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Optical interferometry from the Earth

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Abstract. Ground-based optical interferometers can perform astrometric measurements with a precision approaching $10\,\mu$as between pairs of stars separated by $\sim 10''$ on the sky. These narrow-angle measurements can be used to search for extrasolar planets and to determine their orbital parameters, to characterize microlensing events, and to measure the orbits of stars around the black hole at the center of our Galaxy.

Keywords. instrumentation: interferometers, techniques: high angular resolution, techniques: interferometric, astrometry, gravitational lensing, Galaxy: center

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1. Introduction

Astronomical interferometry at visible and infrared wavelengths has become an important tool in a number of fields, ranging from the measurement of fundamental stellar parameters and the determination of the distribution of circumstellar material to studies of the central regions of active galactic nuclei. Most of these applications rely on measurements of the visibility amplitudes (and sometimes closure phases) with a small number of baselines, and the parametric fitting of models to these data. This paper focuses mostly on a different interferometric technique, namely precise astrometry. Several instruments are currently under construction that will use this method to determine the orbits of extra-solar planets, to observe stars orbiting the black hole at the center of our Galaxy, and to characterize microlensing events. The first of these instruments to enter operation is PRIMA (Quirrenbach *et al.* 1998, Delplancke *et al.* 2000) at the Very Large Telescope Interferometer (VLTI), operated by the European Southern Observatory (ESO) on Cerro Paranal in Chile.

2. Interferometric Astrometry

2.1. *The Basic Principle of Interferometric Astrometry*

Astrometric observations by interferometry are based on measurements of the delay $D = D_{\rm int} + (\lambda/2\pi)\phi$, where $D_{\rm int} = D_2 - D_1$ is the internal delay measured by a metrology system (see Fig. 1), and ϕ the observed fringe phase (see e.g. Quirrenbach 2001 and references therein). D is related to the baseline $\vec{B}$ by

$$D = \vec{B} \cdot \hat{s} = B \cos\theta, \tag{2.1}$$

where $\hat{s}$ is a unit vector in the direction towards the star, and θ the angle between $\vec{B}$ and $\hat{s}$. Each data point is thus a one-dimensional measurement of the position of the star θ, provided that the length and direction of the baseline are accurately known. The second coordinate can be measured with a separate baseline at a roughly orthogonal orientation.

2.2. *Atmospheric Limitations of Ground-Based Astrometry*

The Earth's atmosphere imposes serious limitations on the precision that can be achieved with astrometric measurements from the ground. The first-order terms of the atmospheric wavefront distortions (frequently referred to as *tip* and *tilt*) are global wavefront gradients, which correspond to a motion of the centroid of the stellar light in the two coordinates. Because most of the power of atmospheric turbulence is in these low-order modes, the amplitude of this image motion is similar to the width of the stellar images, i.e., $\approx \lambda/r_0 \approx 0''\!.5 \ldots 1''$. One can obviously reduce this error by taking many exposures and thus averaging over many independent realizations of the atmospheric turbulence, but achieving a precision of a small fraction of a milliarcsecond in this way is clearly not possible.

It helps, however, to make differential measurements over small angles on the sky, i.e., to measure the position of the target star with respect to that of a nearby reference. It can be shown that the variance σ_θ^2 of measurements of the angle θ is given by (Shao & Colavita 1992)

$$\sigma_\theta^2 \approx \frac{16\pi^2}{B^2 t} \int_0^\infty dh\, v^{-1}(h) \int_0^\infty d\kappa\, \Phi(\kappa,h) \cdot [1-\cos(B\kappa)] \cdot [1-\cos(\theta h\kappa)], \tag{2.2}$$

if the integration time $t \gg \max(B,\theta h)/v$. Here $v(h)$ is the wind speed at altitude h, and $\Phi(\kappa,h)$ denotes the three-dimensional spatial power spectrum of the refractive index. It may at first seem surprising that stronger winds should give a smaller measurement error, but within the frozen-turbulence picture a higher wind speed means that one averages faster over independent realizations of the stochastic refractive index fluctuations. Inserting a Kolmogorov power spectrum in Eqn. 2.2 one obtains the two limiting cases

$$\sigma_\theta^2 \approx \begin{cases} 5.25\, B^{-4/3}\theta^2 t^{-1} \int_0^\infty dh\, C_N^2(h) h^2 v^{-1}(h) & \text{for } \theta \ll B/h, \quad t \gg B/v \\ 5.25\, \theta^{2/3} t^{-1} \int_0^\infty dh\, C_N^2(h) h^{2/3} v^{-1}(h) & \text{for } \theta \gg B/h, \quad t \gg \theta h/v \end{cases} \tag{2.3}$$

for long and short baselines, respectively. In particular one can see that for sufficiently small angles θ the important scaling relations $\sigma_\theta \propto \theta$ and $\sigma_\theta \propto B^{-2/3}$ hold for the astrometric error σ_θ. For a good site such as Mauna Kea or Cerro Paranal astrometric measurements with a precision of $\sim 10\,\mu$as are possible over angles of $\sim 10''$. It is also apparent from the factor h^2 under the integral in this equation that the astrometric error is dominated by the turbulence at high altitudes. The low level of high-altitude turbulence at the South Pole would therefore make an astrometric interferometer at a site on the high Antarctic plateau an attractive possibility (Lawrence *et al.* 2004).

2.3. *Dual-Star Interferometry*

Because of the short coherence time of the atmosphere, precise astrometry from the ground requires simultaneous observations of the target and astrometric reference; it is not possible to alternate between the two as in the case of radio interferometry. In a dual-star interferometer, each telescope accepts two small fields and sends two separate beams through the delay lines. The delay difference between the two fields is taken out with an additional short-stroke differential delay line; an internal laser metrology system is used to monitor the delay difference. For astrometric observations, this delay difference ΔD is the observable of interest, because it is directly related to the coordinate difference between the target and reference stars; from Eqn. 2.1 it follows immediately that

$$\Delta D \equiv D_t - D_r = \vec{B} \cdot (\hat{s}_t - \hat{s}_r) = B(\cos\theta_t - \cos\theta_r), \tag{2.4}$$

where the subscript t is used for the target, and r for the reference.

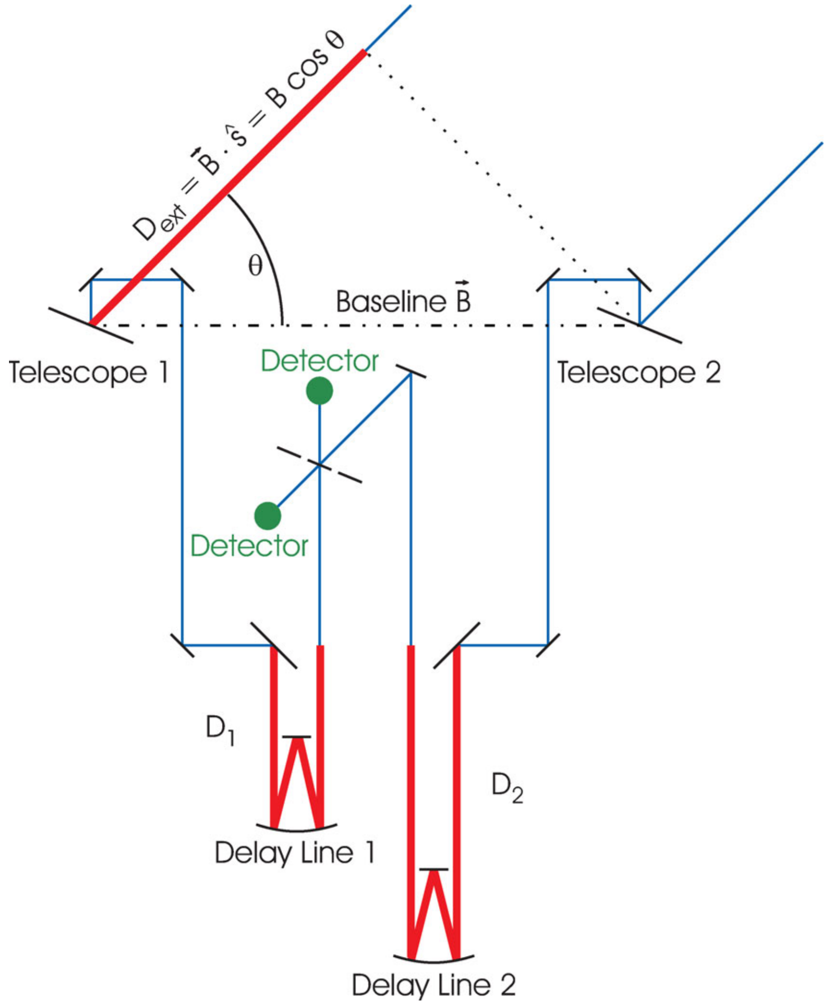

Figure 1. Schematic drawing of the light path through a two-element interferometer. The external delay $D = \vec{B} \cdot \hat{s}$ is compensated by the two delay lines. The pathlengths D_1, D_2 through the delay lines are monitored with laser interferometers. The zero-order interference maximum occurs when the delay line positions are such that the internal delay $D_{\rm int} = D_2 - D_1$ is equal to D.

Measurements of the delay difference between two stars give *relative* astrometric information; this means that the position information is not obtained in a global reference frame, but only with respect to the nearby comparison stars, which define a local reference frame on a small patch of sky. This approach greatly reduces the atmospheric errors, and some instrumental requirements are also relaxed (see below). The downside is that the information that can be obtained in this way is more restricted, because the local frame may have a motion and rotation of its own. This obviously makes it impossible to measure proper motions. Moreover, all parallax ellipses have nearly the same orientation and axial ratio, which allows only "relative parallaxes" to be measured.

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2.4. *Astrometric Precision*

The photon noise limit for the precision σ of an astrometric measurement is given by the expression

$$\sigma = \frac{1}{\mathrm{SNR}} \cdot \frac{\lambda}{2\pi B}. \tag{2.5}$$

Since high signal-to-noise ratios can be obtained for bright stars, σ can be orders of magnitude smaller than the resolution λ/B of the interferometer. With an SNR ~ 50, it is thus possible to attain an astrometric error of $\sim 10\,\mu$as on the longest baselines of the VLTI, comparable to the atmospheric contribution expected for an angular separation of $10'' \ldots 20''$ and half-hour integrations (Shao & Colavita 1992, von der Lühe *et al.* 1995).

The fundamental instrumental requirements can be derived directly from the basic expression of the geometric delay, which can be written as

$$\Delta D \equiv D_t - D_r = \vec{B} \cdot (\hat{s}_t - \hat{s}_r) \equiv \vec{B} \cdot \Delta\vec{s}. \tag{2.6}$$

Here D_t and D_r denote the delay of the target and reference, respectively, $\vec{B}$ is the baseline vector, and $\hat{s}_t$ and $\hat{s}_r$ are unit vectors in the directions towards the two stars. The propagation of systematic errors in measurements of the differential delay $\delta\Delta D$ and of the baseline vector δB to errors in the derived position difference $\delta\Delta s$ can be estimated from the total differential

$$\delta\Delta s \approx \frac{\delta\Delta D}{B} + \frac{\Delta D}{B^2}\delta B = \frac{\delta\Delta D}{B} + \Delta s\frac{\delta B}{B}. \tag{2.7}$$

This formula allows one to draw two important conclusions. First, the systematic astrometric error is inversely proportional to the baseline length. Together with the $B^{-2/3}$ scaling of the atmospheric differential delay r.m.s. this clearly favors longer baselines, up to the limit where the target star gets resolved by the interferometer. The second important conclusion from Eqn. 2.7 is that the relative error of the baseline measurement gets multiplied with Δs; this means that the requirement on the knowledge of the baseline vector is sufficiently relaxed to make calibration schemes possible that rely primarily on the stability of the telescope mount. For a $10\,\mu$as (50 prad) contribution to the error budget for a measurement over a $20''$ angle, with an interferometer with a 100 m baseline, the metrology system must measure $\delta\Delta D$ with a 5 nm precision; the baseline vector has to be known to $\delta B \approx 50\,\mu$m (Quirrenbach *et al.* 1998). For PRIMA it is foreseen that the baseline vector will be determined from repeated observations of stars in the same way that is also customary in radio interferometry.

2.5. *PRIMA Observing and Data Reduction Strategy*

As explained above, astrometric observations with interferometers are equivalent to measurements of delays, i.e., to measurements of the difference in optical pathlength of light from a star at infinity to the two telescopes forming the interferometer†. The accuracy goal of $10\,\mu$as $= 50$ prad corresponding to a total allowable error of 5 nm for a 100 m baseline can only be achieved through a quadruple-differential technique (Quirrenbach *et al.* 2004, Elias *et al.* 2008):

(a) Two stars with small angular separation are observed simultaneously to reduce the effects of atmospheric turbulence.

(b) The optical pathlength within the interferometer is monitored with a laser interferometer. The terms entering the error budgets are thus the differential effects between

† There are additional complications if the delay lines are not evacuated, see Daigne & Lestrade (1999).

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the starlight and metrology beams, due e.g. to misalignments or dispersion between the effective observing wavelength and the wavelength of the metrology system.

(c) The paths of the two stars through the instrument are exchanged periodically by rotating the field by 180°. In this way many systematic errors caused by asymmetries are canceled.

(d) The orbits of extra-solar planets are determined from variations of the positions of their parent stars with time; only differences with respect to the position at some reference epoch matter.

It is important to realize that the raw delays have eleven (!) significant digits; astrometric planet detection implies taking differences of large and nearly equal numbers. The implementation of this quadruple-differential technique therefore requires unusual attention to detail in the understanding and calibration of varied astrophysical, atmospheric, and instrumental effects, in the construction of error budgets, in planning the operations, and in specifying and coding the data reduction software.

In particular, the desired accuracy can only be achieved if all systematic sources that can possibly affect the data are understood properly, and removed in a systematic way. While the magnitude of some astrometric errors can be predicted quite reliably (e.g., those related to atmospheric turbulence), others defy simple analysis and may have to be described with parameterized models (e.g., dynamic temperature gradients in the interferometer light ducts). Experience with other forefront astrometric facilities (e.g., the HIPPARCOS spacecraft, the Mark III Interferometer, the automated Carlsberg Meridian Circle) also shows that completely unanticipated systematic effects almost inevitably show up in the actual data. The ability to detect, diagnose, and remove such unanticipated effects is of paramount importance for the success of astrometric programs.

It is therefore necessary to perform a careful a priori analysis of the errors, and to design and implement systems for a posteriori analysis of remaining trends in the data. One further needs an operation and calibration strategy that takes full advantage of, and optimizes the use of the quadruple-differential technique described above. Finally one needs software to perform the initial steps of the data reduction, including carrying out said differences with appropriate corrections, and conversion of delays to angles on the sky. This data reduction software has to allow inspection of the residuals and to enable searches for remaining systematic trends over several years. The latter capability is required because the integrity of the data can only be checked after the quadruple-differencing process, and because the residuals are dominated by stellar parallax (which has a period of one year) and proper motion.

3. Astronomical Goals of Ground-Based Interferometric Astrometry

3.1. *Astrometric Planet Detection*

The first discovery of a planet orbiting a star similar to our Sun (Mayor & Queloz 1995) has opened a completely new field of astronomy: the study of extra-solar planetary systems. More than 350 planets outside our own Solar System are known to date, and new discoveries are announced at an increasing pace. These developments have started to revolutionize our view of our own place in the Universe. We know now that other planetary systems can have a structure that is completely different from that of the Solar System. Moreover, the existential question whether other habitable worlds exist can for the first time in human history be addressed in a scientific way.

Nearly all known extra-solar planets have been found with an indirect technique, the radial-velocity method. What is actually detected is not the planet itself, but the motion

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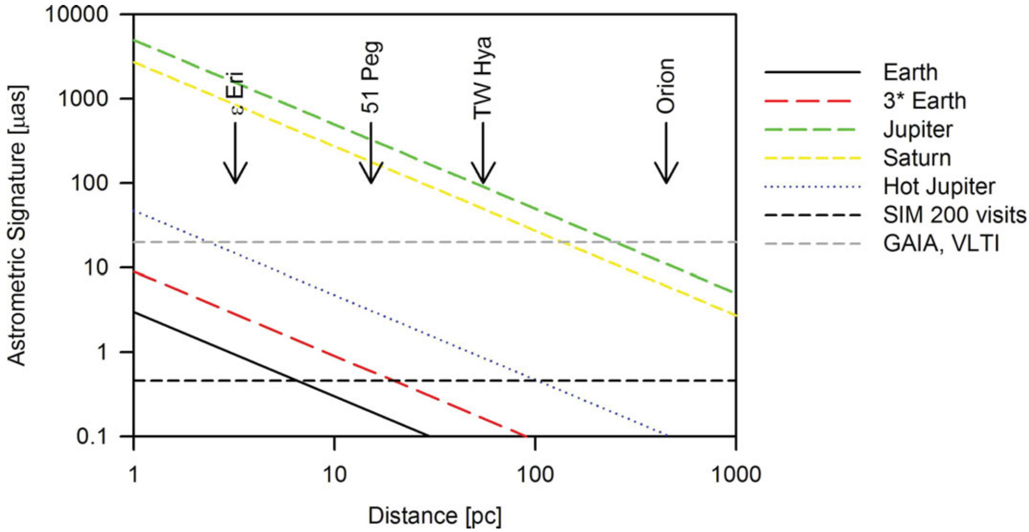

Figure 2. Astrometric signature (semi-amplitude) for five sample planets orbiting a Solar-mass star, as a function of distance. Anticipated detection limits for ground-based (VLTI PRIMA) and space-based (Space Interferometry Mission) instruments are also shown. Adopted from Quirrenbach (2003).

of its parent star around the common center of gravity. The Doppler shift due to the line-of-sight component of this motion can be detected with spectroscopic methods. While radial-velocity surveys have had tremendous successes, it must not be forgotten that they have technical and astrophysical limitations, which necessarily lead to a biased view of exo-planetary astrophysics. It is therefore important to develop complementary techniques, which can give additional information on the systems already detected, and find planets in situations where the radial-velocity technique cannot be used.

The principle of planet detection with astrometry is similar to that behind the Doppler technique: one infers the presence of a planet from the motion of its parent star around the common center of gravity. In the case of astrometry one observes the two components of this motion in the plane of the sky; this gives sufficient information to solve for the orbital elements without $\sin i$ ambiguity. Astrometry also has advantages for a number of specific questions, because this method is applicable to all types of stars, and more sensitive to planets with larger orbital semi-major axes.

From simple geometry and Kepler's Laws it follows immediately that the astrometric signal θ of a planet with mass m_p orbiting a star with mass m_* at a distance d in a circular orbit of radius a is given by

$$\begin{aligned}\theta &= \frac{m_p}{m_*}\frac{a}{d} = \left(\frac{G}{4\pi^2}\right)^{1/3}\frac{m_p}{m_*^{2/3}}\frac{P^{2/3}}{d} \\ &= 3\,\mu\mathrm{as}\cdot\frac{m_p}{M_\oplus}\cdot\left(\frac{m_*}{M_\odot}\right)^{-2/3}\left(\frac{P}{\mathrm{yr}}\right)^{2/3}\left(\frac{d}{\mathrm{pc}}\right)^{-1}.\end{aligned} \tag{3.1}$$

This signature is shown in Fig. 2 for five sample planets (analogs to Earth, a "Super-Earth", Jupiter, Saturn, and a "Hot Jupiter" with $m_p = 1\,M_{\mathrm{jup}}$ and $P = 4$ days) orbiting a $1\,M_\odot$ star.

The specific strengths of the astrometric method enable it to answer a number of questions that cannot be addressed by any other planet detection method. Among the most prominent goals of astrometric planet surveys are the following projects:

• Mass determination for planets detected in radial velocity surveys (without the $\sin i$ factor). The RV method gives only a lower limit to the mass, because the inclination of the orbit with respect to the line-of-sight remains unknown. Astrometry can resolve this ambiguity, because it measures two components of the orbital motion, from which the inclination can be derived.

• Confirmation of hints for long-period planets in RV surveys. Many of the stars with detected short-period planets also show long-term trends in the velocity residuals. These are indicative of additional long-period planets, whose presence can be confirmed astrometrically.

• Inventory of planets around stars of all masses. The RV technique works well only for stars with a sufficient number of narrow spectral lines, i.e., fairly old main-sequence stars with $m_* \lesssim 1.2\,M_\odot$, and around G and K giants. Astrometry can detect planets around intermediate-mass main sequence stars and complete a census of gas and ice giants around stars of all types.

• Detection of gas giants around pre-main-sequence stars, signatures of planet formation. Astrometry can detect giant planets around young stars, and thus probe the time of planet formation and migration. Observations of pre-main-sequence stars of different ages can provide a critical test of the formation mechanism of gas giants. Whereas gas accretion on $\sim 10\,M_\oplus$ cores requires $\sim 10\,$Myr, formation by disk instabilities would proceed rapidly and thus produce an astrometric signature even at very young stellar ages (Boss 1998).

• Detection of multiple systems with masses decreasing from the inside out. Whereas the astrometric signal increases linearly with the semi-major axis a of the planetary orbit, the RV signal scales with $1/\sqrt{a}$. This leads to opposite detection biases for the two methods. Systems in which the masses increase with a are easily detected by the RV technique because the planets' signatures are of similar amplitudes. Conversely, systems with masses decreasing with a are more easily detected astrometrically.

• Determine whether multiple systems are coplanar or not. Many of the known extrasolar planets have highly eccentric orbits. A plausible origin of these eccentricities is strong gravitational interaction between two or several massive planets (Lin & Ida 1997, Papaloizou & Terquem 2001). This could also lead to orbits that are not aligned with the equatorial plane of the star, and to non-coplanar orbits in multiple systems.

• Search for massive terrestrial planets orbiting low-mass stars in the Solar neighborhood. With a $10\,\mu$as precision goal and operating in the K band, PRIMA at the VLTI will be able to look for rocky planets down to a limit of a few Earth masses around nearby M stars.

In summary, astrometry is a unique tool for dynamical studies of extrasolar planetary systems; its capabilities to determine masses and orbits are not matched by any other technique. Astrometric surveys of young and old planetary systems will therefore give unparalleled insight into the mechanisms of planet formation, orbital migration and evolution, orbital resonances, and interaction between planets. The first such program will be carried out with PRIMA at the VLTI, and pave the way towards future more precise astrometric surveys from space.

3.2. *Astrometry of Microlensing Events*

Photometric observations of microlensing events in rich stellar fields — such as the bulge of our Galaxy — have become a widely used tool in several fields of astrophysics.

Microlensing has been used to constrain the mass contained in massive compact halo objects (MACHOs), as suggested by Paczyński (1986), and to search for extra-solar planets (e.g., Beaulieu *et al.* 2006). Parameters of the lensing object can be derived from an analysis of the light curve, but unfortunately there are degeneracies between lens mass, relative proper motion, and relative parallax. These degeneracies can be broken by astrometric observations, determining the position of the center of light as a function of time during the encounter (Miyamoto & Yoshii 1995).

The astrometric signature of microlensing events is of the order of the Einstein radius, i.e., typically $\approx 100\,\mu$as, well within the reach of ground-based interferometry. The differential nature of ground-based astrometry prevents one from measuring absolute parallaxes. This introduces some complications, but such measurements are nevertheless sufficient for the present purpose (Boden *et al.* 1998). For relatively high lens masses (a few $M_\odot$), when the Einstein radius is large, the two images may be resolved by the interferometer, thus producing a binary signature in the observed visibilities (Deplancke *et al.* 2001). The main limitation of ground-based interferometry for observations of microlensing events is the fringe tracking sensitivity, which is sufficient only for events reaching exceptionally high brightness.

3.3. *Astrometry of the Galactic Center Cluster*

Beautiful images and spectral data cubes of the Galactic Center region obtained with speckle techniques and adaptive optics on large telescopes have provided a surprising wealth of information on the central stellar cluster (e.g., Ghez *et al.* 2005, Eisenhauer *et al.* 2005). In addition to providing probes of the gravitational field of the black hole, the stars in the central parsec of the Milky Way pose many interesting questions about their properties, formation, and dynamics.

Interferometry will enable measurements of more subtle effects such as the general-relativistic precession of stellar orbits, as well as the hypothetical precession due to an extended mass distribution (Rubilar & Eckart 2001, Eckart *et al.* 2002). In addition, it has been suggested that flaring infrared emission from the position of the black hole can be used to trace the potential well at a few Schwarzschild radii (Trippe *et al.* 2007). Interferometric astrometry of this source can then used to test general relativity in the strong-field regime. These scientific goals will be pursued with PRIMA (see Bartko *et al.* 2008); the ASTRA project at the Keck Interferometer (Pott *et al.* 2008) and GRAVITY at the VLTI are designed primarily to address these questions. More details can be found in the paper by Eisenhauer (2009) in this volume.

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