CORRESPONDENCE.

ON THE NUMBER OF YEARS IN WHICH PREMIUMS AMOUNT TO TWICE THE TOTAL SUM PAID.

To the Editors of the Journal of the Institute of Actuaries.

DEAR SIRS,—In furtherance of a plan which had proved effective in securing proposals for assurance, a New Business Superintendent of this City sought to discover a method by which he might readily calculate the number of years in which premiums paid on a life assurance policy would at any given rate of interest amount to twice the total of such premiums. By simple experiment with tables of the values of $s_{\overline{n|}}$ he ascertained that, if either the number of years or the interest rate per-cent were given, the other could be found by dividing 125 by the given value.

I am not aware that this simple formula, which gives a remarkably close approximation to the true result, has ever been published. I venture, therefore, to bring it under your notice and to furnish the following algebraical verification

To prove that if $n = 125/100i$, or $100i = 125/n$, $s_{\overline{n+1|}} - 1 = 2n$ approximately.

$$s_{\overline{n+1|}} - 1 = \frac{(1+i)^{n+1} - 1}{i} - 1$$

$$= \frac{1 + (n+1)i + \frac{(n+1)n}{2} \cdot i^2 + \frac{(n+1)n(n-1)}{2 \quad 3} \cdot i^3 + \ldots - 1}{i} - 1$$

$$= (n+1) + \frac{(n+1)n}{2} i + \frac{(n+1)n(n-1)}{2 \cdot 3} i^2 + \ldots - 1$$

$$= n + \frac{n^2+n}{2} \cdot \frac{1{\cdot}25}{n} + \frac{n^3-n}{6}\left(\frac{1{\cdot}25}{n}\right)^2 + \frac{n^4 - 2n^3 - n^2 + 2n}{24}\left(\frac{1{\cdot}25}{n}\right)^3$$

$$+ \frac{n^5 - 5n^4 + 5n^3 + \ldots}{120}\left(\frac{1{\cdot}25}{n}\right)^4 + \frac{n^6 - 9n^5 + 25n^4 \ldots}{720}\left(\frac{1{\cdot}25}{n}\right)^5 + \ldots$$

$$= n(1 + {\cdot}625 + {\cdot}260 + {\cdot}082 + {\cdot}020 + {\cdot}004 + {\cdot}001 + \ldots)$$

$$+ ({\cdot}625 + 0 - {\cdot}163 - 102 - {\cdot}038 - {\cdot}009 \ldots)$$

$$+ \frac{1}{n}(-{\cdot}260 - {\cdot}082 + {\cdot}102 + {\cdot}105 + {\cdot}049 \ldots)$$

$$+ \ldots\ldots$$

$$= n \times 1{\cdot}99 \ldots + {\cdot}31 \ldots + \frac{1}{n}(-{\cdot}09 \ldots) + \ldots$$

$$= 2n \text{ (approx.)}$$

Yours faithfully,

P. D. TOUZEL.

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[Nearly the same result may be obtained by taking $s_{\overline{n+1|}}$, *i.e.*, $1 + (1+i) + (1+i)^2 + \ldots + (1+i)^n$ as roughly equal—on the analogy of Simpson's formula—to $\frac{1}{6}(n+1)\{1 + 4(1+i)^{\frac{1}{2}n} + (1+i)^n\}$. We then have, if $s_{\overline{n+1|}} - 1 = 2n$, $(1+i)^n + 4(1+i)^{\frac{1}{2}n} + 4 = 15\{1 - 2/5(n+1)\}$ whence, approximately, $(1+i)^{\frac{1}{2}n} + 2 = 3{\cdot}873\{1 - 1/5(n+1)\}$;

$$\frac{1}{2} n \log_e(1+i) = \log_e 1{\cdot}873 + \log_e\{1 - 2{\cdot}07/5(n+1)\}$$

$= {\cdot}6275 - 2/5(n+1)$; and $i = \frac{1{\cdot}255}{n} - \frac{4}{5n(n+1)}$ —EDS. *J.I.A.*]

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