INCOMPLETE DIAGONALS OF LATIN SQUARES

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The following question has been asked by J. Dénes [2]: If $n-1$ elements of the diagonal of an $n \times n$ array are prescribed, is it possible to complete the array to form an $n \times n$ latin square?" It is known that if n diagonal elements are given such a completion is not always possible.

That the answer to Dénes' question is yes follows directly from a theorem of M. Hall Jr. [1].

<u>Given elements</u> $a_1, \ldots, a_n$ <u>(possibly with repetitions) of an abelian group</u> G <u>of order</u> n, <u>there exist two permutations</u> $g_1, \ldots, g_n$ <u>and</u> $g'_1, \ldots, g'_n$ <u>of the elements of</u> G, <u>such that</u> $a_i = g_i + g'_i$ $i = 1, 2, \ldots, n$, <u>if and only if</u> $a_1 + \ldots + a_n = 0$.

The application is as follows. Let $a_1, \ldots, a_{n-1}$ be the prescribed diagonal elements and identify distinct a_i with (some) distinct elements of Z_n. Set $a_n = -(a_1 + \ldots + a_{n-1})$ so that $a_1 + \ldots + a_n = 0$. By Hall's theorem select permutations $g_1, \ldots, g_n$ and $g'_1, \ldots, g'_n$ of the elements of Z_n such that $a_i = g_i + g'_i$. The array (b_{ij}) where $b_{ij} = g_i + g'_j$ is then a latin square and $b_{ii} = a_i$, $i = 1, 2, \ldots, n-1$, thus satisfying the requirements given.

We understand that a different construction was found by E. Milner and J. Schaer.

REFERENCE

1. M. Hall, Jr., A combinatorial problem on abelian groups. Proc. Amer. Math. Soc. 3 (1952) 584-587.

2. J. Dénes, Lecture at University of Surrey, 1967.

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