Recent Advances in Supernova Theory

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In this paper, I summarize two new developments in the theory of core-collapse supernovae. The first is the recent establishment of an analytic context for understanding neutrino-driven explosions. Converting the supernova problem into an eigenvalue problem, Burrows & Goshy (1993) have derived a critical condition on neutrino luminosity and mass accretion rate through a stalled bounce shock for instability and explosion. The second development is the recent calculation of Burrows & Fryxell (1993) of the boost in the neutrino luminosities by the Rayleigh-Taylor-like overturn of the shocked mantle of a protoneutron star. This boost may turn duds into explosions and may be the missing ingredient of supernova theory.

1. Introduction

Core-collapse supernova predominate in the supernova bestiary (van den Bergh & Tammann 1991), but have challenged theorists during the entire post-war era of astrophysics. The sparseness of data that directly probe the dynamics of collapse and shock generation has hobbled advances in supernova theory, as has the wider than normal range of physical inputs required from the gravitational, neutrino, hydrodynamic, transport, thermodynamic, and nuclear realms. Extracting the essential elements of the explosion mechanism has not been easy. As a result, supernova theory has been perceived at various times to be confusing, arcane, hopeless, muddled, or vulnerable to the quick fix by a well-meaning Cincinnatus.

However, to those who take the time to understand the science of core-collapse and protoneutron stars, much of supernova theory is mature. In fact, there is a growing feeling that all the pieces of the supernova puzzle are falling into place. In this paper, I summarize two new developments in the theory of supernova explosions. The first development is the establishment by Burrows and Goshy (1993, BG) of an analytic theory of neutrino-driven explosions. These authors have converted the supernova problem into an eigenvalue problem for the shock radius and discovered a critical condition for explosion between the neutrino luminosity and the accretion mass flux ($\dot{M}$). By identifying the early phase of the explosion with a wind and using the results of Duncan, Shapiro, & Wasserman (1986), BG have approximately derived the dependence of the supernova energy on the magnitude of the driving neutrino luminosity and its duration.

The second development is the demonstration by Burrows & Fryxell (1993, BF) that the large neutrino luminosities required to ignite and drive the supernova (BG) can result from the convective overturn of the unstable mantle of the protoneutron star. Within ten milliseconds of shock stagnation, the 2-D hydrodynamic calculations of BF manifest a convective flash that restarts the supernova. By means of this instability, heat and leptons are rapidly advected to the neutrinospheres and radiated at nearly twice the standard rates.

2. A General Theory of Neutrino-Driven Supernovae

Supernova theory has been retarded by the reliance on complicated radiation-hydrodynamic codes that run seldom. The essential elements of the supernova mechanism have been

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discovered only piecemeal. What is lacking is a simple and quantitative theory of the mechanism of supernova explosions. Under the assumption that the bounce shock aborts, BG constructed such a theory. They derived a critical condition on the neutrino luminosity and the mass accretion flux such that Wilson's neutrino-mediated mechanism obtains. Their theory attempts to replace partial differential equations (PDE's) with ordinary differential equations (ODE's) and converts the explosion problem into an eigenvalue problem.

A bounce-shock stalls into accretion within 10-20 milliseconds of its creation. The radius that it achieves (R_s) depends predominantly on the mass accretion flux through it ($\dot{M}$) and the core luminosities of the electron (L_{ν_e}) and anti-electron neutrino species, those most strongly coupled to the shocked matter. R_s is analogous to the shock stand-off distance above the white dwarf in an AM Her system (Chevalier and Imamura 1982) and in the steady-state it does not depend on the radius out to which the bounce-shock was originally thrown by the rebounding core. In BG, R_s is derived using the three equations of hydrodynamics, the neutrino transfer equations, with lepton and energy source and sink terms, and suitable boundary conditions at the shock and the neutrinosphere.

Derived from the hydro equations, the basic ODE's are:

$$\frac{dv}{dr} = \frac{\frac{v}{2r}(v_e^2 - 4c^2) + (\gamma_4 - 1)(H - C)}{c^2 - v^2} \tag{2.1}$$

$$\frac{dT}{dr} = \frac{1}{C_V}\left[\frac{(H-C)}{v}\left(1 - \frac{(\gamma_4 - 1)\chi}{c^2 - v^2}\right) + \chi\frac{2(v^2 - 1/4v_e^2)}{r(c^2 - v^2)}\right] \tag{2.2}$$

$$\frac{dP}{dr} = \frac{-\frac{\rho c}{r}(2v^2 - \frac{v_e^2}{2}) + \rho v(\gamma_4 - 1)(H - C)}{c^2 - v^2} \tag{2.3}$$

$$\frac{dY_e}{dr} = (P - M)/v \tag{2.4}$$

$$\dot{M} = -4\pi r^2 \rho v \tag{2.5}$$

$$L_{\nu_e} = \left(\frac{7}{16}\right) 4\pi R_\nu^2 \sigma T_\nu^4 \, , \tag{2.6}$$

where $\chi = \frac{T}{\rho}\frac{\partial P}{\partial T}\Big|_V$, R_{ν_e} is the ν_e- neutrinosphere radius, C_V is the specific heat at constant volume, $\gamma_4 - 1 = \frac{\partial P}{\partial U}\Big|_V$, c is the speed of sound, $v_e^2 = \frac{2GM}{r}$, H and C are the neutrino heating and cooling rates per mass, respectively, P and M are the electron production and electron-neutrino production rates, respectively, and the other symbols have their standard meanings. H, C, P, and M are obtained using extensions of the same arguments used in Bethe and Wilson (1985).

The luminosity, L_{ν_e}, and T_ν fix the position of the neutrinosphere, R_ν. For a given R_s, equations (2.1)–(2.6) (some are redundant) can be solved between R_ν and R_s. BG's prescription for deriving the eigenvalue, R_s, was to find the R_s at a given L_{ν_e} and $\dot{M}$ at which the ν_e "optical" depth from R_ν to infinity is 2/3.

The crucial discovery of this study is that for a given $\dot{M}$, there is a *critical* L_{ν_e}, above which there is no steady-state solution. BG identify situations with super-critical luminosities with the supernova explosion. Temperature profiles at the critical luminosities for various $\dot{M}$'s are depicted in Figure 1. These profiles are similar to those obtained with a more complete 1-D radiation/hydro code. A critical curve of L_{ν_e} vs. $\dot{M}$ can be derived (Figure 2) that separates success from failure. A power-law fit to the results of

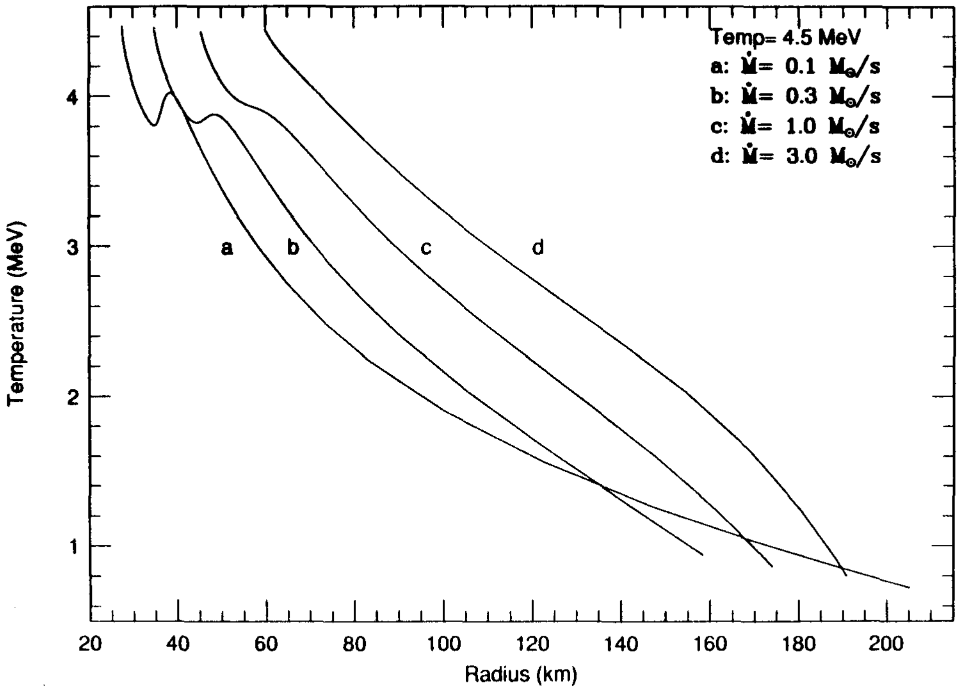

FIGURE 1. The temperature (in MeV) versus the radius (in kilometers) for four values of the accretion mass flux ($\dot{M} = 0.1, 0.2, 1.0, 3.0 M_\odot/\mathrm{s}$). For these curves, L_{ν_e} is at its respective critical value. The neutrinospheric temperature was set equal to 4.5 MeV. The reader should consult Burrows and Goshy (1993) more details.

BG for the critical accretion rate is,

$$\dot{M}_{\rm crit} \sim 1.1 M_\odot/\mathrm{s} \left(\frac{L_{\nu_e}}{5\times 10^{52}\,\mathrm{ergs/s}}\right)^{2.3} . \qquad (2.7)$$

The high power of 2.3 emphasizes the stiff dependence of explosion on the neutrino luminosity. Improvements in the assumptions and approximations, in particular in the neutrino "transport," and different core masses will change the specific numbers. However, the finiteness of the stable branch and the existence of a critical condition seem to be robust.

Using eq. (2.7), BG concluded that accretion alone does not power a spherical supernova explosion. A core luminosity of whatever provenance (diffusion, convection, etc.) is required, unless the iron core envelopes are much denser than in most current progenitors. This conclusion explains why Wilson and Mayle (1992) have needed to evoke doubly-diffusive core mixing to enhance the driving neutrino luminosities that are otherwise too small.

After the condition in eq. (2.7) is achieved, the explosion should develop into a transient neutrino-driven wind (Burrows 1987). Duncan, Shapiro, and Wasserman (1986) have already performed a preliminary investigation of winds from neutron stars. Scaling up their results to the supernova regime, BG obtain for the mechanical luminosity of the wind,

$$L_w \sim 4.2\times 10^{51}\,\mathrm{ergs/s} \left(\frac{L_{\nu_e}}{5\times 10^{52}\mathrm{ergs/s}}\right)^{3.5} \left(\frac{1.3 M_\odot}{M}\right)^2 \left(\frac{30\mathrm{km}}{R_\nu}\right)^{4/3} . \qquad (2.8)$$

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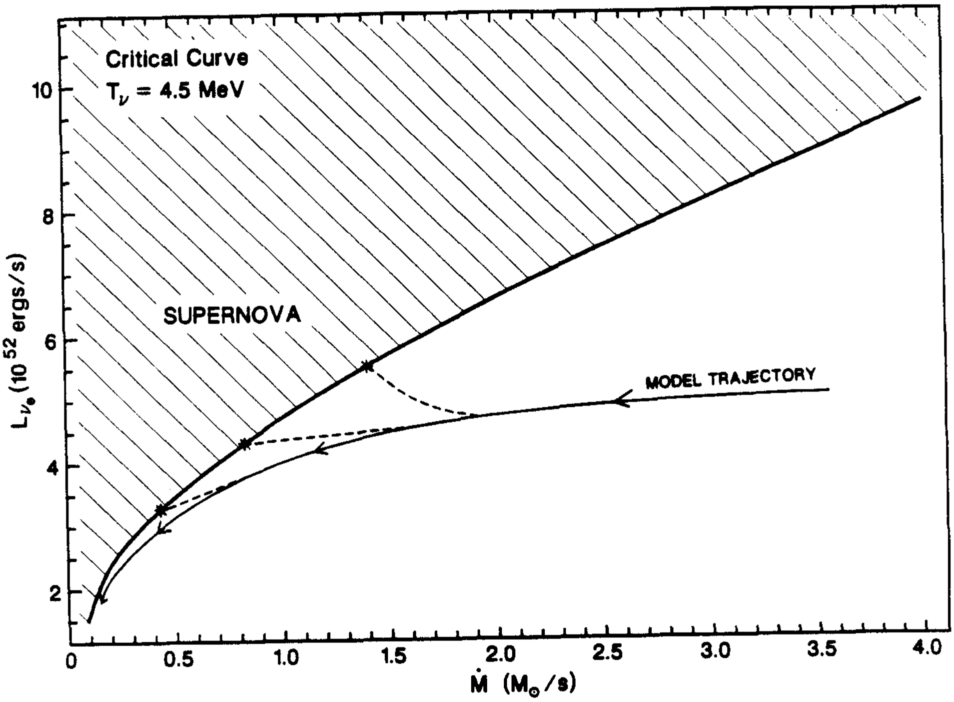

FIGURE 2. The critical curve of L_{ν_e} (in 10^{52} ergs/s) versus accretion mass flux $\dot{M}$ (in $M_\odot$/s). A model in the hatched region should "supernova."

The steep dependence on L_{ν_e}in eq. (2.8) is its most important feature. This high power echoes that in eq. (2.7) and serves to reemphasize the sensitivity of supernova theory to the driving neutrino luminosities. If we integrate L_w over time, we obtain the total mechanical energy pumped into the supernova after the onset of the blast:

$$E_s \sim 4.2 \times 10^{51}\,\text{ergs} \left(\frac{\tau}{1\text{s}}\right) \left(\frac{L_{\nu_e}}{5 \times 10^{52}\text{ergs/s}}\right)^{3.5} \left(\frac{1.3 M_\odot}{\dot{M}}\right)^{2} \left(\frac{30\text{km}}{R_\nu}\right)^{4/3}, \tag{2.9}$$

where we have arbitrarily assumed that L_{ν_e} is constant over a time τ. If we constrain E_s to be equal to 10^{51} ergs, we can derive the τ's and E_ν's required at given L_{ν_e}'s. For $L_{\nu_e} = 8.0 \times 10^{52}$ ergs/s, τ ~45 milliseconds, E_{ν_e} ~3.6 $\times 10^{51}$ ergs, and E_ν(total) $\sim$ 2 $\times$ 10^{52} ergs, all reasonable and within bounds. A slightly higher L_{ν_e} has great leverage in decreasing τ and E_ν, since $\tau \propto E_s/L_{\nu_e}^{3.5}$. Note that the long delays of 100's of milliseconds to seconds seen in the detailed hydro calculations could be artifacts of the missing neutrino flux and that the "long-term" mechanism may be a misnomer.

3. The Convective Trigger

The sputtering shock and electron-capture before and after bounce together create such wildly varying entropy and composition profiles that Rayleigh-Taylor-like instabilities are inevitable (Burrows & Lattimer 1988). The salient discoveries of the earlier work of Burrows & Fryxell (1992) were that the instability does encompass the neutrinospheres, is grossly aspherical, achieves high Mach numbers (~1.0), and starts almost immediately after the shock stalls. Hence, the rapid advection of heat from opaque to transparent regions and the enchancement in the driving neutrino luminosities are naturally achieved.

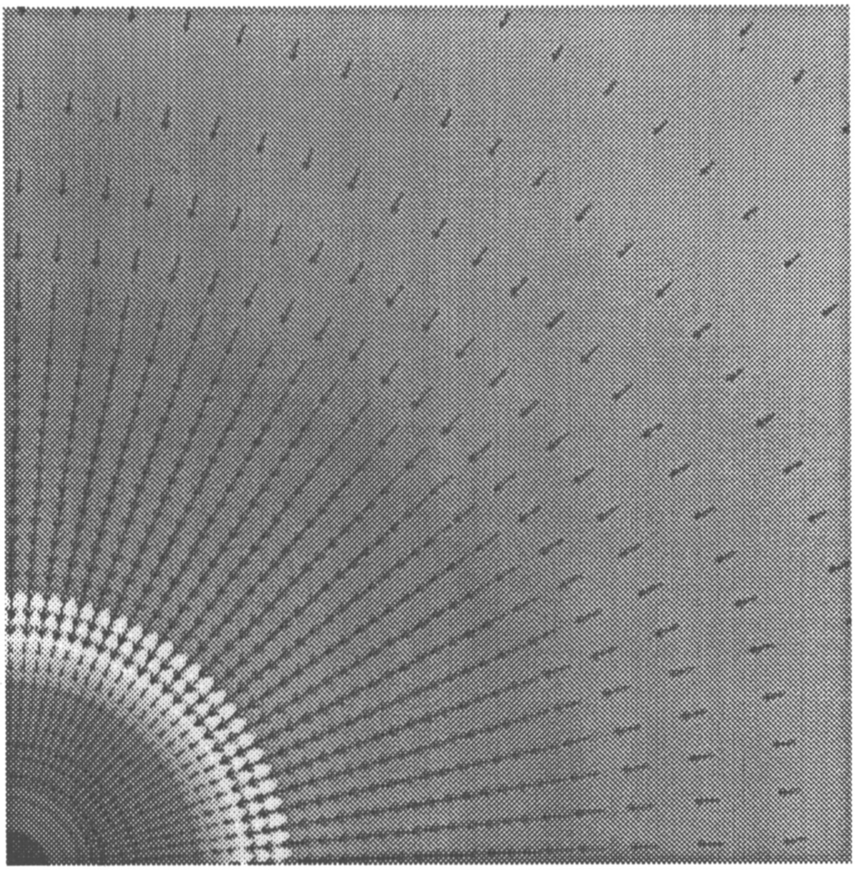

FIGURE 3. The temperature distribution in a protoneutron star just after the bounce shock stalls (2 milliseconds). The inner radius is at 20 kilometers. Refer to Burrows & Fryxell (1993) for details.

With a one-dimensional, transport algorithm coupled to a two-dimensional hydrodynamics code, BF recently showed that when 2-D convection without transport is not adequate, that 2-D hydro with transport with exactly the same initial model and EOS is adequate for explosion. These calculations demonstrated the existence of the convective boost, and showed that it can turn a failure into a success.

The specific algorithms employed by BF will not be discussed here and the reader is referred to that work for details. Figures 3 – 6 depict the temperature distributions during the first 30 milliseconds (at 2, 14, 20, and 30 milliseconds) after the shock stalls, with velocity vectors superposed. As one can see, particularly in Figures 4 and 5, the rapid overturn of the zones from 20 to ~100 kilometers dredges heat outward, creating a grossly asymmetrical luminosity field. In the calculations of BF, the enhanced neutrino luminosities blow a large bubble that is driven into explosion. After only 30 milliseconds, the shock has moved from ~120 kilometers to 360 kilometers. The same structure with the same neutrino-transport algorithm, initial model, and EOS does not explode within this time if constrained to 1-D. L_{ν_e} is enhanced between 10 and 30 milliseconds from $\sim 4 \times 10^{52}$ ergs/s in the 1-D calculations to between 4 and 8.5×10^{52} ergs/s in the 2-D calculations. During the same time, the $\bar{\nu}_e$ luminosity ($L_{\bar{\nu}_e}$) is boosted from $2–4 \times 10^{52}$

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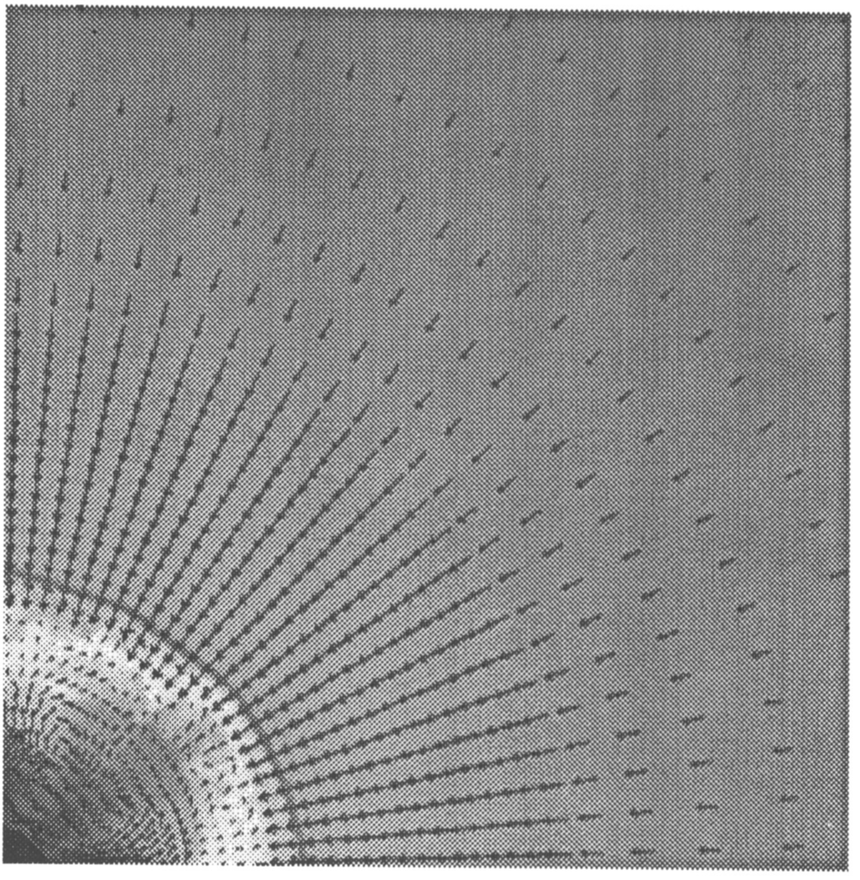

FIGURE 4. Same as Fig. 3, but at $t = 14$ milliseconds. The overturning motions are clearly seen.

ergs/s (1-D) to 2–6 × 10^{52} ergs/s (2-D). Heat and leptons are rapidly dredged up from the opaque inner zones and radiated from the neutrinospheres (60–80 kilometers) at an accelerated rate. Turning on transport and capture does not suppress the instability. In fact, the more rapid loss of electron and thermal pressure causes the inner zones to sink deeper into the potential well to higher gravities, which increases the overturn speeds and decreases the overturn timescales. The factor-of-two enhancements in L_{ν_e} and $L_{\bar{\nu}_e}$ (the convective "flash") is just what is needed to explode the supernova within only tens of milliseconds of stalling (Burrows & Goshy 1993). This short time suggests that the neutrino-driven mechanism of supernova explosions needn't be as "long-term" (hundreds of milliseconds to seconds) as was originally formulated (Wilson 1985).

If such high luminosities as are quoted above can be maintained for $\sim$ 50 milliseconds, using eq. (2.9) we see than an explosion energy of $\sim 10^{51}$ ergs can easily be achieved. In fact, since for a pre-explosion accretion rate near $3M_\odot$/s the outer mantle may be unbound (BG), and ^{56}Ni production may provide ~10% of the supernova energy, the requirements on the driving luminosity after explosion can be relaxed. Hence, an explosion that starts within tens of milliseconds and is driven for only a quick 50 milliseconds is plausible.

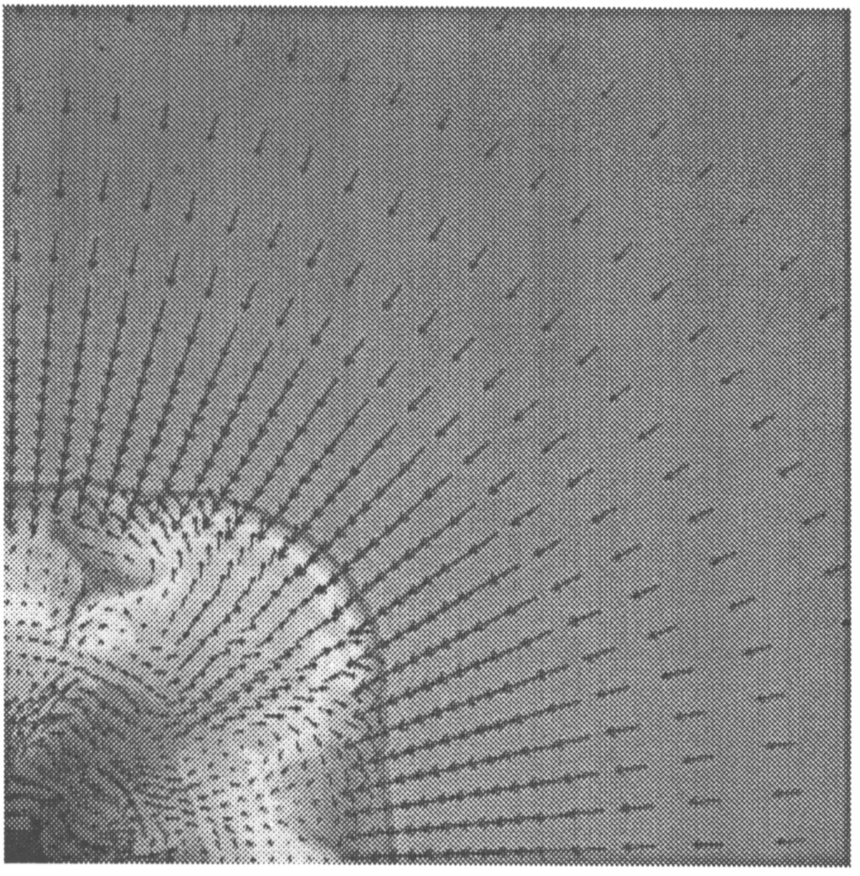

FIGURE 5. Same as Fig. 4, but at $t = 20$ milliseconds.

4. Conclusions

The neutrino signature of the convective flash (its rise time, duration, magnitude, etc.) should be distinctive in the multitude of underground detectors being readied for the next galactic core collapse. Table 1 from Burrows, Klein, & Gandhi (1992) provides some characteristics of these facilities and an estimate of the total integrated event number due to a collapse at 10 kpc.

LVD, MACRO, SNO, and Super Kamiokande (SK) are particularly noteworthy as neutrino telescopes and should all be online (in some capacity) by 1996. SNO and SK should catch at least 50 and 500 events, respectively, at 10 kpc, during the convective flash.

LVD and MACRO may each acquire during this convective flash more events than were garnered in total from SN 1987A by all Earth's detectors.

Clearly, a neutrino light curve from a galactic collapse will speak volumes about the supernova phenomenon. However, the mantle instability that we have highlighted in this paper may also generate sufficient gravitational radiation (GR) to be detected by the second-generation LIGO (Thorne 1987). If the collapsing core is not generally rotating rapidly, the major GR signature of supernova will be due to the overturning convective motions. Frequencies of 50–1000 Hz (peaking near a few hundred Hz?) may predominate and if at least $10^{-10}M_{\odot}c^2$ of this is radiated within 100 milliseconds of bounce, strains

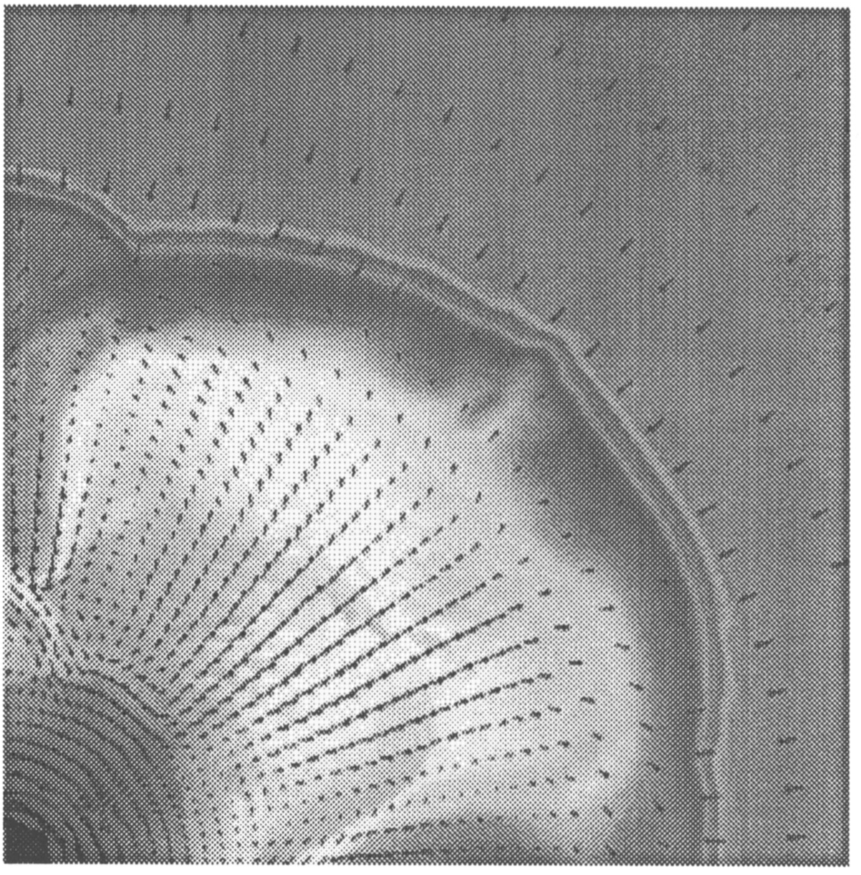

FIGURE 6. Same as Fig. 4, but at $t = 30$ milliseconds. The supernova explosion is well developed.

of $\sim 10^{-21}$ at 10 kpc may be expected. Using the arguments of Thorne (1987), we find a range for supernova detection by the advanced LIGO of 10–30 kpc. This implies that a supernova anywhere in the galaxy may be within reach via gravity waves. Whether we can legitimately conclude this awaits more detailed calculations, but the possibility of probing the multi-dimensional nature of protoneutron stars with GR should not be taken lightly.

The large neutrino luminosity contrasts of BF (as much as a factor of two) lend credence to the possibility that net neutrino emission anisotropies can give the residual neutron star a significant kick. If we assume that 10% of the total neutrino energy radiated is radiated during the convective flash and take the blob sizes and timescales derived in BF, we can derive net anisotropies of $\sim$10%. This yields a neutron star recoil speed of 200–300 km/s, and implies that overturn instabilities may solve more than one problem in the physics of compact objects.

For nucleosynthetic reasons (Thielemann, this volume), it is natural to associate the mass cut with the inner edge of the oxygen-burning shell. It is there that one usually (though not always) finds ledges in $\eta(= 1 - 2Y_e)$ and density. If we assume that this edge falls in at $\frac{1}{\sqrt{2}}$ times the free-fall rates after the collapse rarefaction reaches it, we derive

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TABLE 1. Supernova Neutrino Telescope Characteristics

Detector	Total mass (tons) (Fiducial Mass)	Composition	Threshold (MeV) at 10 kpc	# Events
		CERENKOV:		
KIII	3000 (2140)	H_2O	5	370
Super Kamiokande	40,000 (32,000)	H_2O	5	5500
SNO	1600/1000	H_2O/D_2O	5	780
		SCINTILLATION:		
LVD	1800 (1200)	Kerosene	5–7	375
MACRO	1000	"CH_2"	10	240
Baksan	330 (200)	"White Spirits" "CH_2"	10	70
LSND	200	"CH_2"	5	70
Borexino	300	$(BO)_3(OCH_3)_3$	~ 0.2	200
CalTech	1000	–	2.8	290
		DRIFT CHAMBER:		
ICARUS	3600	^{40}Ar	5	120
		RADIOCHEMICAL:		
Homestake ^{37}Cl	610	C_2Cl_4	0.814	4
Homestake ^{127}I	–	NaI	0.664	25
Baksan ^{37}Cl	3000	C_2Cl_4	0.814	22
		EXTRAGALACTIC:		
SNBO	100,000	$CaCO_3$	–	10,000
JULIA	40,000	H_2O	–	10,000

that it takes approximately 100 milliseconds $\left(\frac{R_{ox}}{10^8\text{cm}}\right)^{3/2}\left(\frac{1.4M_\odot}{M_{ox}}\right)^{1/2}$ for the oxygen shell to achieve the inner zones. With realistic values of R_{ox} and M_{ox}, we see that this time may be between 100 and 500 milliseconds. This is long compared to the timescale of the convection-assisted re-ignition, but short compared to the duration of the traditional long-term mechanism. How everything associated with collapse and explosion is timed remains to be seen, but all that is relevant is coming into increasingly sharper view. The basic mechanism of the supernova explosion may soon become clear to all.

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