UNIVERSITY COLLEGE, GOWER STREET, LONDON, W.C.,
11*th December*, 1916.

DEAR SIR,

The letter from Prof. Lodge and the late Mr. Jackson, p. 311 of the *Gazette*, raises a number of points.

With regard to the first and second paragraphs of the letter, the difference between the writers and myself is that they in effect deny that the meaning of the expression $a \times b$ can be affected by what precedes it. I am unable to agree with them, and there is no more to be said. My view of the matter is stated on p. 284.

In the third paragraph the writers say that "the whole question before us is whether $a+b\times c$ is or is not ambiguous under present conventions." It appears to me that a great deal more is involved, as I have endeavoured to show on pp. 282-283.

In particular it is absolutely necessary, if it is permissible to omit brackets, to explain in what manner the sign of division is to be dealt with, and I do not therefore understand why the writers in the sixth paragraph charge me with "dragging a red herring across the trail by discussing the evaluation of a *term* in which division as well as multiplication occurs."

And after all, what are the present conventions? The matter is mentioned in the fourth paragraph, but is dealt with more at length in Mr. Jackson's letter on p. 246 (endorsed by Prof. Lodge on p. 247). Mr. Jackson says that he has looked "somewhat carefully into the conventions as to the sequence in which arithmetical operations are to be performed. Many arithmetical books, in their chapter on Fractions, lay down three rules of interpretation. Stated in their baldest form these rules are:

"1. Multiplications and divisions must be performed before additions and subtractions.

"2. Multiplications and divisions must be performed in order (from left to right).

"3. The word 'of' is, however, equivalent to a bracket.

"It will be convenient to state at once the conclusions I have reached, before entering into the arguments on the subject. These are, that the Rule 1, though not always happily expressed, is a rule of fundamental importance, and is essential to the harmony of arithmetic and algebra; but that Rules 2 and 3 are of an artificial character, that they are not necessary and cannot be defended."

On p. 281 I showed that Rule 1 cannot stand if Rule 2 is abandoned, a fact which Prof. Lodge and Mr. Jackson have not yet disproved. In view of what they now say in their sixth paragraph I ask Prof. Lodge to explain to the readers of the *Gazette* how Rule 1 can stand if Rule 2 be abandoned. If there is any flaw in my reasoning on p. 281, will Prof. Lodge kindly explain it? This is a very simple and unambiguous question, and I submit that I am entitled to an answer. If he cannot show that I am wrong, the claim made by Mr. Jackson and himself to clearness in treating this subject cannot be upheld.

The fifth and seventh paragraphs assert the existence of a convention alleged to be firmly established for more than a century. Whether that is the case or not I do not know, and the point is immaterial, for an appeal to tradition reads very strangely in a mathematical journal. The whole of the argument in these paragraphs falls to the ground unless Prof. Lodge is able to answer the questions I have put to him.

The whole matter is one of great gravity in the teaching of Arithmetic. The teacher who uses unnecessary rules develops in the minds of his pupils a spirit which is antagonistic to progress. During upwards of

thirty years of experience in teaching Mathematics I have never used the convention or conventions alleged to exist. I regard them as absolutely useless, and under no circumstances would I inflict them on my pupils.

M. J M. HILL.

THE LIBRARY.

CHANGE OF ADDRESS.

THE Library is now at 9 Brunswick Square, W.C., the new premises of the Teachers' Guild.

The Librarian will gladly receive and acknowledge in the *Gazette* any donation of ancient or modern works on mathematical subjects.

SCARCE BACK NUMBERS.

Reserves are kept of A.I.G.T. Reports and Gazettes, and, from time to time, orders come for sets of these. We are now unable to fulfil such orders for want of certain back numbers, which the Librarian will be glad to buy from any member who can spare them, or to exchange other back numbers for them :

Gazette No. 8 (very important).
A.I.G.T. Report No. 11 (very important).
A.I.G.T. Reports, Nos. 10, 12.

GLASGOW: PRINTED AT THE UNIVERSITY PRESS BY ROBERT MACLEHOSE AND CO. LTD.