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Dark Matter in Groups and Clusters of Galaxies

Jaan Einasto and Maret Einasto

Tartu Observatory, Estonia

Abstract.
We compare the characteristics of stellar populations with those of dark halos. Dark matter around galaxies, and in groups, clusters and voids is discussed. Modern data suggest that the overall density of matter in the Universe is $\Omega_M = 0.3 \pm 0.1$, about 80 % of this matter is non-baryonic dark matter, and about 20 % is baryonic, mostly in the form of hot intra-cluster and intragroup gas, the rest in stellar populations of galaxies. All bright galaxies are surrounded by dark matter halos of external radii $200 - 300$ kpc; halos consist mostly of non-baryonic matter with some mixture of hot gas. The Universe is dominated by dark energy (cosmological constant) term. Dark matter dominates in the dynamical evolution of galaxies in groups and clusters.

1. History of the dark matter concept

The story of dark matter is a classical example of a scientific revolution (Kuhn 1970, Tremaine 1987). It is impossible in this review talk to discuss all aspects of dark matter. We start with a historical introduction, followed by a comparison of ordinary stellar populations and the nature of dark matter. Thereafter we consider dark matter in galaxies, groups and clusters of galaxies, and in voids; we also discuss the mean density of matter in the Universe.

First hints for the presence of a mass paradox in galaxies and clusters of galaxies came over 60 years ago. Oort (1932) noticed that there may exist a discrepancy between the dynamical estimate of the local density of matter in the Solar neighborhood in the Galaxy, and the density of luminous matter. Known stellar populations may be insufficient to explain the vertical gravitational attraction in the Galaxy which causes motions of stars perpendicular to the plane of the Galaxy. Zwicky (1933) measured radial velocities of galaxies in the Coma cluster and found that the mass of the cluster exceeds the summed mass of its galaxies more than tenfolds. These studies raise two problems, the one of the local dark matter in the disk of the Galaxy, and the global dark matter pervading clusters of galaxies. In the 1930s astronomers were very busy to understand the evolution of stars, and dark matter problems escaped the attention of the astronomical community.

The next essential step in the dark matter story was made by Kahn & Woltjer (1959). They noticed that the Andromeda galaxy and our Galaxy approach each other, whereas almost all other galaxies recede from us. The total mass of the Local Group, inferred from ascribing the approach velocity to mutual attrac-

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tion, exceeds the conventional mass of M31 and Galaxy approximately tenfold. This discovery again did not attract much attention. During the discussion of the stability of clusters of galaxies in the 1960s, Ambartsumian suggested an opposite view that clusters may be recently formed and expanding systems. This suggestion contradicts, however, data on ages of cluster galaxies (see e.g. van den Bergh 1999).

In the late 1960s and early 1970s it was realized that the mass paradox may be a global problem for all bright galaxies. Einasto (1969), Sizikov (1969) and Freeman (1970) noticed that rotation velocities of galaxies decrease more slowly in the outskirts of galaxies than expected from the distribution of light. Two possibilities were discussed to explain this discrepancy – systematic deviations from circular motion or the presence of some massive but invisible population in the outskirts of galaxies.

One approach that has led to the conclusion of the presence of dark matter around galaxies was the modeling of galaxies using a combination of all available observational data on stellar populations in galaxies of different morphological type. Such combined models were reported during the First European Astronomy Meeting in Athens in September 1972 (Einasto 1974). It was shown that ordinary stellar populations cannot explain almost flat rotation curves of the outer parts of spiral galaxies. To explain flat rotation curves the presence of a new invisible population, a "dark corona", was suggested. Independent evidence for the presence of dark matter around galaxies was inferred by Ostriker and Peebles (1973) based on disk stability arguments. Available data were, however, not sufficient to determine the total mass and dimension of the hypothetical dark population.

To derive the mass distribution at larger distances from galactic centers the teams at Tartu and Princeton investigated the dynamics of companions of bright galaxies. They demonstrated that the internal mass, inferred from the motion of companion galaxies, increases with distance from centers of bright galaxies up to several hundred kiloparsec, thus increasing the hitherto assumed dimensions and masses of galaxies by an order of magnitude (Einasto, Kaasik & Saar 1974, Ostriker, Peebles & Yahil 1974). These studies suggest that the presence of dark matter is a general property of galaxies and systems of galaxies; this matter has a dominant contribution to the mass budget in the Universe. Difficulties connected with this interpretation of rotation curves and dynamics of companion galaxies were discussed by Burbidge (1975). These three studies triggered the boom of the dark matter studies.

Dark matter was discussed during the Third European Astronomical Meeting in Tbilisi, in July 1975. This Meeting was the highlight of the dark matter discussion where supporters (Bertola & Tullio 1976, Einasto et al. 1976) and opponents (Karachentsev 1976, Oleak 1976, Materne & Tammann 1976, Fesenko 1976) of the concept of dark matter had a hot debate. The majority of speakers argued against the dark matter concept; in the summary of the Meeting Kharadze noted that the dark matter concept did not find support.

The next public discussion of the dark matter problem was during the IAU General Assembly in Grenoble in August 1976. Here the focus was the nature of the dark population. Ostriker, Peebles & Yahil (1974) assumed that dark halos consist of faint stars; this concept was discussed by Maarten Schmidt.

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Population studies led the group at Tartu to conclude that dark matter cannot be made of ordinary stars but must have a different origin (Jaaniste & Saar 1975). To make a clear distinction between known halo population (which consists of old stars) and the new population the term "corona" was suggested (Einasto 1974). The difference between ordinary galactic populations and the dark matter population was summarized by Einasto, Jõeveer & Kaasik (1976). Ivan King from the audience noted "perhaps really there are two halos in galaxies, stellar and dark". Initially hot gas was considered as a possible candidate for the dark matter (Einasto 1974). However, subsequent studies by Komberg & Novikov (1975), and Chernin et al. (1976) demonstrated that only a fraction of the corona may be gaseous. X-ray observations have confirmed that the mass of hot gas in coronae is comparable with the mass of stellar populations, however, hot gas is not sufficient to explain the total "missing" mass. Thus the origin of coronae remained unclear.

The final acceptance of the presence of dark matter around galaxies came after Morton Roberts, Vera Rubin and their collaborators had shown that the outer parts of practically all spiral galaxies have flat rotational curves (Roberts & Whitehurst 1975, Rubin, Ford & Thonnard 1978, 1980, Rubin 1987). However, theorists accepted the presence of dark matter only after its role in the evolution of the structure of the Universe was realized. This illustrates Eddington's test: "No experimental result should be believed until confirmed by theory" (Turner 1999b). It was clear that, if nature created so much dark matter, it must have some purpose. Rees (1977) noticed that neutrinos can be considered as dark matter particles; and Chernin (1981) showed that, if dark matter is non-baryonic, then this helps to explain the paradox of small temperature fluctuations of background microwave radiation. Density perturbations of non-baryonic dark matter start growing already during the radiation-dominated era whereas the growth of baryonic matter is damped by radiation. If non-baryonic dark matter is dynamically dominating, the total density perturbation can have an amplitude of the order 10^{-3} at the recombination epoch, which is needed for the formation of the observed structure of the Universe. Baryonic matter flows after recombination to gravitational wells formed by non-baryonic matter. Chernin considered neutrinos with non-zero rest mass as a possible candidate, but other non-baryonic particles do the job as well. This result was discussed in a conference in Tallinn in spring 1981. In the summary speech of this conference Zeldovich concluded: "Observers work hard to collect data, theorists interpret observations; are often in error, correct their errors and try again; and there are only very rare moments of clarification. Today it is one of such rare moments when we have holy feeling of understanding Nature. Non-baryonic dark matter is needed to start structure formation early enough".

Soon it was realized that neutrino-dominated or hot dark matter generates almost no fine structure of the Universe – galaxy filaments in superclusters (Zeldovich, Einasto & Shandarin 1982), and that the structure forms too late (White, Davis & Frenk 1984). A much better candidate for dark matter is some sort of cold particles as axions (Blumenthal et al. 1984). The dark matter concept as a solid basis of the contemporary cosmology was incorporated in full details in a series of lectures by Primack (1984), and was discussed in the IAU Symposium on Dark Matter (Kormendy & Knapp 1987). Thus in the end it took over fifty years from the first discoveries by Oort and Zwicky until the new paradigm was generally accepted. However, the story is not over. The nature of the dark matter is still unclear – we do not know exactly what the cold dark matter is, and whether it is mixed with hot dark matter (neutrinos).

2. Galactic populations

Dark matter is invisible, and the only possibility to determine its mass, radius and shape in galaxies and clusters of galaxies is modeling of populations present in these systems, using all available observational data on the distribution of populations and on dynamics of the system. Thus our first task is to find the main parameters of stellar populations in galaxies.

Models of stellar populations in galaxies were constructed by Einasto (1974), and more recently by Einasto & Haud (1989), Bertola et al. (1993) and Persic, Salucci & Stel (1996), among others. Models use luminosity profiles of galaxies, rotation curves, velocity dispersions of central stellar clusters, and other relevant data. Parameters can be determined for the stellar halo, the bulge and the disk, and for the dark population. To determine the amount of dark matter in and around galaxies, the mass-to-luminosity ratio of the stellar population, M/L_B, is of prime importance. The available data show that the mean M/L_B is surprisingly constant; for the stellar halo it is of the order of unity, for the bulge approximately 3, and only for the metal-rich cores of massive galaxies the value approaches 10. The mean mass-to-luminosity ratio for all visible matter, weighed with the luminosities of galaxies, is $M/L_B = 4.1 \pm 1.4$. We can summarize properties of ordinary and dark populations as follows:

1) Stellar populations have $1 \leq M/L_B \leq 10$, while dark population has $M/L_B \gg 1000$.

2) There is a continuous transition of stellar populations from stellar halo to bulge, from bulge to old disk, from old to young disk; and intermediate populations are clearly seen in our Galaxy. All stellar populations contain a continuous sequence of stars of different mass, some of these stars have ages and masses which correspond to ages and masses of luminous red giants, thus all stellar populations are visible, in contrast to the dark population.

3) The density of stellar populations rapidly increases toward the plane or the center of the galaxy, while the dark matter population shows a much lower concentration of mass to the galactic plane and center.

These arguments show quantitatively that dark matter must have an origin different from stellar populations. Since old and young stellar populations form a continuous sequence, the dark population must have originated much earlier. There must be a large gap between the formation time of dark halo and oldest visible stellar populations, since there appears to be no intermediate populations between the dark and stellar populations (Einasto, Jõeveer & Kaasik 1976).

3. Dark matter in galaxies

The possible existence of dark matter near the plane of the Galaxy was advocated by Oort (1932, 1960), who determined the density of matter in the Solar vicinity and found that there may be a discrepancy between the dynamical density and the density calculated from the sum of densities of known stellar populations. This discrepancy was studied by Kuzmin (1952), Eelsalu (1959) and Jõeveer (1972, 1974); all three independent analyses demonstrated that there is practically no local mass discrepancy in the Galaxy. A much higher value of the local dynamical density was found by Bahcall (1984a, 1984b, 1987). It is clear that non-baryonic dark matter cannot contract to a flat population needed to explain the presence of the local mass discrepancy. For this reason it is natural to expect that the local dark matter, if present, must be of stellar origin. The local dark matter problem has been analyzed by Gilmore (1990 and references therein). Most recent data suggest that dynamically determined local density of mass is approximately 0.1 $M_{\odot}\ pc^{-3}$, in good agreement with direct estimates of the density. Thus there is no firm evidence for the presence of local dark matter in our Galaxy.

Flat rotation curves of galaxies suggest that there must be another dark population in galaxies. This global dark matter must have a more-or-less spherical distribution to stabilize the flat population (Ostriker & Peebles 1973). As discussed above the dark population has properties completely different from properties of known stellar populations. It is generally believed that this population is non-baryonic.

The mass and volume occupied by dark matter halos around galaxies can be determined only on the basis of relative motions of visible objects moving within these dark halos. Almost all bright galaxies are surrounded by dwarf companion galaxies, and in this respect they can be considered as poor groups of galaxies (Einasto et al. 1974). Such clouds of satellites have radii 0.1 to 1 h^{-1} Mpc. The relative motions of companion galaxies indicate that the total mass within the radius of orbits grows approximately linearly with distance. This suggests that dark halos of main galaxies have approximately isothermal density profiles. The outer radius of isothermal halos of giant galaxies is, however, not well determined, since there are no objects which can test the relative velocity at large distance from the main galaxy. The Local Group of galaxies yields an unique possibility to measure the relative radial velocity of two subgroups, located around our Galaxy and the Andromeda galaxy. These measurements show that the total mass of the Local Group is $\approx 5 \times 10^{12}\ M_{\odot}$ (Kahn & Woltjer 1959, Einasto & Lynden-Bell 1982). Masses determined from velocities of companions within both subgroups are $2 \times 10^{12}\ M_{\odot}$ and $3 \times 10^{12}\ M_{\odot}$, for our Galaxy and M31, respectively. In these determinations it is assumed that dark halos of M31 and Galaxy have external radii about 200 – 300 kpc (Haud & Einasto 1989, Tenjes, Haud & Einasto 1994). We see that individual masses are in good agreement with to total mass derived from the approach velocity; in other words this agreement confirms that estimated external radii and masses of dark halos are correct.

4. Dark matter in clusters

The distribution of mass in clusters of galaxies can be determined by three independent methods: from the distribution of relative velocities of galaxies, from the distribution and temperature of hot X-ray emitting gas, and from the gravitational lensing effect. All three methods can be applied in the case of clusters of galaxies and rich groups of galaxies, so we start our discussion from these systems.

4.1. Rich clusters of galaxies

The classical method to determine the mass distribution in clusters is based on the measurements of the velocity dispersion of galaxies in clusters. The method may be biased since the number of clusters with measured redshifts is usually small and it is difficult to exclude foreground and background clusters, especially in regions of high density of galaxies (superclusters). During the last decade X-ray measurements have supplied a more accurate method to determine masses and mass profiles of clusters of galaxies. The method is based on the observation that hot gas and galaxies are in hydrostatic equilibrium within a common cluster potential, i.e. both move under gravity in the potential well of the cluster. The mass distribution of the cluster can be derived from the mean temperature of the gas and radial gradients of the temperature and density (Watt et al. 1992, Mohr et al. 1999). The intensity of the X-ray emission gives information on the mass distribution of hot gas, thus X-ray observations yield simultaneously the distribution of the total mass and gas mass in the cluster. Galaxies give additional information on the distribution of mass in galaxies, thus altogether three distributions can be found. ROSAT X-ray satellite data are presently available for many clusters and rich groups of galaxies. As an example of the integrated mass distribution in the Perseus cluster of galaxies we refer to Böhringer (1995). In other clusters studied so far the distributions are rather similar. The main conclusions from these studies are the following:

1) the radial distributions of the total mass, gas mass, and galaxy mass are similar;

2) intra-cluster hot gas constitutes 14 ± 2 % of the total mass of clusters (for Hubble constant $h = 0.65$);

3) the mass in visible populations of galaxies is ≈ 3 % of the total mass of the cluster.

X-ray data also yield the mass-to-luminosity ratio of the cluster: $M/L_V = 150\ h\ M_\odot/L_\odot$ (David, Jones & Forman 1996). This mean value is valid for the whole range of temperatures and masses of clusters and groups. Modern data based on velocity dispersions of galaxies in clusters yield $M/L_V = 213 \pm 60\ h\ M_\odot/L_\odot$ (Carlberg et al. 1997).

Gravitational lensing yields another independent method to derive the mass of clusters of galaxies. This method has been applied for several clusters, and the results are in agreement with masses determined from X-ray data (Mellier, Fort & Kneib 1993, Schindler et al. 1995).

ROSAT data have been used to investigate the mass distribution in a clusters filament in the core of the Shapley supercluster (Kull & Böhringer 1999). Data show that there exist a continuous X-ray emission along the filament joining three rich clusters of galaxies. This emission indicates the presence of a potential well along the filament filled with dark matter and hot gas. A similar distribution of galaxies along the main chain of the Perseus supercluster is known long ago (Jõeveer, Einasto & Tago 1978). These data indicate that galaxies, hot gas and dark matter form similar condensations along filaments joining clusters and groups of galaxies.

4.2. Poor groups of galaxies

Most galaxies in the Universe belong to poor groups with one or few bright galaxies and a number of faint dwarf companions. The Local group is an example of poor groups with two major concentration centers. The basic difficulty in the study of the mass distribution in poor groups lies in the weakness of the X-ray emission and absence in most groups bright companions to measure the relative velocity on large distance from the groups center.

Available X-ray data suggest that poor groups have a lower fraction of hot gas than do rich clusters of galaxies; the mass of hot gas is approximately ten times smaller than the stellar mass (Ponman & Bertram 1993, Pildis, Bregman, & Evrard 1995). According to X-ray data dark matter extends significantly beyond the apparent configuration of bright galaxies in good agreement with optical data on the distribution of faint companion galaxies; the total mass-to-luminosity ratio is in agreement with optical data, $M/L_B \approx 120\ h\ M_\odot/L_\odot$ (Ponman & Bertram 1993). Galaxies in compact groups show signs of distortions which indicate that these groups are formed as a result of orbital decay; galaxies merge within a few billion years to form a giant elliptical galaxy in the center of the group; this process is rather rapid. The presence of such groups indicates that there should exist fossil groups, consisting only of the central giant elliptical galaxy surrounded by the dark matter of the previous group. Such fossil groups are actually observed, examples are NGC 315 in the Perseus supercluster chain – a massive radio galaxy with very large radio lobes (Jõeveer, Einasto & Tago 1978), and NGC 1132 (Mulchaey & Zabludoff 1999).

4.3. Dynamics of main galaxies in groups and clusters

If the dominant galaxies in groups were formed by merging of its former companions one would expect the internal velocity dispersion of these galaxies to be comparable with the velocity dispersion of galaxies in the group before the merger event. In Fig. 1 we plot the velocity dispersion of central galaxies, σ_{gal}, in groups and clusters as a function of the velocity dispersion of galaxies in respective systems, σ_{clust}. We see that the internal velocity dispersion of dominant galaxies in rich clusters is much lower than the velocity dispersion in clusters, but comparable to the velocity dispersion of galaxies in subgroups often found in clusters. This observation suggests that central galaxies formed already in the early stages of cluster evolution, before subgroups merged with the presently observed cluster. A similar conclusion has been reached by Dubinski (1998) using N-body simulations of cluster evolution.

5. Dark matter in voids

In the mid-1970s it was discovered that field galaxies are not randomly distributed in space but form long filaments and chains between clusters and groups; clusters and groups themselves are concentrated to superclusters of galaxies (Jõeveer & Einasto 1978, Jõeveer, Einasto & Tago 1978). Between galaxy filaments there are big volumes devoid of any visible form of matter. One of the first questions asked was: are these voids really empty or do they contain some hidden matter? This problem was investigated by Einasto, Jõeveer & Saar (1980).

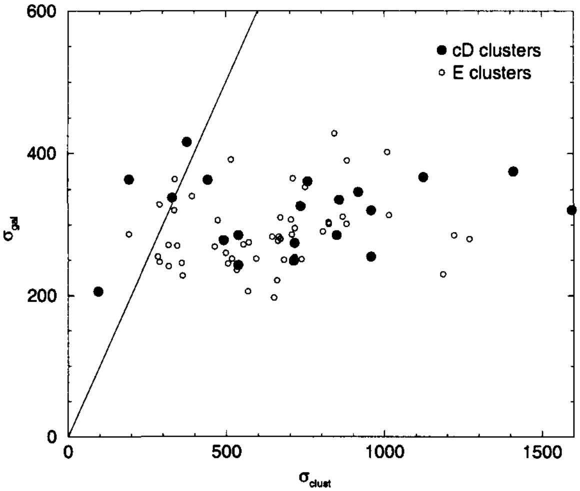

Figure 1. The relation between the velocity dispersion of the dominant galaxy, σ_{gal}, and the velocity dispersion of the host cluster, σ_{clust}. The relation is shown separately for clusters with central cD galaxy and central E galaxy. Straight line marks the equality of both dispersions, $\sigma_{gal} = \sigma_{clust}$.

The study was based on the well-known theory of the growth of density perturbations developed by Zeldovich (1970) and Press & Schechter (1974). According to Zeldovich matter flows away from under-dense regions towards high-density ones until over-dense regions collapse and form galaxies. The density of matter in under-dense regions decreases approximately exponentially and never reaches zero. In order to form a galaxy or cluster the over-density within a radius of r must exceed a certain limit, about 1.68 in case of spherical perturbations (Press-Schechter limit). On the basis of these considerations one can make two important conclusions: first, there must exist some primordial matter in voids, and second, galaxy formation is a threshold phenomenon. Recent hydrodynamical simulations of the evolution of the density field and formation of galaxies have confirmed these theoretical expectations (Cen & Ostriker 1992, 1999, Katz et al. 1992, 1996).

Quantitative estimates of the total fraction of matter in voids have been made by Einasto et al. (1994, 1999). These estimates are based on N-body calculations of the evolution of under- and over-density regions for a variety of cosmological models. The density field was calculated using a small smoothing length, about 1 h^{-1} Mpc, which corresponds to the mean size of small groups of galaxies, dominant structural elements of the Universe. A problem in these calculations is the identification of the present epoch of simulations. This epoch can be determined using σ_8 normalization of the density field. The present mean value of density perturbations in a sphere of radius 8 h^{-1} Mpc can be determined directly from observations. Initially half of all matter was located in regions below the mean density (this follows from the simple fact that initial density perturbations are very small). During the evolution matter flows away

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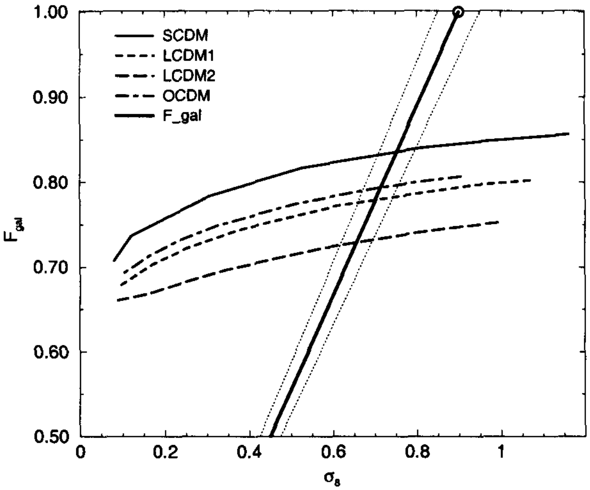

Figure 2. The relation between the fraction of matter in galaxies, F_{gal}, and σ_8. The thick bold solid line shows the definition relation $(\sigma_8)_m = F_{gal}(\sigma_8)_{gal}$, while the bold solid, the dashed and the dot-dashed lines give the relation obtained from numerical simulations of how voids are emptied in different cosmological models.

from low-density regions (see Fig. 2). The present fraction of matter in voids is somewhat model-dependent, 25 ± 10 % of the total amount of matter (Einasto et al. 1999).

6. Mean density of matter in the Universe

There are several independent methods to derive the mean density of matter in the Universe. The first method is based on primordial nucleosynthesis data, which indicate that the baryon density is $\Omega_b h^2 = 0.019 \pm 0.002$ (Schramm & Turner 1998, Turner 1999a). If we use a Hubble parameter of $h = 0.65 \pm 0.05$, and apply the ratio of baryon to overall density as suggested by X-ray data, we obtain for the mean density of matter $\Omega_M = 0.31(h/0.65)^{-1/2} \pm 0.04$. The second method uses mass-to-luminosity ratios of groups and clusters, and the mean luminosity density. This method gives the density of the clustered matter. If we add the density of matter in voids as suggested by void evacuation data, we get $\Omega_M = 0.25 \pm 0.05$ (Bahcall 1997, Einasto et al. 1999). The distant supernova project (Perlmutter et al. 1998, 1999, Riess et al. 1998) allows to measure the curvature of the Universe and to distinguish between the matter density, Ω_M, and the cosmological constant parameter, Ω_Λ; this method suggests that the Universe is dominated by the cosmological term, the density of matter is $\Omega_M = 0.28 \pm 0.1$. Similarly, the comoving maximum of the galaxy power spectrum permits the measurement of the cosmological curvature (Broadhurst & Jaffe 1999), and favours a Universe with $\Omega_M = 0.4 \pm 0.1$. As demonstrated by Bahcall & Fan (1998) and Eke et al. (1998), the rate of the evolution of cluster abundance depends strongly on the mean density of the Universe. The cluster abundance method yields for the density a value $\Omega_M = 0.3 \pm 0.1$. Finally, the dynamics of the Local Group and its vicinity, using the least action method, also yields a low density value (Shaya et al. 2000).

The weighed mean of these independent methods is $\Omega_M = 0.30 \pm 0.05$; i.e. the overall density of matter in the Universe is sub-critical by a wide margin. The quoted error is intrinsic, if we add possible systematic errors we get an error estimate ± 0.1. Supernova and CMB data exclude the possibility of an open Universe: the dominating component in the Universe is the dark energy – the cosmological constant term or some other term with negative pressure (Turner 1999b, Perlmutter et al. 1998, 1999).

7. Summary

The present knowledge of the dark matter in the Universe can be summarized as follows.

1) The evidence for the presence of local dark matter in the disk of the Galaxy is not convincing; if present, it must be of baryonic origin as non-baryonic matter cannot form a flat disk.

2) The mean mass-to-luminosity ratio of stellar populations in galaxies is $M/L_B \approx 4\ M_\odot/L_\odot$; the mean mass-to-luminosity ratio in groups and clusters of galaxies is $100 - 200\ h\ M_\odot/L_\odot$.

3) The presence of dark matter halos of galaxies, and of dark common halos of groups and clusters is well established; the bulk of the dark population consists of some sort of cold dark matter. About 5 % of mass in poor groups, and 15 % in rich clusters is in the form of hot X-ray emitting gas.

4) There exists dark matter in voids; the fraction of matter in voids is ≈ 25 %, and in high-density regions ≈ 75 % of the total matter.

5) The total density of matter is, $\Omega_M = 0.3 \pm 0.1$, and the density of dark energy (cosmological constant) is $\Omega_\Lambda = 0.7 \pm 0.1$.

6) On all scales larger than sizes of galaxies the dynamics is determined by dark matter.

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