These dates enable the observer's position to be plotted relative to the track, thus providing full information of the satellite's path relative to the observer.

The determination of a full representative orbit is a much more complicated task; of the six elements (inclination, eccentricity, longitudes of node and perigee, period and initial time) necessary to determine an instantaneous orbit, all but two change fairly rapidly. Many accurate observations are required before such an orbit can be obtained, and even then it cannot be used for predictions for very long.

Captain F. J. Wylie, R.N.(ret.), the Institute's new President.

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