CIRCUMSTELLAR DUST SHELLS

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Abstract. The optical properties of circumstellar graphite particle shells are considered. In the case of the optically thin shell (e.g. the shell of a RCB star after minimum), the temperature of the grains is calculated. It is shown that the dust shell always has a sharp inner border. The RCB phenomenon may be considered as a sudden moving of this border toward the star and the consequent increase in its optical thickness.

1. Introduction

There are stars of different types which have dust shells (Ney, 1972). Hereinafter we will deal with the graphite shells which are supposed to surround the RCB stars (Loreta, 1934; O'Keefe, 1939; Alexander *et al.*, 1972; Stein, 1972; Feast and Glass, 1973). It is well known that infrared observations confirmed the dust shell hypothesis on the RCB phenomenon, but there are some difficulties in understanding this mechanism in detail.

We have no explanation how the graphite grains can be formed under the actual photospheric temperature of a star. However, observation of the chromospheric spectrum at the minimum of R CrB forces us to the conclusion that the condensation of carbon into graphite takes place close to the photosphere of the star.

One can see two ways of removing this difficulty. First, the RCB stars may have extended chromospheres so the condensation takes place at a sufficiently large distance from the photosphere. Second, it is possible that a sufficiently low temperature, necessary for the condensation of carbon, exists above cooler spots which are similar to sunspots.

Since these difficulties are well known, nobody has built an adequate model of the RCB phenomenon. Even the simplest part of the problem, namely the problem of radiation transfer inside the dust shell of RCB stars (e.g. Herbig, 1970) needs to be developed to the stage when one can use the theory for the interpretation of the infrared observations. This problem is treated in the present paper.

2. The Statement of the Problem

The shell considered here is assumed to be at rest, uniform and having two sharp boundaries: the outer one and the inner one. We also suppose that the shell consists of grains of a uniform size. Since the condensation of carbon is expected to produce small size particles, it is likely that most of them satisfy the condition

$$2\pi a/\lambda_{\text{eff}} \ll 1\,, \tag{1}$$

where a is the radius of a grain and λ_{eff} is the effective wavelength of the radiation of the central star. Condition (1) gives the restriction $a \ll 0.1\ \mu$ for the shell of R CrB.

Sherwood and Plaut (eds.), Variable Stars and Stellar Evolution, 165–171.

The scattering of light by such particles is obviously negligible so we have to consider the true absorption only. The cross-section of the absorption is given by the expression

$$\sigma_{ab} = \pi a^2 \cdot \frac{8\pi a}{\lambda} \frac{6m'm''}{[2 + (m')^2 - (m'')^2]^2 + (2m'm'')^2}, \tag{2}$$

where

$$m = m' - im'' \tag{3}$$

is the complex refraction index. Formula (2) can be rewritten in the equivalent form

$$\sigma_{ab} = \pi a^2 \cdot \frac{8\pi a}{\lambda} \frac{6\tilde{\sigma}\lambda/c}{(\tilde{\varepsilon} + 2)^2 + 4\tilde{\sigma}^2\lambda^2/c^2} \tag{4}$$

where c is the light velocity, $\tilde{\sigma}$ is the conductivity of graphite, and $\tilde{\varepsilon}$ is its dielectric constant. Since $\tilde{\sigma} = 1.5 \times 10^{15}\ \mathrm{s}^{-1}$ for graphite, one can obtain for $\lambda > 1\mu$ the simple approximation

$$\sigma_{ab} \approx \pi a^2 \frac{12\pi a c}{\tilde{\sigma}\lambda^2}. \tag{5}$$

We shall use this approximation for the infrared radiation of the shell. On the other hand it is more convenient to describe the absorption of the visible light by the cross-section from equation (2) with the constants m' and m''.

We suppose here, therefore, that all the grains satisfy (2) and (5) and their number density may be written as

$$n = \begin{cases} 0 & r < r_i \\ n \quad \text{if} & r_i \leqslant r \leqslant r_o \\ 0 & r > r_o \end{cases} \tag{6}$$

where r is the distance from the centre of the star and subscripts i and o refer to the inner and outer border accordingly.

3. Radiation from the Star

Since we neglect the scattering, the intensity of the radiation of the central star can be obtained from the relation

$$I_\lambda^*(r, \vartheta) = I_{0\lambda}^* \exp\left[-\int_{-\infty}^{s} \sigma_{ab} n(s)\, \mathrm{d}s\right], \tag{7}$$

where s is the coordinate measured along the line of sight, ϑ is the angle between the line of sight and the radius at the distance r from the centre of the star. The relation between r and s is:

$$s = r \cos\vartheta. \tag{8}$$

Therefore inside the shell the intensity of the light of the star is equal to

$$I_\lambda^*(r, \vartheta) = I_{0\lambda}^* \exp\{-\sigma_{ab} n [r \cos\vartheta - \sqrt{r_i^2 - r^2 \sin^2\vartheta}]\} \tag{9}$$

if $\vartheta < \arcsin(r_*/r)$

and $I_\lambda^* = 0$ if $\vartheta > \arcsin(r_*/r)$.

The radiation flux from the star received on the Earth can be calculated from

$$H_\lambda^* = \frac{\pi r_*^2}{r_\oplus^2} I_{0\lambda}^* \int_0^1 \exp\{-\sigma_{ab} n [\sqrt{r_0^2 - r_*^2 y} - \sqrt{r_i^2 - r_*^2 y}]\}\, dy \tag{10}$$

where r_* is the stellar radius and $r_\oplus$ is the distance from the Earth.

4. The Infrared Radiation of the Shell Itself

The equation of radiation transfer in the present case has the form

$$\cos\vartheta \frac{\partial I_\lambda}{\partial r} + \frac{\sin\vartheta}{r} \frac{\partial I_\lambda}{\partial \vartheta} = \sigma_{ab} n (B - I_\lambda), \tag{11}$$

where

$$B = 2hc^2 \lambda^{-5} (\exp(hc/\lambda k T_g(r)) - 1)^{-1} \tag{12}$$

and $T_g(r)$ is the temperature of a grain at the distance r from the centre of the star.

If the temperature $T_g(r)$ were known we could solve Equation (11) which has the following different forms for the four different ranges of the angle ϑ.

If

$$0 < \vartheta < \arcsin(r_*/r), \tag{13}$$

then

$$I_\lambda^{(1)}(r, \vartheta) = n \int_{r_0}^{r} \frac{\sigma_{ab} B x\, dx}{\sqrt{x^2 - r^2 \sin^2\vartheta}} \exp\{-\sigma_{ab} n (r \cos\vartheta - \sqrt{x^2 - r^2 \sin^2\vartheta})\} \tag{14}$$

If

$$\arcsin(r_*/r) < \vartheta < \arcsin(r_i/r), \tag{15}$$

then

$$\begin{aligned} I_\lambda^{(2)}(r, \vartheta) &= \exp[-\sigma_{ab} n (r \cos\vartheta - \sqrt{r_i^2 - r^2 \sin^2\vartheta})] + \\ &+ n \int_{r_i}^{r_0} \frac{\sigma_{ab} B x\, dx}{\sqrt{x^2 - r^2 \sin^2\vartheta}} \exp[-\sigma_{ab} n (\sqrt{x^2 - r^2 \sin^2\vartheta} - \sqrt{r_i^2 - r^2 \sin^2\vartheta})] + \\ &+ n \int_{r_i}^{r} \frac{\sigma_{ab} B x\, dx}{\sqrt{x^2 - r^2 \sin^2\vartheta}} \exp[-\sigma_{ab} n (r \cos\vartheta - \sqrt{x^2 - r^2 \sin^2\vartheta})]. \end{aligned} \tag{16}$$

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If

$$\arcsin(r_i/r) < \vartheta < \pi/2, \tag{17}$$

then

$$I_\lambda^{(3)}(r,\vartheta) = n \int_{r\sin\vartheta}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - r^2\sin^2\vartheta}} \times$$

$$\times \exp\left[-\sigma_{ab} n\left(r\cos\vartheta + \sqrt{x^2 - r^2\sin^2\vartheta}\right)\right] +$$

$$+ n \int_{r\sin\vartheta}^{r} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - r^2\sin^2\vartheta}} \exp\left[-\sigma_{ab} n\left(r\cos\vartheta - \sqrt{x^2 - r^2\sin^2\vartheta}\right)\right]. \tag{18}$$

And if

$$\pi/2 < \vartheta < \pi, \tag{19}$$

then

$$I_\lambda^{(4)}(r,\vartheta) = n \int_{r}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - r^2\sin^2\vartheta}} \exp\left[-\sigma_{ab} n\left(r\cos\vartheta - \sqrt{x^2 - r^2\sin^2\vartheta}\right)\right] \tag{20}$$

The infrared flux at the Earth is equal to

$$H_\lambda^{IR} = \frac{\pi r_0^2}{r_\oplus^2} n \Bigg\{ \int_0^{(r_*/r_0)^2} \mathrm{d}y \int_{r_i}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - r_0^2 y}} \times$$

$$\times \exp\left[-\sigma_{ab} n\left(r_0\sqrt{1-y} - \sqrt{x^2 - y r_0^2}\right)\right] +$$

$$+ \int_{(r_*/r_0)^2}^{(r_i/r_0)^2} \mathrm{d}y \Bigg[\!\!\Bigg[\exp\left[-\sigma_{ab} n\left(r_0\sqrt{1-y} - \sqrt{r_i^2 - r_0^2 y}\right)\right] \times$$

$$\times \int_{r_i}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - y r_0^2}} \exp\left[-\sigma_{ab} n\left(\sqrt{x^2 - r_0^2 y} - \sqrt{r_i^2 - r_0^2 y}\right)\right] +$$

$$+ \int_{r_i}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - y r_0^2}} \exp\left[-\sigma_{ab} n\left(r_0\sqrt{1-y} - \sqrt{x^2 - y r_0^2}\right)\right] \Bigg]\!\!\Bigg] +$$

$$+ \int_{(r_i/r_0)^2}^{1} \mathrm{d}y \int_{r_0\sqrt{y}}^{r_0} \frac{\sigma_{ab} Bx\,\mathrm{d}x}{\sqrt{x^2 - r_0^2 y}} \exp\left(-\sigma_{ab} n r_0\sqrt{1-y}\right) \times$$

$$\times \left[\exp\left(\sigma_{ab} n\sqrt{x^2 - y r_0^2}\right) - \exp\left(-\sigma_{ab} n\sqrt{x^2 - y r_1^2}\right)\right] \Bigg\}. \tag{21}$$

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5. The Heat Equilibrium of the Grain

A grain absorbs the energy in the visible region of the spectrum and re-emits it in the infrared. Using Equations (5) and (12) one can get the expression for the emitted power

$$P_- = 4\pi \int_0^\infty \sigma_{ab} B \, d\lambda = 64\pi^3 hc^2 a^3 \frac{180c}{\tilde{\sigma}} \left(\frac{kT_g}{hc}\right)^6 \zeta(6), \tag{22}$$

where

$$\zeta(n) = \sum_{i=1}^{\infty} i^{-n}. \tag{23}$$

The power of the stellar radiation absorbed by the grain is given by the relation

$$P_+ = \int\int \sigma_{ab} I_\lambda^* \, d\lambda \, d\omega = 4\pi C hc^2 a^3 \left(\frac{kT_*}{hc}\right)^5 \int_0^\infty \frac{x^4 \, dx}{e^x - 1} \int_{\sqrt{1-(r_*/r)^2}}^{1} \times$$
$$\times \exp\left[-\frac{Cna^3 kT_*}{hcx}\left(r\mu - \sqrt{r_0^2 - r^2(1-\mu^2)}\right)\right] d\mu, \tag{24}$$

where

$$C = \frac{48\pi^2 m' m''}{[2 + (m')^2 + (m'')^2]^2 + (2m'm'')^2}. \tag{25}$$

A more complex expression represents the absorbed power of the infrared radiation:

$$P_+^{IR} = \int_{(4\pi)} d\omega \int I_\lambda^{IR} \, d\lambda = 2\pi n \int_0^\infty d\lambda \left\{ \int_{\sqrt{1-(r^*/r)^2}}^{1} d\mu \int_{r_1}^{r} \frac{\sigma_{ab} Bx \, dx}{\sqrt{x^2 - r^2(1-\mu^2)}} \times \right.$$
$$\times \exp[-\sigma_{ab} n (r\mu - \sqrt{x^2 - r^2(1-\mu^2)})] + \int_{\sqrt{1-(r_1/r)^2}}^{\sqrt{1-(r^*/r)^2}} \times$$
$$\times d\mu \Big[\Big[\exp[-\sigma_{ab} n (r\mu - \sqrt{r_i^2 - r^2(1-\mu^2)})] \int_{r_1}^{r_0} \frac{\sigma_{ab} Bx \, dx}{\sqrt{x^2 - r^2(1-\mu^2)}}$$
$$\times \exp[-\sigma_{ab} n (\sqrt{x^2 - r^2(1-\mu^2)} - \sqrt{r^2 - r^2(1-\mu^2)})] +$$
$$+ \int_{r_1}^{r} \frac{\sigma_{ab} Bx \, dx}{\sqrt{x^2 - r^2(1-\mu^2)}} \exp[-\sigma_{ab} n (r\mu - \sqrt{x^2 - r^2(1-\mu^2)})] \Big]\Big] +$$
$$+ \int_0^{\sqrt{1-(r_1/r)^2}} d\mu \Big[\Big[\int_{r\sqrt{1-\mu^2}}^{r_0} \frac{\sigma_{ab} Bx \, dx}{\sqrt{x^2 - r^2(1-\mu_2)}} \times$$

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$$\times \exp[-\sigma_{ab}n(r\mu + \sqrt{x^2 - r^2(1-\mu^2)})] +$$

$$+ \int_{\sqrt{1-\mu^2}}^{r} \frac{\sigma_{ab}Bx\,\mathrm{d}x}{\sqrt{x^2 - r^2(1-\mu^2)}} \times$$

$$\times \exp[-\sigma_{ab}n(r\mu - \sqrt{x^2 - r^2(1-\mu^2)})]\Big]\Big] +$$

$$+ \int_{-1}^{0} \mathrm{d}\mu \int_{r}^{r_0} \frac{\sigma_{ab}Bx\,\mathrm{d}x}{\sqrt{x^2 - r^2(1-\mu^2)}} \times$$

$$\times \exp[-\sigma_{ab}n(r\mu - \sqrt{x^2 - r^2(1-\mu^2)})]\Big\}. \tag{26}$$

It is clear that first of all one must solve the equation

$$P_- = P_+^* + P_+^{IR} \tag{27}$$

to calculate the $T_g(r)$ function and then it will be possible to calculate the values of the energy flux from the star and from the shell observed on the Earth.

6. The Post-Minimum Shell

It is shown by the observations of Feast and Glass (1973) that the shells of RCB stars exist for a long time after minimum. It is probable that they exist during the whole quiet stage of a RCB star. It is likely that most of the time the shell is optically thin, so that

$$\tau = \sigma_{ab}n(r_o - r_i) \ll 1 \tag{28}$$

in the visible and, therefore, in the infrared. Since

$$P_- \sim \tau; \qquad P_+^* \sim \tau; \qquad P_+^{IR} \sim \tau^2 \tag{29}$$

one can neglect the heating of grains by the infrared emission of the shell itself. This means that

$$P_+^* = 4\pi Chc^2a^3\left(\frac{kT_*}{hc}\right)^5 (1 - \sqrt{1-(r_*/r)^2})\,\zeta(5) \tag{30}$$

and, therefore

$$\frac{r}{r_*} = \left\{1 - \left[1 - \frac{5k\zeta(6)\,T_g^6\,[[2+(m')^2+(m'')^2]^2 + (2m'm'')^2]}{2h\tilde{\sigma}\zeta(5)\,T_*^5 m'm''}\right]^2\right\}^{-1/2} \tag{31}$$

The numerical results obtained from this relation are shown in Figure 1 where the temperature of a grain is plotted vs its distance from the star's centre for the different temperatures of the stellar photosphere. The dashed line shows the boundary where carbon becomes oversaturated and so condensation is possible. The position of this

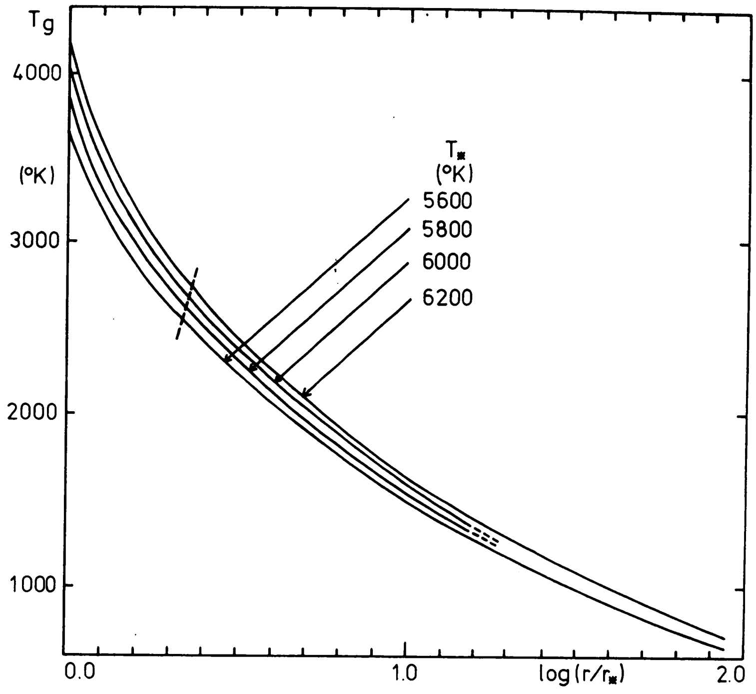

Fig. 1. Temperature of a grain plotted against the distance from the star's centre.

boundary is not very certain because the upper atmosphere of the star was assumed to be isothermal. However the sharp inner border of the dust shell undoubtedly exists. The position of this boundary depends on the carbon density and the temperature of the photosphere. If this conception is correct, then the RCB phenomenon is the result of a sudden moving of this boundary toward the star and the consequent growth of the optical thickness of the shell.

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