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CORRECTION TO

'SPREADABLE ARRAYS AND MARTINGALE STRUCTURES'

B. GAIL IVANOFF and N. C. WEBER

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The statement of Lemma 3.6 in Ivanoff and Weber [2] is incorrect. In fact, conditions (2) and (3) of Lemma 3.6 are equivalent, and necessary but not sufficient for (1). Lemma 3.6 should be stated as follows.

LEMMA 3.6. *Let X be a finite or infinite weak $\mathcal{F}$-SS array. Then:*

(1) *X is $\mathcal{F}$-stationary; and*
(2) *μ_{ij} forms an $\mathcal{F}$-martingale.*

The following counterexample satisfies both (1) and (2) but is not separately spreadable. Let X be a 4×2 array ($X = (X_{ij})$, $i = 1, 2, 3, 4$; $j = 1, 2$) that takes the following values, each with probability $\frac{1}{4}$:

$$\begin{pmatrix} -1 & 1 & -1 & 1 \\ 1 & -1 & 1 & -1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & 1 & -1 \\ -1 & 1 & -1 & 1 \end{pmatrix},$$
$$\begin{pmatrix} -1 & 1 & -1 & 1 \\ -1 & 1 & -1 & 1 \end{pmatrix}, \quad \begin{pmatrix} 1 & -1 & 1 & -1 \\ 1 & -1 & 1 & -1 \end{pmatrix}.$$

We recall the shift operator $\theta_{ab} \circ X = (X_{ij} : i \geq a+1, j \geq b+1)$, $a, b \geq 0$. We see that (1) (and hence (2)) above is satisfied, that is, for $i, j \geq 0$ and all $h \geq i$, $k \geq j$,

$$\theta_{hk} \circ X =_{\mathcal{D}|\mathcal{F}_{ij}} \theta_{ij} \circ X,$$

where $\mathcal{F}_{ij}$ denotes the minimal σ-field generated by (X_{ab}) for $a \leq i$, $b \leq j$.

However the array X is not separately spreadable and so not weak $\mathcal{F}$-separately spreadable. If column 2 is deleted, then the first two columns of the resulting 3×2 array are identical, which is clearly not the case with the original array X.

In fact, necessary and sufficient conditions for the weak $\mathcal{F}$-separately spreadable property follow from the observation that X is weak $\mathcal{F}$-separately spreadable if and only if for every $h, k \geq i$ and $l, m \geq j$ and $s, t, u, v \geq 0$,

$$\phi_{uv} \circ \theta_{hl} \circ X =_{\mathcal{D}|\mathcal{F}_{ij}} \phi_{st} \circ \theta_{km} \circ X,$$

where $\phi_{uv} \circ X$ is the matrix X with column u and row v deleted $(u, v \geq 0)$.

THEOREM 1. *The following are equivalent for a finite or infinite array X.*

(1) *X is weak $\mathcal{F}$-separately spreadable.*

(2) *If $(S, T) \geq (0, 0)$ and $(U, V) \geq (0, 0)$ are any bounded random times such that for every $(i, j) \geq (0, 0)$ and $(h, l) \geq (0, 0)$,*

$$\{(S, T) = (i, j), (U, V) = (h, l)\} \in \mathcal{F}_{ij},$$

then $\phi_{UV} \circ \theta_{ST} \circ X =_{\mathcal{D}} X$.

The proof of the theorem is similar to that of [2, Lemma 4.5] and [1, Theorem 1].

The foregoing can be expressed in terms of a four-dimensional martingale structure by defining $\mathcal{H}_{ijkl} := \mathcal{F}_{ij}$ for all i, j, k, l with associated prediction array

$$\nu_{ijkl} := P(\phi_{kl} \circ \theta_{ij} \circ X \in \cdot|\mathcal{H}_{ijkl}).$$

Next $((S, T), (U, V))$ is an $\mathcal{H}$-adapted random time if

$$(S = i, T = j, U = k, V = l) \in \mathcal{H}_{ijkl} = \mathcal{F}_{ij} \quad \forall i, j, k, l.$$

The four-dimensional martingale property can be defined in a manner analogous to the two-dimensional version in [2, Section 4.2]. As in [2, Lemma 4.5], it is straightforward to show that (1) and (2) of Theorem 1 are equivalent to:

(3) *(ν_{ijkl}) is an $\mathcal{H}$-martingale.*

References

[1] B. G. Ivanoff and N. C. Weber, 'Some characterizations of partial exchangeability', *J. Aust. Math. Soc. (Series A)* **61** (1996), 345–359.

[2] B. G. Ivanoff and N. C. Weber, 'Spreadable arrays and martingale structures', *J. Aust. Math. Soc.* **79** (2005), 277–296.