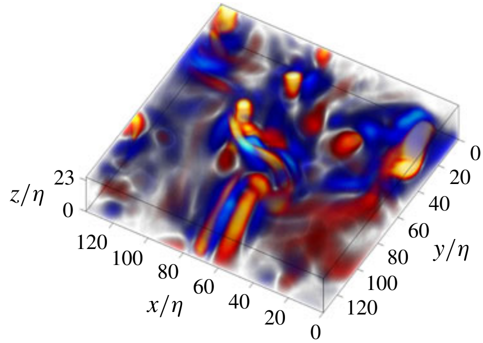
Focus on Fluids

journals.cambridge.org/focus

New insights into the fine-scale structure of turbulence



Michael Wilczek†

Max Planck Institute for Dynamics and Self-Organization, 37077 Göttingen, Germany

In a recent study, Lawson & Dawson (*J. Fluid Mech.*, vol. 780, 2015, pp. 60–98) present experimental results on the fine-scale structure of turbulence, which are obtained with a novel variant of particle image velocimetry, to elucidate the relation between the small-scale structure, dynamics and statistics of turbulence. The results are carefully validated against direct numerical simulation data. Their extensive study focuses on the mean structure of the velocity gradient and the pressure Hessian fields for various small-scale flow topologies. It thereby reveals the dynamical impact of turbulent strain and vorticity structures on the velocity gradient statistics through non-local interactions, and points out ways to improve low-dimensional closure models for the dynamics of small-scale turbulence.

Key words: intermittency, turbulence modelling, turbulent flows

1. Introduction

The investigation of the fine-scale structure of fully developed turbulence has been at the heart of turbulence research ever since the early days of the field. The kinetic energy dissipation, for example, is at the centre of the celebrated Kolmogorov phenomenology. The considerable spatial fluctuations of the energy dissipation, taken into account in the extended phenomenology from 1962, are a signature of the intermittent nature of turbulence. Intermittency has been observed in countless experimental and numerical studies and marks one of the central challenges in deriving a statistical theory of fully developed turbulence. It is also thanks to pioneering numerical work that we now know that intermittency manifests itself in coherent small-scale structures, such as vortex tubes and strain sheets. In fact, turbulence can be envisioned as the entangled dynamics of such structures throughout their lifetime, starting from their births out of instabilities, through mutual advection and straining until their deaths by viscous decay.

† Email address for correspondence: michael.wilczek@ds.mpg.de

doi:10.1017/jfm.2015.536

Comprehensive information on the fine-scale structure of three-dimensional turbulence is encoded in the velocity gradient field $A_{ij}(\boldsymbol{x}, t) = \partial u_i(\boldsymbol{x}, t)/\partial x_j$. Understandably, velocity gradients are notoriously hard to measure experimentally owing to the dimensionality of the field and the necessity to accurately resolve the finest scales of turbulent motion. The dynamical equation of the velocity gradient field is obtained by taking the spatial gradient of the Navier–Stokes equation, thereby highlighting smaller-scale features of the flow. The gradient dynamics, of course, displays the same mathematical intricacies as the velocity dynamics such as nonlinearity, non-locality and their interplay with viscous dissipation. However, the amplification or depletion of velocity gradients induced by the nonlinear advective term in the Navier–Stokes equation as well as the isotropic part of the pressure Hessian takes a particularly simple, local form of a quadratic nonlinearity. In two seminal works by Vieillefosse (1982) and Cantwell (1992) it was shown analytically that the local nonlinearity, when considered as the only dynamical effect, leads to an unphysical blowup of the velocity gradients in a finite time. An important lesson learned from these works is that non-local pressure and viscous effects are necessary to regularize the dynamics and to prevent the finite-time singularity. While the viscous contribution is qualitatively well understood, for example in terms of the linear diffusion model by Martin, Dopazo & Valino (1998), it remains one of the central challenges to understand the non-local pressure contribution to the velocity gradient tensor dynamics arising through the deviatoric part of the pressure Hessian.

Recent years have brought significant progress in terms of experimental measurements of velocity gradients as well as in terms of phenomenological models capturing features of the dynamics, as summarized in the reviews by Wallace & Vukoslavcevic (2010) and Meneveau (2011), respectively. With their recent work, Lawson & Dawson (2015) push the state of the art of this exciting subfield of turbulence research one step further in three respects. By a sophisticated experimental set-up, they were able to obtain high-fidelity, volumetric and time-resolved measurements of the velocity gradient and pressure Hessian fields which give novel insights into the statistical signatures of the small-scale structure of turbulence. All of their results are furthermore carefully compared to publicly available direct numerical simulation (DNS) data from the Johns Hopkins turbulence database (Li *et al.* 2008). Finally, they use their results to confirm a recent closure theory for the pressure Hessian introduced by Wilczek & Meneveau (2014) and furthermore make suggestions for future improvements.

2. Overview

The main question pursued by Lawson & Dawson (2015) is how the small-scale structure, dynamics and statistics of turbulence are interrelated, a long-standing and fundamental question in the field. To contribute to the answer, they acquire experimental data from a comparably large French washing machine set-up, consisting of a dodecagonal water tank of 2 m in height and 2 m in diameter equipped with two slowly turning, counter-rotating impellers at the top and at the bottom. This generates an axisymmetric turbulent flow at $Re_\lambda \approx 180$ with a Kolmogorov length scale of $\eta \approx 0.93$ mm and a Kolmogorov time scale of $\tau_\eta \approx 0.88$ s. The advantage of such a large experimental set-up is that even the finest scales of turbulence are still accessible to particle image velocimetry (PIV) techniques. For the current study, a hybrid PIV method recently introduced by Lawson & Dawson (2014) is used, which combines scanning PIV with tomographic reconstruction. The experimental results

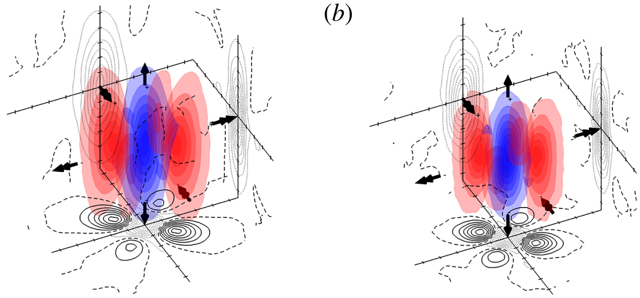


FIGURE 1. Conditional Q field for a strain-dominated region in the eigenframe of the strain tensor. (*a*) DNS data and (*b*) experimental data. Figures from Lawson & Dawson (2015), reused with permission of the authors.

are compared with those obtained from a publicly available dataset of homogeneous isotropic turbulence obtained by DNS at $Re_\lambda \approx 433$ (Li *et al.* 2008), showing good qualitative and quantitative agreement despite the Reynolds-number disparity.

To reduce the complexity of the information contained in the velocity gradient field, it is common practice to study two of the, in the case of isotropic turbulence, five independent invariants of the velocity gradient tensor, $Q = -\mathrm{Tr}(\boldsymbol{A}^2)/2$ and $R = -\mathrm{Tr}(\boldsymbol{A}^3)/3$. Here Q can be understood as characterizing the competition between enstrophy and strain, whereas R represents the competition between enstrophy production and strain production. The second invariant, Q, is intimately related to the pressure, as can be seen from the Poisson equation $\Delta p = 2Q$ obtained from the Navier–Stokes equation. The pressure field therefore can be understood as a non-local functional of the Q field.

With their technique, the authors were able to acquire a large number, $O(10^3)$, of statistically independent spatio-temporal measurements. Each of these short 'movies' of a measurement volume of $135\eta \times 134\eta \times 25.4\eta$ consists of 10 snapshots in time, separated by $0.068\tau_\eta$. One such snapshot is visualized in the figure accompanying the title. It shows vorticity-dominated ($Q > 0$) regions in red as well as strain-dominated ($Q < 0$) regions in blue. The visualization demonstrates that the tube- and sheet-like structures of the enstrophy and strain fields live in close neighbourhood.

To make contact to the statistics of small-scale turbulence, the authors pick a number of points in the R–Q plane, representative of various flow topologies, and investigate the mean Q field under these conditions. One such conditional Q field is depicted in figure 1 for a flow configuration that is dominated by strain and strain production. As expected, the central region in this plot is occupied by a strain structure (blue), which is elongated along the intermediate strain eigendirection. Interestingly, such straining regions are, on average, flanked by an asymmetric quadruplet of vorticity-dominated structures (red) elongated in the same direction, which form a 45° configuration with the compressional and extensional axes. A striking agreement between experiment and simulation is observed. This statistical analysis not only confirms the above qualitative observation that strain and enstrophy live side by side, but also gives a precise quantification of their average arrangement. The authors then investigate in great detail how these turbulent structures prevent the blowup induced by the local nonlinearity through the non-local part of the velocity gradient dynamics.

The direct, non-parametric estimates of the conditional Q field are critically compared with a parametrized estimate, which retains terms up to second order in the velocity gradient weighted by scalar 'correlation functions' encoding the spatial dependence. This so-called stochastic estimation put forward by Adrian (1994) and coworkers is extremely helpful to improve the statistical convergence of such complex statistical quantities. Lawson & Dawson (2015) extend this idea further by making contact to a recent closure theory suggested by Wilczek & Meneveau (2014), which quantifies non-local pressure Hessian effects on the velocity gradient statistics starting from the assumption of Gaussian velocity fields and subsequently taking into account non-Gaussian features by calibration against DNS data. When restricted to time-reversible terms, as dictated by the Gaussian closure assumption, their experimental data confirm the recent theoretical result. At the same time, the more general stochastic estimation also suggests the inclusion of irreversible terms, which points at a subtle but potentially important refinement of the closure model.

3. Future

Over the past 25 years we have seen a rapid evolution of experimental and computational techniques to study turbulent flows. Experiments now can gather fine-scale information on turbulence which previously was only accessible to numerical simulations. With the increase of computational power, numerical simulations on the other hand now can forge into Reynolds-number ranges previously only accessible to experiments. The contribution by Lawson & Dawson (2015) is an inspiring example for these recent developments and highlights the synergetic potential of experimental, numerical and theoretical techniques that are now in the hands of the community. It appears likely that precise comparisons of numerical and experimental data will further evolve to become a standard procedure in the field, providing not only confidence in the characterization of the subtle details of turbulent flows, but also a testbed for novel theoretical approaches and reduced models. With these latest developments, establishing a quantitative connection between the small-scale coherent structure and the intermittent and non-Gaussian statistical properties of turbulence may have finally come within reach.

References

ADRIAN, R. J. 1994 Stochastic estimation of conditional structure: a review. *Appl. Sci. Res.* **53** (3–4), 291–303.

CANTWELL, B. J. 1992 Exact solution of a restricted Euler equation for the velocity gradient tensor. *Phys. Fluids* A **4** (4), 782–793.

LAWSON, J. M. & DAWSON, J. R. 2014 A scanning PIV method for fine-scale turbulence measurements. *Exp. Fluids* **55** (12), 1857.

LAWSON, J. M. & DAWSON, J. R. 2015 On velocity gradient dynamics and turbulent structure. *J. Fluid Mech.* **780**, 60–98.

LI, Y., PERLMAN, E., WAN, M., YANG, Y., MENEVEAU, C., BURNS, R., CHEN, S., SZALAY, A. & EYINK, G. 2008 A public turbulence database cluster and applications to study Lagrangian evolution of velocity increments in turbulence. *J. Turbul.* **9**, N31.

MARTIN, J., DOPAZO, C. & VALINO, L. 1998 Dynamics of velocity gradient invariants in turbulence: restricted Euler and linear diffusion models. *Phys. Fluids* **10** (8), 2012–2025.

MENEVEAU, C. 2011 Lagrangian dynamics and models of the velocity gradient tensor in turbulent flows. *Annu. Rev. Fluid Mech.* **43** (1), 219–245.

VIEILLEFOSSE, P. 1982 Local interaction between vorticity and shear in a perfect incompressible fluid. *J. Phys. France* **43** (6), 837–842.

WALLACE, J. M. & VUKOSLAVCEVIC, P. V. 2010 Measurement of the velocity gradient tensor in turbulent flows. *Annu. Rev. Fluid Mech.* **42** (1), 157–181.

WILCZEK, M. & MENEVEAU, C. 2014 Pressure Hessian and viscous contributions to velocity gradient statistics based on Gaussian random fields. *J. Fluid Mech.* **756**, 191–225.

https://doi.org/10.1017/jfm.2015.536 Published online by Cambridge University Press