

SUGGESTION ON THE PROBLEM OF TIME

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ABSTRACT. Having analysed the real reason why the TDT second differs the TAI second, the TDT's property, unit as well as its definition, and the radical disadvantages existed in the problem of time, we present a new program on the problem of time.

1 INTRODUCTION

Several recommendations on the problem of time were presented at the 21st General Assembly of the IAU. We think the recommendations still need to be improved. This is why this paper is presented.

If a symbol, except the TDT and TT, used here is the same as the recommendations, its meaning is just the same as it is in the recommendations. We define : the TDT denotes the apparent geocentric ephemeris time argument and the TT the ideal form of the TAI.

Throughout the paper, we discuss the problem always under the accuracy of $O(c^{-2})$.

2 THE GEOCENTRIC UNITS AND THE SI UNITS

Let y^α ($\alpha = 0, 1, 2, 3$, $y^0 = ct, y^i = y^1, y^2, y^3$) express the geocentric quasi Cartesian space-time coordinate system, its metric tensors $g_{\alpha\beta}$ are:

$$\begin{cases} g_{00} = -1 + 2U_e/c^2 \\ g_{0i} = g_{i0} = 0 & (i = 1, 2, 3) \\ g_{ij} = \delta_{ij}[1 + 2U_e/c^2] & (i, j = 1, 2, 3) \end{cases} \quad (1)$$

where the U_e is the sum the Earth's gravitational potential and the tide potential of all other bodies.

From Eq.(1), we can see the metric tensors $g_{\alpha\beta}$ are dimensionless quantities. That means the space-time characteristic of the y^α system is completely irrelative to the measuring units of space-time used by the y^α system, so any set of space-time measuring units

can be used by y^α system.

As well known, the coordinate time t of y^α system is defined by the proper time of a standard clock rested at infinite distance far from the geocenter. So the proper time unit of the standard clock rested at infinite distance inevitably defines the coordinate time unit of the y^α system. Since various standard clocks, the units of which differ from each other, can be used to define the coordinate time scale, so there are many coordinate time scales which can be used by y^α system. The difference between any two of all the different coordinate times must be a proportional constant. On the contrary, if there are many time scales, each of which differs from another by a proportional constant, and if some one of them can be used as the coordinate time of some space-time coordinate system, then any one of them can also be used as the coordinate time scale of the space-time coordinate system. And all of these different coordinate time scales should be understood to be defined by the different proper times of various standard clocks rested at infinite distance far from the geocenter. The TT and the TCG can be regarded just so for the geocentric space-time coordinate system.

On the geoid of the Earth, the proper time of the standard clock used to define the TCG at infinite distance is just equal to the TAI, so the unit of the TCG is defined by the proper time unit of the atomic clock rested at infinite distance far from the geocenter. This condition is also called the geocentric state, so the measuring units of space-time corresponding to using the TCG are generally called the geocentric units. So when we consider the TAI as the proper time on the geoid of the Earth, we must use the geocentric units in the y^α system. On the geoid of the Earth, the proper time of the clock used to define the TT at the infinite distance is not equal to TAI but $(1 - L_G)TAI$, so the clock used to define the TT is not the atomic clock but a hypothetical standard clock. When it is rested at infinite distance far from the geocenter, its coordinate time is just equal to the proper time of the atomic clock on the geoid of the Earth, so proper time unit, SI second, can be a replacement of the unit coordinate time of the hypothetical clock rested at infinite distance far from the geocenter. So though theoretically the unit of the TT should be defined by the unit coordinate time of the hypothetical clock rested at infinite distance far from the geocenter, in fact it is realized by its replacement, SI second of the atomic clock on the geoid of the Earth. So the measuring units of space-time corresponding to using the TT are generally called the SI units. The difference between the TT and the TAI is only a constant. So when we consider the TAI as the coordinate time of the y^α system, we must use the SI units in the y^α system.

3 TDT'S PROPERTY, UNIT AND ITS DEFINITION

Since the apparent geocentric ephemerides are always derived from the planet ephemerides established in the barycentric system of the solar system by means of the four dimensional space-time transformation between the geocentric system and the barycentric system of the solar system, so we can get the conclusions as follows:

1) the four dimensional space-time transformation produced the apparent geocentric ephemerides must be performed between the geocentric space-time coordinate system and a barycentric space-time system of the solar system in which the planet ephemerides have

been established;

2) we should understand the TDT from the process of the four dimensional space-time transformation. That means when the geocentric system coordinate time is a known quantity in this transformation process, the TDT is that of the geocentric system coordinate time; otherwise, it is a geocentric system coordinate time derived from the planet ephemeris time argument TB (TDB) at the geocenter.

Let $x^\alpha (x^0 = cT, x^i = x^1, x^2, x^3)$ express the barycentric quasi-Cartesian space-time coordinate system of the solar system, its metric tensors $G_{\alpha\beta}$ are:

$$\begin{cases} G_{00} = -1 & + \frac{2U}{C^2} \\ G_{i0} = G_{0i} = 0 \\ G_{ij} = \delta_{ij}[1 + \frac{2U}{C^2}] \end{cases} \quad (i, j = 1, 2, 3) \quad (2)$$

when the geocentric units are used, i.e. the TAI is considered as a proper time, the coordinate time t of the y^α system is the TCG, and the coordinate time T of the x^α system is the TCB. According to following relationship between TCB and TB given by the IAU

$$TB = (1 - L_B)TCB \quad (3)$$

we can conclude the barycentric space-time coordinate system of the solar system in which the planet ephemerides have been set up is not the x^α system but the $x_1^\alpha (x_1^0 = CT_1, x_1^i = x_1^1, x_1^2, x_1^3)$ space-time system defined as follows:

$$x_1^\alpha = (1 - L_B)x^\alpha \quad (\alpha = 0, 1, 2, 3) \quad (4)$$

The T_1 in Eq.(4) is the TDB. So at this time, the relationship between the coordinate time TCG of the y^α system and the planet ephemeris time argument TDB should have been derived based on the space-time transformation between the y^α system and the x_1^α system. Under the condition that the geocentric units are used in all space-time systems, the result should be as follows:

$$\begin{aligned} \Delta TCG &= \int (1 - \frac{U_{ext}(x_e) - \overline{U_{ext}(x_e)}}{C^2} - \frac{V_e^2 - \overline{V_e^2}}{2C^2})(1 - L_G)^{-1} dTDB \\ &- V_e \cdot (1 - L_G)^{-1}(x - x_e)/C^2 \end{aligned} \quad (5)$$

where \overline{A} denotes a average over a long enough period.

So when the TCG is a unknown quantity, the TDT should be

$$\begin{aligned} \Delta TDT &= \Delta TCG|_{x=x_e} \\ &= \int (1 - \frac{U_{ext}(x_e) - \overline{U_{ext}(x_e)}}{C^2} - \frac{V_e^2 - \overline{V_e^2}}{2C^2})(1 - L_G)^{-1} dTDB \end{aligned} \quad (6)$$

If the TB is a unknown quantity and the TCG is a known quantity, then the TDT is that of the TCG. So the relationship between the TDT and TAI, at this time, should be

$$TDT = TCG = (1 - L_G)^{-1}(TAI + 32.184s) \quad (7)$$

So the TDT is either the TCG or a special TCG derived from the TB at the geocenter, its unit must differ from the time unit of the TAI. That is to say, the problem that the TDT's second differs from the TAI's second is the inevitable result that we in practice, have used the geocentric units in the y^α system as well as x^α system and x_1^α system at the same time.

When the SI units are used, i.e. the TAI can be considered as the coordinate time of the geocentric space-time system, the coordinate time t of y^α system is the TT. The planet ephemerides can be established in such a barycentric space-time system of the solar system $x_2^\alpha(x_2^0 = CT_2, x_2^i = x_2^1, x_2^2, x_2^3)$ that can be defined as follows:

$$x_2^i = (1 - L_c)x^i \quad (i = 1, 2, 3) \tag{8}$$

$$T_2 = (1 - L_c)T \tag{9}$$

Since the TCB is the coordinate time of x^α system when the geocentric units are used and the T in Eq.(9) is also the coordinate time of the x^α system but the SI units are used, so the relationship between the TCB and the T in Eq.(9) should be

$$T = (1 - L_G)TCB \tag{10}$$

Based on Eqs.(3), (9) and (10), we have

$$\begin{aligned} T_1 &= TB = (1 - L_B)TCB = (1 - L_C - L_G)TCB \\ &= (1 - L_c)(1 - L_G)TCB = (1 - L_c)T = T_2 \end{aligned} \tag{11}$$

So at this time and under the condition that the SI units are used in all space-time systems, the relationship between the coordinate time TT of the y^α system and planet ephemeris time argument $T_2(TDB)$ can be derived as follows:

$$\begin{aligned} \Delta TT &= \int (1 - \frac{U_{ext}(x_e) - \overline{U_{ext}(x_e)}}{C^2} - \frac{V_e^2 - \overline{V_e^2}}{2C^2}) dTDB \\ &- V_e \cdot (x - x_e) / C^2 \end{aligned} \tag{12}$$

So when the TT is a unknown quantity, the TDT should be

$$\begin{aligned} \Delta TDT &= TT|_{x=x_e} \\ &= \int (1 - \frac{U_{ext}(x_e) - \overline{U_{ext}(x_e)}}{C^2} - \frac{V_e^2 - \overline{V_e^2}}{2C^2}) dTDB \end{aligned} \tag{13}$$

If the TB is a unknown quantity and the TT is a known quantity, then the TT is not only the coordinate time of the y^α system but also the TDT. So only at this time, we can have

$$TDT = TT = TAI + 32.184s \tag{14}$$

So at this time the ,the unit of the TDT is still the same as the unit of the coordinate time of the y^α system, of course, the problem that the TDT's second deffers from the TAI's

second disappears.

4 SUGGESTIONS ON THE PROBLEM OF TIME

From the above analysis, we can see the radical disadvantages on the problem of time are:

1) In the recommendations mentioned above, two time-like arguments (in this paper, coordinate time means time-like argument) TCG and TT have been defined in y^α system, so two kinds of apparent geocentric ephemeris time arguments corresponding to them respectively should have been defined based on either Eqs.(6),(7) or Eqs.(13),(14) respectively, but in fact, the apparent geocentric ephemeris time argument defined by IAU only has one form which is the TT in Eq.(14) in this paper. According to the recommendations mentioned above, when the TCG is used as the coordinate time of the y^α system, the TDT is still the TDT in Eq.(14), this must deny the reasonable difference between the TDT's unit and the TAI's unit under the case that the geocentric units are used.

2) On the one hand under the condition that the geocentric units are used, the TDB has been defined based on Eq.(3), on the other hand we have required that there are only periodic variations not only between the TDT and the TDB but also between the TAI and the TDB. Under this condition, the TAI is the proper time on the geoid of the Earth, and the TDT is the geocentric system coordinate time corresponding to this proper time. They are forever different from each other by a proportional constant, So it is impossible to realize this kind of requirement.

3) On the one hand, we have defined the relationship between the TCB and TB (TDB) based on Eq.(3), that means we have considered the TAI as the proper time and actually used the geocentric units in all space-time systems at the same time; on the other hand, when we need to get the TDT from the TB, we have been using the Eq.(13) for a long time, that means that the TAI has been considered as the coordinate time of the geocentric system and the SI units have also been using in all the space-time systems at the same time. This must lead to the confusion on the using of measuring units of space-time. Just because of this, on the one hand, people define the TDT based on Eq.(14), on the other hand people say that the TDT's second differs from the TAI's second.

4) The planet ephemerides have been set up in the x_1^q system, but the four space-time transformation produced the apparent geocentric ephemerides has been performed between the y^α system and x^α system.

In general relativity, the space-time transformation is always performed under the condition that the same measuring units of space-time must be used in all space-time systems. Whether for one space-time system or for many space-time systems, theoretically only one set of measuring units of space-time is enough. Why now are there two kinds of measuring units of space-time, the geocentric units and the SI units? Because we have been neglecting the depending relationship between the TAI's different properties and the measuring units of space-time. So in order to have a perfect program on the problem of the time, we must consider the TAI either as the coordinate time of the geocentric system or as the proper time. That is we can never consider it both the coordinate time and the proper time at the same time. Thus we can avoid using two kinds of measuring units of space-time at the same time. We must define the TDT from process of four space-time transformation

produced the apparent geocentric ephemerides. This transformation must be performed between the geocentric system and the barycentric system of the solar system in which the planet ephemerides have been established. Under the condition that the TAI is considered as the coordinate time of the geocentric system coordinate time, we present a new program on the problem of time as follows:

- 1) definitely announce: the unique measuring units of space-time used by all space-time systems are the SI units.
- 2) the geocentric system coordinate time is the $TT = TAI + 32.184s$;
- 3) the planet ephemeris time argument $TB(T_2)$ should be defined based on Eq.(9).
- 4) the TDT is defined either by Eq.(13) when TT is a unknown quantity, or by Eq.(14) when TB is a unknown quantity, the four space-time transformation produced apparent geocentric ephemerides must be performed between the y^a system and the x_2^a system.

Though we have modified the definition of the TB , its numerical value is just equal to its original numerical value. So this modification will hardly produce any significant influence we must consider to present planet ephemerides. In this program, there are only periodic variations not only between the TAI and the TB but also between the TDT and the TB . Here doing so is under the condition that the same measuring units of space-time are used in all space-time systems, it will not cause the so-called changes of the astronomical constants with the variation of the space-time systems.

Of course, under the condition that the TAI is considered as the proper time, in other words, under the condition that the geocentric units are the unique measuring units of space-time used by all space-time systems, we can present other program on the problem of time, but it is inferior to the one presented above.

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