THE ANISOTROPY OF THE UNIVERSE AT LARGE TIMES

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The most important cosmological observation in the last forty years has undoubtedly been the discovery of the microwave background. As well as confirming the existence of a hot early phase of the Universe, by its spectrum, its remarkable isotropy indicates that the Universe must be very nearly spherically symmetric about us. Because of the revolution of thought brought about by Copernicus, we are no longer vain enough to believe that we occupy any special position in the Universe. We must assume, therefore, that the radiation would appear similarly isotropic in any other place. One can show that the microwave radiation can be exactly isotropic at every point only if the Universe is exactly spatially homogeneous and isotropic, that is to say, it is described by one of the Friedmann models. (Ehlers *et al.*, 1968). Of course, the Universe is neither homogeneous nor isotropic locally. This must mean that the background radiation is not *exactly* isotropic, but only isotropic to within the very good limits set by the observations (about 0.1%). One would like to know, however, what limits the observations place on the large-scale anisotropies and inhomogeneities of the Universe. One would also like to know why it is that the Universe is so nearly, but not exactly, isotropic.

Because the large-scale structure of the Universe must be so close to that of a Friedmann model, it seems reasonable to study the above questions by analysing the behaviour of small perturbations from a Friedmann model and calculating what anisotropy they would produce in the background radiation. The perturbations can be divided into two classes: the inhomogeneous perturbations and the homogeneous anisotropic ones. The former kind have been considered by Sachs and Wolfe (1967), Rees and Sciama (1968) and other authors. Such inhomogeneous perturbations would produce small-scale anisotropy in the background radiation. From the fact that no such anisotropy has been detected, one can place limits of about one part in 100 on the relative size of density inhomogeneities of mass greater than 10^{15} $M_{\odot}$ at the time of decoupling. What Collins and I have done, on the other hand, is to consider the behaviour of homogeneous but anisotropic perturbations from a Friedmann model and consider what limits can be set from the observations on large-scale anisotropies such as rotation or shear of the Universe. One can divide spatially homogeneous anisotropic perturbations into various classes according to the types of symmetry that they possess. This classification scheme was first developed by Bianchi and has been extended by Estabrook *et al.* (1968) and by Ellis and McCallum (1969). In the case of the $k=1$ (closed) Friedmann model, the perturbations have to be of type IX. For this model the observational upper limits on the microwave anisotropy places limits on the rotation of 3×10^{-11} s of arc per century, if the radiation was last scattered at a redshift z of about 7, and 2×10^{-14} s of arc per century if the radiation has not

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been scattered since a redshift z of 1000. In other words, a set of axes fixed to distant galaxies would not be rotating with respect to a set of inertial axes defined by gyroscopes to within this accuracy. These remarkable results could be regarded as an observational vindication of Mach's principle which states the local inertial frame should be determined by some sort of average over all the matter in the Universe. One can also place an upper limit of one part in 1000 on the shear or anisotropy in the rate of expansion of the Universe. From this one can deduce that the Universe must have been nearly isotropic back to a redshift of at least 600 (if the radiation was last scattered at a redshift z of 7) and isotropic back to a redshift of 100000 (if the radiation was last scattered at a redshift z of 1000).

The results for the $k=0$ (parabolic) Friedmann model are somewhat similar, though not so spectacular. In this case the perturbations have to be of Bianchi types I or VII_0. The type I perturbations are the simplest and correspond to the Universe expanding at different rates in the three orthogonal directions in the Euclidean space sections. However the type VII_0 perturbations are a more general class in which the direction of the rotation and the principal axes of shear have a sort of spiral behaviour.

The ratio of the length-scale of this spiral to the present Hubble radius is an arbitrary parameter and will be denoted by x. It does not make much sense to consider homogeneous perturbations whose length-scales are less than the length-scales of local inhomogeneities such as clusters and superclusters of galaxies. We therefore took 1/25 as a lower limit for x though 1 might seem a more natural value. With $x=1/25$ the upper limit on the rotation is about 2.5×10^{-5} s of arc per century (for $z=7$) and 1.5×10^{-7} (for $z=1000$). With this extreme value of x the limit one can place on the anisotropy of the Hubble constant is only one part in 10 (for $z=7$) or one part in 25000 (for $z=1000$). These limits imply that the Universe could have been highly anisotropic at redshifts greater than 12 or 25000 respectively.

For the $k=-1$ (hyperbolic) Friedmann model, the perturbations can be of Bianchi types V or VII_h. Type V is the simpler but type VII_h is the more general class. Like type VII_0 it has an arbitrary parameter x which is the ratio of the length-scale of the spiral behaviour of the perturbations to the present Hubble radius. With the extreme value of 1/25 for x, one obtains an upper limit to the rotation of 8×10^{-5} s of arc per century and to the anisotropy of the Hubble constant of one part in 8.

From the above one can see that the observed isotropy of the microwave background implies that, on a large scale the Universe must be nearly isotropic at the present time. The question then arises: why should the Universe be so isotropic in the large scale, even though it is certainly not isotropic locally? In attempts to answer this various dissipative processes have been suggested, such as neutrino viscosity (Misner, 1968a, b) and particle creation (Zel'dovich, 1970), which could reduce the anisotropy in the early stages of the Universe. However such processes could not remove the anisotropy completely, so the Universe would remain isotropic at later times only if the Universe were stable against small anisotropic perturbations. Collins and I have therefore analysed the stability of Friedmann models to homogeneous anisotropy perturbations. For the $k=-1$ (hyperbolic) model, the type V perturbations all die away with time but some of the type VII_h perturbations grow in the later stages of the expansion when the matter density becomes so low that it is no longer dynamically important. This result holds regardless of the exact nature of the matter content of the universe, providing only that it satisfies certain physically reasonable conditions. For the $k=0$ (parabolic) model, both the type I and VII_o perturbations die away while for the $k=1$ (closed) model, the perturbations decrease in amplitude until the model reaches its maximum radius and starts to recollapse.

In view of these results, one might expect that the Universe would be nearly isotropic at late times if and only if it was expanding with nearly the minimum velocity required to avoid recollapse, i.e. if it were nearly a $k=0$ Friedmann model. If it was expanding much faster, the matter would have become dynamically unimportant at an early stage and there would have been time for anisotropic perturbations to grow large. If it was expanding much slower, it would have recollapsed before reaching the present radius and there would not have been time for the anisotropy to be damped out. Thus the explanation of the present isotropy of the Universe is that the present rate of expansion or Hubble constant H is nearly equal to the critical value $(8\pi G\varrho/3c^2)^{1/2}$ required to avoid recollapse (ϱ is the density of the Universe). In other words, ϱ is nearly equal to $3c^2H^2/8\pi G$. The density of observed luminous matter satisfies this relation to within a factor of 100 and most if not all the discrepancy may be made up by forms of matter such as intergalactic gas, neutrinos or black holes that have not been observed yet.

One now has to face the question of why the Universe should be expanding at so nearly the critical rate to avoid recollapse. It seems difficult to explain this in terms of processes in the early stages of the Universe because the differences would be so small at these epochs: a reduction of the rate of expansion by one part in 10^{12} at the time when the temperature of the Universe was 10^{10} K would have resulted in the Universe starting to recollapse when its radius was only 1/3000 of the present value and the temperature was still 10000 deg. The only 'explanation' we can offer is one based on a suggestion of Dicke (1961) and Carter (1970). The idea is that there are certain conditions which are necessary for the development of intelligent life: out of all conceivable universes, only in those in which these conditions occur will there be beings to observe the Universe. Thus our existence requires the Universe to have certain properties. Among these properties would seem to be the existence of gravitationally bound systems such as stars and galaxies and a long enough time-scale for biological evolution to occur. If the Universe were expanding too slowly, it would not have this second property for it would recollapse too soon. If it were expanding too fast, regions which had slightly higher densities than the average or slightly lower rates of expansion would still continue expanding indefinitely and would not form bound systems. Thus it would seem that life is possible only because the Universe is expanding at just the rate required to avoid recollapse.

The conclusion is, therefore, that the isotropy of the Universe and our existence are both results of the fact that the Universe is expanding at just about the critical rate. Since we could not observe the Universe to be different if we were not here, one can say, in a sense, that the isotropy of the Universe is a consequence of our existence.

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DISCUSSION

Heller: Do you intend to extend your results to models with non-vanishing cosmological constant? I think the situation will be much more complicated. Will the results be quantitatively different?

Hawking: If Λ is large and negative, anisotropy does not damp out in the course of time. The universe collapses much sooner than in models without the Λ term and there would not be time for life to develop. If Λ is large and positive anisotropy does damp out but galaxies do not form in the late stages of evolution. Therefore the only universes which contain human beings are those in which Λ is very small or zero.

Grishchuk: Anisotropic homogeneous cosmological models constitute a rather narrow class. I think that all the homogeneous models are the sum of a symmetric background model and some simple perturbation modes. For example, the Bianchi type IX model is identical to the sum of a closed Friedmann background and the longest gravitational wave corresponding to wave number $n=3$ in the Lifshitz classification. The Bianchi type V model contains gravitational waves and rotation and so on. I believe that to get a homogeneous model one can proceed in the following way: start with a symmetric background (e.g. a 3-space of constant curvature or a product of S^2 and $\mathbb{R}^1$) and perturb it in such a way that one does not destroy homogeneity. Therefore I do not think that it is possible to get out of these models reliable information about some important physical quantities like the time of isotropisation because when inhomogeneous perturbations are included these answers may be drastically changed.

Hawking: I agree that the homogeneous modes are only a subset of all possible modes but they have the advantage that they are simple to analyse. Inhomogeneous modes could have had greater amplitudes at redshifts smaller than those considered above. For all models it is sufficient to show that only one homogeneous mode is unstable in order to show that the isotropy of the Friedmann universe is unstable. The homogeneous modes in the $k=0$ and -1 models contain all possible perturbations due to gravitational waves.