# ABOUT THE SECULAR ACCELERATION OF MIMAS

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**Abstract.** We explain the high values of the acceleration ($\approx 2°$ cy$^{-2}$) found in the longitude of Mimas by Kozai and Dourneau when they fit to observations their current theory of the Mimas' motion. In fact, we have found that very long-period terms are missing in these theories; their expansion in powers of $t$ well agrees with the observed acceleration. Effects of tidal dissipation are far smaller and could be determined only after accounting of these long-period terms.

**Key words:** Mimas - analytical theory - secular perturbations

## 1. Introduction

Mimas and Tethys are strongly connected by an 1:2 inclination-type resonance: Observations show that the librating argument : $\phi = 2\lambda_{Mim} - 4\lambda_{Tet} + \Omega_{Mim} + \Omega_{Tet}$ oscillates slowly around zero with a great amplitude : $\phi \approx 95.^{\circ}4 \sin\omega_1$ where $\dot{\omega}_1 = 5.^{\circ}067$ per year (period 70.6 years). Besides the resonance, Mimas and Tethys are affected by the large oblateness of Saturn. Thus, their orbits are given in the current representation (Dourneau 1987) by slowly precessing ellipses with semi-major axes, eccentricities and inclinations assumed as constants. The resonance modify the mean longitudes only. In the Mimas' one, Dourneau gives :

$$\lambda_1 \;=\; \lambda_{o1} + \quad N_1\, t + \quad (s_1\, t^2) - 43.^{\circ}57 \sin\omega_1 - 0.^{\circ}720 \sin 3\omega_1 - 0.^{\circ}021 \sin 5\omega_1$$

When we consider the gravitational effects only, there is no secular acceleration ($s_1 = 0$). But, when Kozai (1957) and Dourneau (1987) have compared their theory to observations, they have found an acceleration in the mean longitude of Mimas equal to $2.^{\circ}8$ cy$^{-2}$ and $2.^{\circ}1$ cy$^{-2}$ respectively. In fact, Duriez (1990) has shown that very long-period terms are missing in the expression of $\lambda_1$ given above, and Vienne (1991) has explicited them extensively in a new analytical theory of the Mimas' motion. The greatest ones are given in Table 1. In fact, their expansion in power of $t$ agrees approximately with the observed acceleration, while the similar effects coming from tidal dissipation are far smaller : $s_1 \leq 0.^{\circ}07$ cy$^{-2}$, and could be really determined only after accounting of these long-period terms.

## 2. New analytical theory of the Saturn's satellites

A new general analytical theory was elaborated by Vienne (1991) for all major Saturn's satellites (except Hyperion). The aim of this work is to give a coherent representation for the motions of all satellites, considered all together, with a precision compatible with spatial observations. The method is described in (Duriez & Vienne 1991): Like in general planetary theory, the Lagrange equations are constructed analytically up to degree six in eccentricities and inclinations and to order 2 in the masses and in the oblateness parameters ($J_2$, $J_4$ and $J_6$). All terms greater than 1 km have been retained in the critical system which collects all secular, resonant and solar perturbations. Then, the result of a numerical integration with a



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TABLE I

Solution for $A_1 \times \Delta\lambda_1$ (mean longitude of Mimas). The fundamental arguments $\omega_1$, $\phi_1$, $\phi_3$ and $\Phi_1$ concern the libration Mimas-Tethys, the pericenter of Mimas, the pericenter of Tethys and the node of Mimas respectively. The series is expressed in sinus of arguments.

| n° | phase (deg) | period (years) | frequency (rad/year) | identification | amplitude (km) |
|---|---|---|---|---|---|
| 1 | 39.059 | 70.636 | .0889514 | $\omega_1$ | 141196.85* |
| 2 | 117.182 | 23.545 | .2668535 | $3\omega_1$ | 2303.97* |
| 3 | 82.656 | 109.536 | .0573619 | $2\Phi_1 + \phi_3 + \omega_1$ | 1217.79 |
| 4 | 355.641 | 52.127 | .1205358 | $-2\Phi_1 - \phi_3 + \omega_1$ | −840.38 |
| 5 | 316.370 | 198.892 | .0315910 | $-2\Phi_1 - \phi_3$ | −628.46 |
| 6 | 120.914 | .616 | 10.1976787 | $\phi_1$ | 424.77 |
| 7 | 81.967 | .622 | 10.1087233 | $\phi_1 - \omega_1$ | −197.69 |
| 8 | 159.860 | .611 | 10.2866323 | $\phi_1 + \omega_1$ | 190.68 |
| 9 | 78.160 | 35.319 | .1778996 | $2\omega_1$ | 133.93 |
| 10 | 121.530 | 42.939 | .1463284 | $2\Phi_1 + \phi_3 + 2\omega_1$ | −66.76 |
| 11 | 194.921 | 14.127 | .4447679 | $5\omega_1$ | 64.13* |
| 12 | 74.615 | 21.056 | .2983993 | $-2\Phi_1 - \phi_3 + 3\omega_1$ | −50.46 |
| 21 | 93.402 | 710.788 | .0088397 | $\phi_1 + 2\Phi_1 - \phi_3$ | 30.68 |

4d-step over 1200 y for the inner satellites (over 9000 y with a 100d-step for Rhea, Titan and Iapetus) is analysed by Fourier techniques (based on the routine MFT elaborated by Laskar, 1988); the frequency of each term is then reconstructed as integer combinations of the fundamental frequencies, leading to a semi-numerical representations of all orbital elements. The internal precision of this representation has been estimated to a few kilometers. The partial derivatives of this representation with respect to all physical parameters and to initial conditions have also been computed, which will allow to adjust them when fitting this new theory to observations. Besides, all short-period terms that lead to perturbations on positions greater than 0.1 km have been computed (Vienne & Duriez 1991a-b).

The representation of the mean longitude of Mimas as Fourier series is given in Table 1 (limited here to perturbations greater than 50 km). Terms with an asterisk are the only ones already considered in the current representation (Dourneau 1987). We see numerous terms which are far to be negligible. Some of them were already found in previous works : the terms n° 3, 4, 5, 10 and 21, and that with argument $\phi_1$ were already detected by Duriez (1990); the last one was also shown by Jefferys and Ries (1979), as well as the terms with arguments $\phi_1 \pm \omega_1$; at last, Stellmacher (1982) had detected the argument $2\omega_1$.

We must emphasize that the arguments $\phi_1$, $\phi_3$ and $\Phi_1$ have periods in the order of year (respectively 0.616 y, 1.237 y and 2.459 y) and that many terms have periods greater than 100 years (up to 700 years). So, at the scale of this system, these periods are very long, and perhaps some secular resonances exist here.

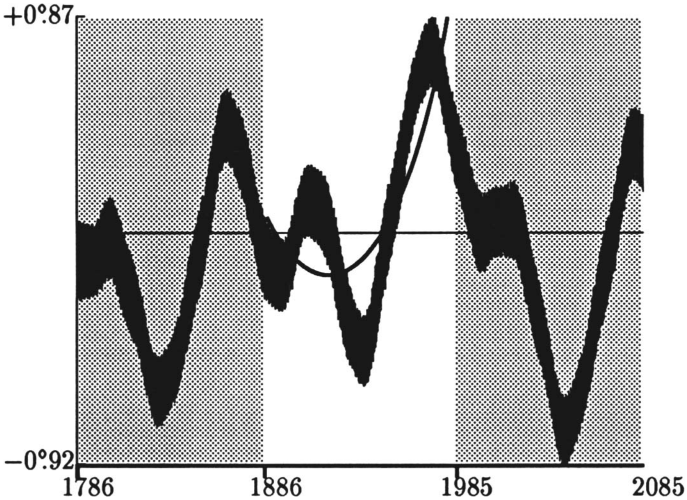


Fig. 1. "Acceleration" in the mean longitude of Mimas equal to 2.33 ± .11 deg $cy^{-2}$, obtained by fitting, over one century, a parabola to the curve which represents the sum of the terms absent from the current theories.

The argument $2\Phi_1 + \phi_3$ (period 200 years) is particularly interesting. It appears in the mean longitude, in combination with the libration argument $\omega_1$. Because this argument contains the pericenter of Tethys, the amplitudes of the corresponding terms depend on its eccentricity, here taken equal to $\approx 0.001$ like Sinclair (1977); however Dourneau and other authors consider the Tethys' orbit as circular. Hence, it would be usefull to determine the precise value of the Tethys' eccentricity. Because the Tethys' eccentricity seems to have here more influence on the long period terms of the mean longitude of Mimas than on the motion of Tethys itself, a good determination of these terms could be used to determine the Tethys' eccentricity.

## 3. Acceleration in the mean longitude of Mimas

As these long-period terms are not negligible, let us see how they affect the determination of the acceleration done by Kozai (1957) and Dourneau (1987).

In this aim, we have fitted a parabola over the series representing the mean longitude of Mimas after removing the terms present in the current theories (noted with an asterisk in Table 1). As shown in Fig. 1, with a fit computed over the century corresponding to the observations used by Dourneau, we found an "acceleration" of 2°3 $cy^{-2}$. Other fits computed over other time-spans would give other values, but in all cases we have found an acceleration of the same order. Of course, none of these experiments really correspond to the fits done by Kozai or Dourneau, because the observations they used included errors and were not distributed uniformly over the time-span. But obviously, we must now take into account these long period terms in the mean longitude of Mimas, before hoping to determine well an eventual acceleration issued from other causes, like tidal dissipation.

In fact, the variation of the mean motion of a satellite due to the tidal dissipation in the planet is given by an expression depending mainly on $Q$, the unelasticity coefficient of the planet. This coefficient is very uncertain but Dermott & al (1988) have shown that for Saturn : $Q \geq 1.6\ 10^4$. Then Sarlat (1990) deduced that the secular acceleration of Mimas should be limited to : $s_1 \leq 0\overset{\circ}{.}07\ cy^{-2}$, which is much smaller than the observed value deduced from current theories.

## 4. Conclusion

The secular acceleration observed by Kozai and Dourneau in the longitude of Mimas does not seem to be explain by tidal dissipation but by the absence, in their theories, of the long-period inequalities revealed in the new theory of the Mimas' motion. These must now be considered before attempting to determine a tidal acceleration of Mimas. Analogous very long-period terms exist also in the longitude of Tethys with amplitudes greater than 100 km. We emphasize that these terms depend on the Tethys' eccentricity which is not really known at present. At last, it seems that these long-period inequalities result from secular resonances which might be considered in the future studies of the evolution of these satellites.

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# Discussion

***H.Kinoshita*** **– Does the Fourier series expression of the longitude of Mimas satisfy the d'Alembert characteristic?**

***L.Duriez*** **– Yes, except that resonance introduces arguments that are no longer related to inclination or eccentricity, but to amplitude of the libration.**