ON THE SPATIAL DENSITY OF W UMa TYPE STARS

E. Budding
Department of Astronomy
University of Manchester
England

ABSTRACT

Attention is directed to the anomalous incidence of W UMa stars, which can be regarded as coming from not only a disproportionately large accumulation among close binary systems with primaries later than around mid-F spectral type, but also as a deficit at early types.

Doubt is placed on the necessity of a straightforward identification of W UMa type light curves with contact binaries; and this allows some reduction in the estimated spatial incidence of contact binaries, from the figure of Van't Veer (1975), to 8×10^{-4} of all stars.

The incidence is considered, with the aid of some simplifying assumptions, as an example of the general evolution of the distribution of binary systems in the primary spectral type - orbital period plane, subject to some known mechanisms of binary evolution.

1. INTRODUCTION

There are three main empirical points concerning W UMa type systems which have caused them to be at the focus of considerable attention. These relate to (i) their incidence, which is generally taken to be spatially relatively high (Shapley, 1948), (ii) the form of their light curves, which has been closely associated with a proximity so close as to actually imply physical contact or "over-contact" of the two photospheres (Kopal, 1955; Lucy, 1968), and (iii) the existence of a correlation between their periods and colours (Eggen, 1967).

There have been numerous attempts to reconcile the information summarized by such empirical points within the framework of established physical theory, especially, perhaps, with regard to the contact condition - its mechanism and stability - though, according to Mochnacki's (1981) recent survey, still with only partial success.

Z. Kopal and J. Rahe (eds.), Binary and Multiple Stars as Tracers of Stellar Evolution, 351–370.

The purpose of this paper is to look more closely again at the underlying empirical points in the hope of eliciting fresh guidelines for theory.

2. IS THERE A PECULIARITY ASSOCIATED WITH W UMa TYPE LIGHT CURVE INCIDENCE?

In Figure 1 is presented some information on the incidence of Main Sequence stars - brighter and fainter single stars, as well as the primaries of some commonly observed kinds of close binary system. This is taken from a recent collection of data (Budding, 1981) on close binary systems of short period. For comparison some modified Salpeter function curves, showing theoretical distributions of dwarf stars brighter than magnitude 10(1) and 15(2), are shown, where the transformation from mass to spectral type was made on the basis of curves representing unevolved Main Sequence stars drawn from the empirically based compilation of Popper (1980).

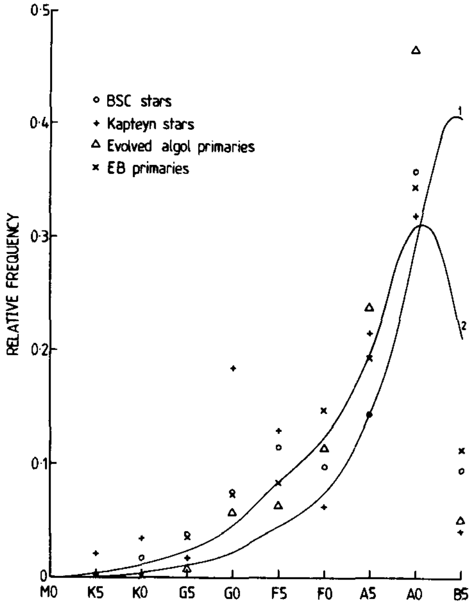

Figure 1. The relative frequency of dwarf stars brighter than some given magnitude in relation to spectral type.

The general trend of fewer stars at types later than A0 can be discerned, though there are complications associated with such things as aging effects, extinction and homogeneity of source material.

In any case, there is a sharp contrast between the distribution of normal (brighter) Main Sequence single stars and primaries in relatively unevolved close binary systems, and the primaries of W UMa systems. This can be seen from Figure 2 where, in addition to the variation of frequency of incidence with primary spectral type, the distribution of the systems in the period - spectral type plane is shown. Whereas from Figure 1 we might expect to observe three or four times the number of early A to early G systems if the primaries were distributed like the other Main Sequence stars, we actually find ten times as many early G as early A W UMa type systems.

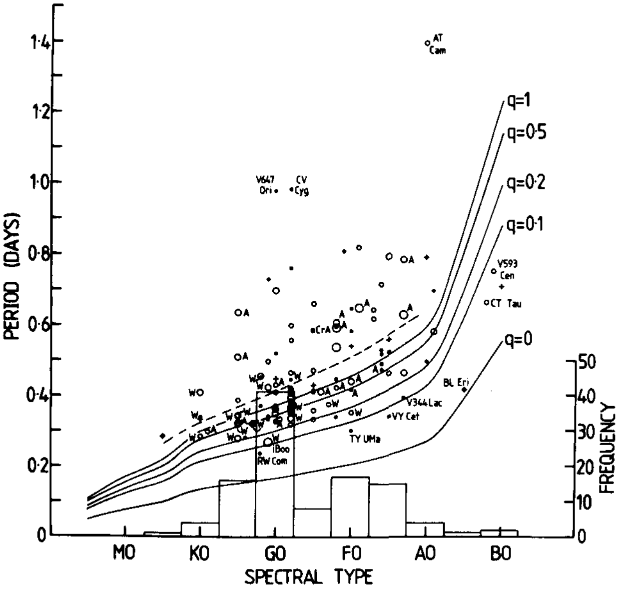

Figure 2. The distribution of W UMa type stars in the orbital period - primary spectral type plane. The dashed line follows the trend of mean periods. The continuous lines give periods at which ZAMS primaries of given spectral type accompanied by stars of relative mass q are in contact with their Roche lobes.

Before leaving Figure 2 some other relevant points may be noted as follows:

1. The correlation between period and colour of these systems, noted by Eggen, is broadly confirmed in the period spectral-type diagram, though there is an appreciable scatter. Though spectral type has been quoted for more systems than those studied by Eggen and is free of reddening complications, it may reflect some qualitative factors in the matter of type assignation, and the sample shown, which comes from various recent catalogues and papers, is certainly inhomogeneous. Comparison of such sources reveals, however, that the type assignation is rarely likely to be in error by more than five type class subdivisions.

2. There is an overlap in the spectral type-period plane between systems of W UMa type and other types of close binary system, i.e. a system like V753 Cyg of primary type F8 and period $0^d.476$ is actually classified as having an Algol type light-curve (Kukarkin et al., 1969) though it lies close to the centroid of the distribution of W UMa-type systems in the spectral type-period plane. Another similar case might be the variable BD And, also of type F8 and period 0.463 but with light curve classified as β Lyr type.

3. Though there is a pronounced peak in the distribution around spectral type GO, light curves which have been classified as of W UMa type do occur at all spectral subgroups from BO to K5.

4. There are a number of exceptional systems, with either anomalously long or short periods. These include: AT Cam, CV Cyg, V647 Ori, AZ Gem and DV Peg (all with periods around 1 day or greater), and V593 Cen, CT Tau, BL Eri, V344 Lac, VY Cet, TY UMa and RW Com (all with periods so short as to suggest "deep contact" if the primaries are at all comparable to Main Sequence stars of the same type). It seems quite possible that some, if not all, of the long period anomalies are not really W UMa stars at all, but RRc type variables (c.f. Eggen, 1967).

3. TOO MANY COOL OR TOO FEW HOT W UMa SYSTEMS?

Having confirmed the existence of a remarkable overabundance of W UMa systems with spectral type around GO, we may next wish to consider whether this anomaly arises from a surfeit among the later type stars, a deficit among the early types, or perhaps both.

In order to examine this we consider first the distributions, with period, of close binary systems of unevolved type of different spectral type groups. In Figures 3 and 4 the frequencies of unevolved close binaries of spectral types A, F and G are plotted normalized against the quoted sample sizes. For comparison we have plotted a curve which could be approximately proportional to the frequency of detected eclipsing systems on the basis of some simplifying assumptions.

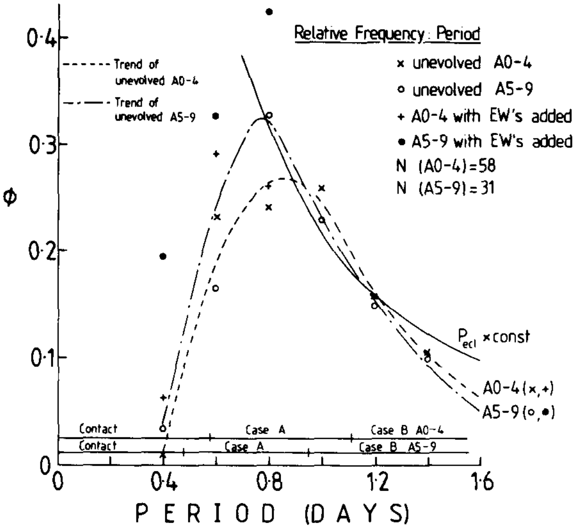

Figure 3. Short period binaries of primary spectray type A.

Such a curve has the form

$$\phi = \text{const.} \times \left(1 + \frac{\Delta P}{P_*}\right)^n \qquad (3.1)$$

where the constant factor has been chosen so that the curve lies close to the observed points in some selected range. P_* corresponds to the period at which a pair of unevolved Main Sequence stars of equal mass, representative of the type range in question would be in "contact" and ΔP is the difference between any particular period P and P_*. The spatial incidence of such binaries in a given interval around P would be proportional to $P^{n+2/3}$, i.e. if spatial incidence was constant, observed incidence would fall off with a $-\frac{2}{3}$ power dependence on period. The form which has been chosen in Figures 3 and 4 sets n = -5/3, which would correspond to a power law distribution of semi-major axes A of the form $g(A) \propto A^{-1}$. Such a power law form has been considered appropriate by, for instance, Tutukov and Yungelson (1980), though they had in mind somewhat wider systems than those considered here.

What can be judged from such diagrams are the following:

1. Though there is some indication of more of a bunching together at shorter periods for the later type stars, the distributions are feasibly of the same form in dependence on period for different mass binaries, i.e. there could exist some function which when suitably scaled for masses or

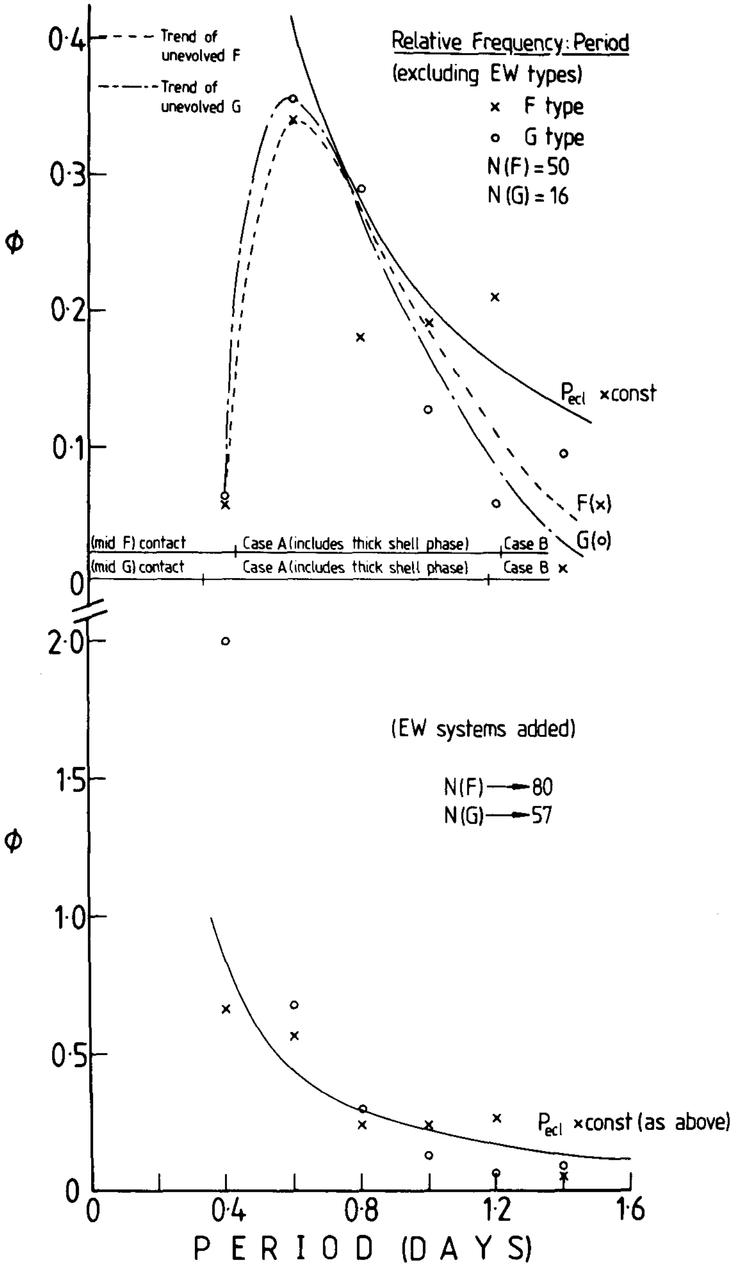

Figure 4. Short period binaries of spectral types F and G.

sizes of the constituent stars could describe the distribution of observed eclipsing unevolved systems of all spectral types with period (c.f. Farinella et al., 1979).

2. The maxima of these distributions occur at period values rather in excess of the contact values, i.e. there is some falling away of the incidence of unevolved close binaries which are not W UMa type systems, but close to contact. This suggests the possibility that some systems, classified as W UMa type on the basis of their light curves, are actually not in contact but just very close unevolved pairs. This is perhaps more noticeable at earlier type.

3. There appears to be some fall-off at longer periods in the number of observed systems relative to the comparison curves. This could be due to some failure of any of the foregoing assumptions, which at least in the spherical star respect are oversimplified, while with respect to the power law distribution have no necessary physical basis. Farinella and Paolicchi (1978) for instance, consider an underlying distribution of truncated Gaussian form in the mass per unit volume of binaries.

4. When the W UMa type stars are added into the distributions, in the case of the A-type systems there is, of course, some rise at the short period end; but the early A systems still lie appreciably below the comparison curve at periods just greater than the contact value. The later A systems exhibit also a deficit at periods just greater than the contact value, though with suitable averaging to include the incidence at slightly longer period, the discrepancy is not as great. Also it is clear that there is a build up at periods less than* the shortest period for unevolved (equal mass) contact for these stars.

5. For F type systems, inclusion of the W UMa stars allows the distribution to match the comparison curve tolerably well right up to the contact period - though, of course, this need not mean that we are dealing with a uniform class of object in which proximity only is varying.

6. However, it is among the G systems, where the previously noted difference in the distribution from that of normal Main Sequence stars was strongest, that we observe a conspicuous overabundance over the comparison curve.

Hence the overall answer to the question posed at the beginning of this section is that the discrepancy in the distribution of W UMa type primaries compared with normal unevolved Main Sequence primaries in binary systems (which distribute like Main Sequence single stars) clearly involves a surplus of late spectral type stars, but could also imply a deficit of earlier spectral type systems if there should be a uniform distribution of close binaries.

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* Systems of still earlier type exhibiting a similar trend were considered by Wilson and Rafert (1981).

4. DO W UMa TYPE LIGHT CURVES NECESSARILY IMPLY CONTACT?

In considering possible interpretations of the foregoing information on W UMa system incidence, and the relationship of these systems to the more conventional unevolved detached kind, the question of whether such stars are necessarily in contact arises (e.g. point (2) in Section (3)). In this connection let us note in Figure 5 a W UMa type light curve which has been generated from a pair of stars not actually in contact.

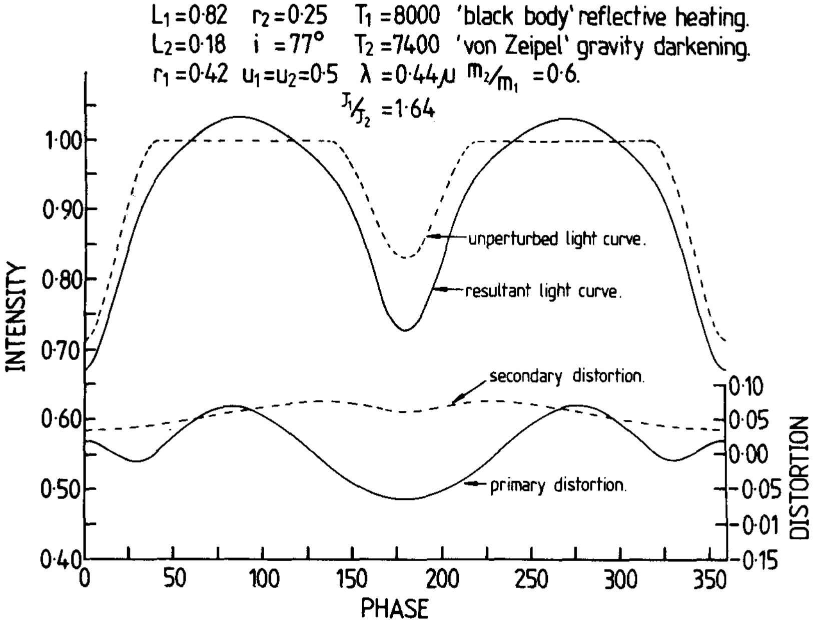

Figure 5. A light curve produced by a pair of close, but not energy transferring "over-contact" stars, which looks like that of a W UMa system.

The difference in luminosities for the given mass ratio (0.6) would be appropriate for a Main Sequence primary, originally of early A type, but having evolved somewhat from ZAMS, for which the ratio of radii has appropriately decreased (from an initial 0.72 down to 0.6). The otherwise generally standard modelling parameters are indicated on Figure 5. The main cause of the W UMa like light curve comes from the large scale of the "ellipticity" distortion of the primary star, which adds to the depth of the secondary minimum, though it is largely eclipsed out at primary minimum. Though the depths of both minima are approximately equal the mean surface flux of the primary can then be more than 1.6 times that of the secondary at the wavelength of observation.

The possibility of this kind of light curve was presented in somewhat general and approximate terms in Budding's (1981) article, where it was argued that close but non-contact pairs for which the primary was of type earlier than, say, mid F could give rise to an EW type photometric variation, though no actual case was cited. In Figure 6, however, we have, with RR Cen, a possible real example of a system of similar type. In Table 1, we present our optimal parameter set for this light curve - values obtained by procedures discussed by Budding and Najim (1980).

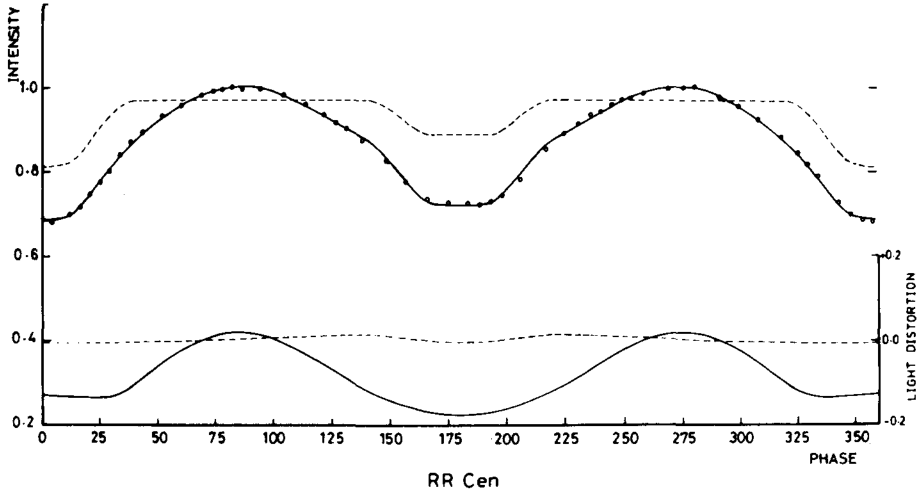

Figure 6. Knipe's λ5280 light curve of RR Cen as modelled by the parameters given in Table 1. (The reference light level is first normalized to unity to provide the quantities L_1, L_2.).

The luminosities L_1, L_2 are constrained so that their sum is unity. Radii r_1 and r_2 correspond to the volumetrically defined quantities (r^*), in Kopal's (1959) terms, whose sum, being approximately constant at 0.75 in the condition of "contact" of the two stars, can be used as a discriminant about this condition. The totality at the second minimum acts as good constraint on geometrical elements, serving, for instance, to show that the inclination i could not be much less than the derived 80°. The close to unity value of reduced chi-squared (ν represents the number of degrees of freedom, which amounts to 50 for Knipe's (1965) normal points), indicates that the observations are in excellent accord with the fitting function on the basis of Knipe's accuracy assessment Δℓ. The mass ratio m_2/m_1, limb darkening (u), gravity darkening (τ) and reflection coefficients (E) are adopted quantities. Some other details of notation or method may be found in Budding and Najim (1980). A slight, probably in-

$L_1 = 0.913$	$\pm\ 0.02$	$u_1 = 0.58$
$L_2 = 0.087$		$u_2 = 0.5$
$r_1 = 0.43$	$\pm\ 0.01$	$\tau_1 = 1.0$
$r_2 = 0.19$	$\pm\ 0.03$	$\tau_2 = 1.1$
$i = 80^\circ$	$\pm\ 1^\circ$	$E_1 = 1.0$
$\Delta\theta_o = 0^\circ\!.2$	$\pm\ 0^\circ\!.3$	$E_2 = 1.1$
$m_2/m_1 = 0.6$		$\Delta\ell_{(s.d.)} = 0.0055$ (Knipe's value)
	$\chi^2/\nu = 0.93$	

Table 1. Optimal parameter set for Knipe's (1965) light curve of RR Cen ($\lambda_{eff} = 5280$ Å).

significant, correction to the zero point of the listed phases, $\Delta\theta_o$, was also found.

At first sight it seems difficult to reconcile the observed ratio of radii (~ 0.45) with the assumed mass ratio (0.6) on the basis of the "standard" Main Sequence mass radius relation. However, if we compare a pair of stars like, for example, the 1.5 and 1 $M_\odot$ models whose evolution tracks were computed by Iben (1967), it can be observed that the ratio of radii has dropped from an initial 0.77 to 0.37 by the time of the end of the (Main Sequence) thick shell burning phase of the more massive star. By this time the bolometric luminosity ratio would have reached 14, which, allowing for a slight bolometric excess to the more massive star, which would appear as of early F type, still surpasses somewhat the derived value for RR Cen of about 10.5. The observed parameters could, however, be feasibly matched by a pair of stars still evolving in the Main Sequence band, the primary, of mass about 1.5 $M_\odot$ towards the end of this stage, with a secondary not far off 1 $M_\odot$ and relatively little evolved.

A possible difficulty rests with the fractional radius of the primary which, for the adopted mass ratio 0.6, already seems too big for its Roche lobe mean radius ($r_1^* \sim y_5$ - in Kopal's 1959 notation - = 0.42). There are various remarks one might make about this; concerning, for example, the appropriateness of underlying approximations, the Roche model formulae, effects of truncation of series of terms, or the sizes of probable errors. However, the main point of the present section is not to show that one or other model is the definitive one, but that the scale of uncertainty when dealing with light curves such as this is such as to allow models of inherently quite different kinds to be able to reproduce the observations plausibly. In a word, the contact model need not be unique.

In this way it could be argued that a good many, perhaps most, of the W UMa systems with primary spectral type earlier than about F5 and periods greater than half a day need not be in contact at all. A much stronger case for contact comes with the W UMa systems of later type and low periods, as was argued by Budding (1981). Such stars may well represent the bulk of the classical contact W UMa systems, which form the basis of numerous special studies.

5. AN ASPECT OF THE PARAMETER DETERMINACY QUESTION FOR W UMa LIGHT CURVES

Though the results of the foregoing section imply that light curves of W UMa type do not necessarily imply contact, there is clearly an ambiguity since numerous authors have generated light curves of the same general form from models of stars which are in over-contact. The general problem of determinacy and uniqueness in curve fitting is rather broad and cannot be fully dealt with here, but there is one particular aspect of W UMa light curve generation which, as more data becomes available, might be capable of receiving further empirical testing. This refers to the differing possibilities with regard to gravity darkening (or brightening), about which different authors have presented different ideas.

The main point of present relevance about this is that the generation of light curves requires some description of the extent of gravity darkening (usually by means of a single pair of parameters), but the adopted position on this will influence the resulting values of other quantities treated as unknowns. This affects the degree of observational support for the contact hypothesis (Anderson et al., 1980; Kopal, 1968). Anderson et al. (op cit.) urge detailed consideration of spectrographic evidence to help resolve this question. Alternative suggestions may be offered, as follows.

Firstly, the main geometrical elements determined at different observation wavelengths should be sensibly the same. Any systematic variation with wavelength may reflect model inadequacies such as an imposed incorrect gravity darkening parameter. Secondly, the promising new method of differential polarimetry might be applied to advantage to a few of the brighter W UMa systems. It could, in this way, be possible to provide some independent check on orbital inclinations, whose values can be seen to correlate with assumed gravity darkening parameters in published lists of parameter values.

In connection with this latter point a further test is, in principle, possible. In Figure 7 we compare a distribution of the quantity x^2, where $x = (\sin i - \sin i_{min}) \div (1 - \sin i_{min})$, $\sin i_{min}$ being given by $\{1 - (r_1 + r_2)^2\}^{\frac{1}{2}}$, for a sample of 49 essentially uncomplicated and well determinable detached pairs, derived originally from Svechnikov's (1969) compilation, but with some additions and modifications based on more recent analyses of 13 of these systems, with that coming from the tabulation of geometric elements of 35 W UMa systems published by Mochnacki (1981).

The range of inclinations available to a system composed of spherical stars of relative radii (in terms of the mean separation) r_1 and r_2 in

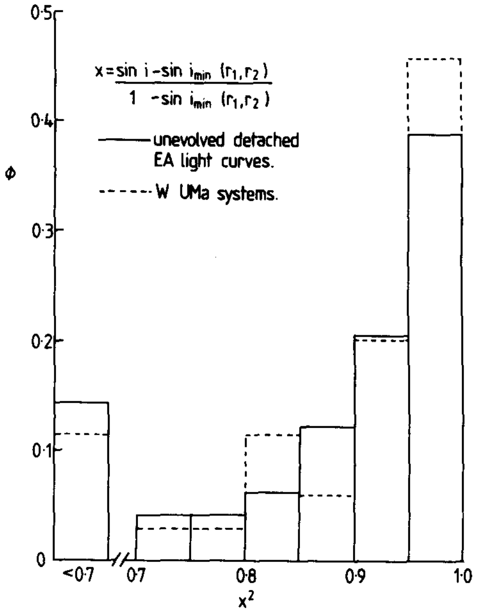

Figure 7. Relative frequency ϕ of systems of different inclination.

which they can be seen to eclipse is from 90° to $\cos^{-1}(r_1 + r_2)$. The probability of finding a system at inclination i within this range would, in principle, be proportional to sin i, assuming an arbitrary distribution of orbital revolution axes over the sphere and if there were no selection effects related to the amplitude of the light variation associated with the eclipses. If we are comparing distributions whose range of possible inclinations is different, due to different proximity, we can standardize by use of the aforementioned variable x (Note $0 \leq x \leq 1$). The more realistic approach to the probability of a certain value of x is therefore to write $P(x) = x\,S(x)$, where $S(x)$ expresses the selection effect referred to. It does not seem plausible to suppose that $S(x)$ can be precisely specifiable for the general context we are considering, in which the spherical star geometry is, in any case, an oversimplification. However, in the combined interests of simplicity and clarity we have, in Figure 7, scaled the abscissae to values of x^2.

Though the sample sizes are rather small to allow any decisive interpretation at this stage, it could be regarded as odd that the distribution of derived inclination values for W UMa systems is relatively somewhat more compressed towards 90° than that of the detached systems. In fact, since the underlying arguments have nowhere referred to proximity effects, which should enhance the chance of discovery over a purely spherical case at lower inclination, one would expect the biassing to be in the opposite sense to that observed. A possible explanation for such a discrepancy could be due to a systematic error of procedure in relation to the assigned gravity darkening parameter used in the sources quoted by Mochnacki (1981). Other things being equal, an assigned low value of gravity darkening coefficient would require a higher inclination to produce the same "photometric ellipticity".

6. THE SPATIAL INCIDENCE OF CONTACT BINARIES

It is clear, in a general way, from the high relative incidence of W UMa stars among cooler stars, and the known high spatial density of low mass stars that the number of W UMa systems as a whole, in the galactic field, must be comparatively large. About 400 EW variables (≡ W UMa type) are listed among 4062 eclipsing variables of all types in the "General Catalogue" of Kukarkin et al. (1969) (Van't Veer, 1975; Yamasaki, 1975). Lucy's (1976) data suggests that he counted about 120 such systems brighter than magnitude 12, a sample which is probably close to the 118 systems of known spectral type used to form Figure 2.

Comparing the distribution of such stars having photographic magnitudes brighter than a given value, N_{EW}, with the average numbers of stars N_* given by Allen (1973), we find ratios as given in Table 2.

m_{pg}	$(N_{EW}/N_*) \times 10^{-4}$	$(N_{EAU}/N_*) \times 10^{-4}$
7	4.7	0.6
8	5.6	1.5
9	3.8	1.3
10	2.4	1.9
11	1.7	1.3
12	1.0	0.6

Table 2. Incidence of W UMa (EW) and unevolved, detached binaries (EAU).

A selection effect operating against the discovery of fainter variables can be assumed to become significant, at least by the ninth magnitude, and a reasonable estimate for the discovered incidence frequency α_d would appear to be about 5×10^{-4} (c.f. Van't Veer, 1975).

What this means in terms of the actual spatial incidence α_s of genuine common envelope W UMa systems depends on how light curves are interpreted. If we assume that this configuration really only refers to the aforementioned considerable excess at around spectral type G0, a conservative estimate of $f_1 = 50\%$ may be put for systems classified as having EW type light curves to be of over-contact type. Figure 7 suggests that the selection effect in inclination f_2 is so severe that perhaps only the range $1 \geqslant x \geqslant 0.98$ of the eclipsing sample is actually complete. It will be assumed, however, that this range, which should account for 12%

of the entire group if their orbital axes are distributed randomly to the line of sight, actually corresponds to 39% of the observed set. This percentage really refers to the more populated comparison group of detached binaries - the percentage of EW systems in this range is somewhat more than 39% according to Figure 7, but the point of that comparison was to suggest possible systematic error in the photometrically derived inclinations of W UMa systems.

We finally derive for the spatial incidence of contact W UMa type binaries a proportion

$$\alpha_s = f_1\ f_2\ \alpha_d = 5 \times 0.5 \times (0.39/0.12) \times 10^{-4} = 8 \times 10^{-4} \qquad (6.1)$$

of all stars. This figure is rather less than that proposed by Van't Veer (though subject to essentially the same uncertainty, i.e. ∿ 50%), chiefly because of the more conservative estimate of what is likely to be a contact system (f_1), and also because of some difference in the estimated proportion of W UMa stars actually seen (f_2).

7. POSSIBLE LINES OF EXPLANATION

In considering the origin and high incidence of contact W UMa systems two major lines of approach have been followed:

- (i) some stars may originate in the contact condition (incomplete fission) and remain in, or indistinguishably close to, such a state for nuclear timescales; or
- (ii) stars may become like this from originally detached binaries through some process involving a loss of angular momentum. The high incidence should then be related to relevant properties of the supposed antecedents of the contact systems.

Of the two approaches the second would appear to be more pragmatic than the first, in the sense that the first requires us to explain why just binaries should be formed in this way, and why such binaries should be confined to a particular spectral range. The stability of the configuration through the various stages of its formation should also be examined in order to establish its required duration. It may be possible to do this; but in the second approach less presumption is possible, since both binaries and angular momentum loss are known to exist independently of the existence of W UMa stars. Moreover, since angular momentum loss would normally imply also mass loss, some path may be open to account for the excess of low mass systems, perhaps together with the deficit of comparable high mass systems. Of course, many more possible contact W UMa stars exist than could directly be accounted for by the observed deficit of very close spectral type A systems. The situation might be interpreted in terms of a relatively rapid degradation of close and more massive binaries, through some of the considered kinds of interactive evolution, to a slower accumulation of the remnants of such evolution at lower mass and period values.

It has been argued that the numbers of unevolved binaries at lower mass are insufficient to account for the large incidence of W UMa systems (e.g. Kraft, 1969). Some relevant quantities will be considered shortly. Let us first note possible difficulties in comparing the expected numbers of "protomorphs" of contact systems with the observed population of unevolved binaries in the presence of light curve ambiguities, selection effects relating to discoveries, and general processes associated with mass and angular momentum loss and aging. Thus, for example, magnetic braking (Huang, 1966; Mestel, 1968) might be of key significance in explaining why a close low mass pair could spend only a fraction of the primary core hydrogen burning lifetime as a detached system (Van't Veer, 1976; Vilhu, 1981; Mochnacki, 1981).

Before introducing such an extra degree of freedom into the problem, however, certain points can be made from considering the situation in which only the normal processes of binary evolution are involved, but with the well known possibility that such processes can lead to a common envelope phase, around which time significant mass and angular momentum loss may occur. Such a mechanism could certainly enhance the persistence of the contact or close to contact condition; without it, i.e. in a purely conservative regime, comparable numbers of pre and post-mass transfer binaries, in which the close to contact phase appears as a relatively short episode, should be expected. The comparisons of Kraft (1969), at least if we assume that the separations of low mass binaries should initially distribute like those of higher mass, appear sufficient to disallow such a line of explanation.

The line of explanation that we now seek to investigate is that the number of contact binaries N_w, brighter than a given apparent magnitude m_o, which, in view of the Eggen correlation can be essentially associated with unit variation of a single independent variable, which we shall choose to be primary absolute magnitude M, can be related to a corresponding number of protomorphs N_p, via some relation of the type

$$N_w(M) = \int_{\Delta'M} N_p(M - \delta M)\ \nu(M, \delta M)\ \tau(M, \delta M)\ dM \quad , \tag{7.1}$$

where the protomorphs come from a range $\Delta'M$ of, by implication, somewhat more massive close binaries, with compensating factors ν and τ accounting for the greater volume occupied by the brighter protomorphs, and the relative timescales which they spend in detached and contact conditions, respectively. If the required distribution $N_p(M)$ could be obtained from such a relation it might be compared with the observed incidence of unevolved systems.

To do this in a more complete way involves a number of possibly complicated factors, about which consideration is deferred. We proceed, at this stage, by making a linearized trial solution to the foregoing integral equation. In order to make comparisons we retain the notion of a uniform distribution of unevolved binaries, of the form $\phi = \text{const.} \times f(M)P^n$ and also suppose a more or less constant rate of binary formation of all kinds. Keeping in mind the range of total mass loss (0 - 40%) considered plausible by Refsdel et al. (1974) for the much discussed example AS Eri, we consider the possibility that the protomorphs of the anomalous accumulation of contact binaries of spectral type G0-6 may be essentially found among low period systems with F3-8 type primaries, estimating that ∿ 20% of the original mass of the system may be lost when the separation of centres is small. If such a matching can be successful it might be applied in a parallel way to more massive systems. Let us assume that faintness affects the detection of both kinds of system to the same extent, (in fact, the detection of EW systems falls away somewhat more rapidly with magnitude than that of unevolved EA stars as may be seen from Table 2, though the effect does not have proportionately serious consequences). Equation (7.1) can now be approximated by

$$N_p(M - \Delta M)\ \Delta'M = N_w(M)\ \frac{V(M - \Delta M)}{V(M)}\ \frac{T_p(M - \Delta M)}{T_w(M)} \quad , \qquad (7.2)$$

where V(M) is the volume of space out to which a star at M appears brighter than m_o and $T_{p/w}$ is the expected lifetime in the protomorph/contact condition. Now, in parallel with (6.1),

$$N_w = f_1 f_2 N_d \quad . \qquad (7.3)$$

From the data indicated in Figure 2, with m_o taken to be 12, we have $N_d = 40$, while, since genuine contact was considered more likely among the cooler type EW light curves, we can set $f_1 = 1$. As before, $f_2 = 3.3$ so that $N_w = 132$. If N_p refers, in the present example, to the foregoing type range, the increment $\Delta'M$ can be taken as effectively unity, from which it probably differs little in any case. Taking M (blue) and ΔM to be 5.0 and 1.0 respectively (remembering the generally increased proportion of secondary light in W UMa systems), we find $V(M - \Delta M)/V(M)$ to be about 4, though with a strong dependence on the assumed mass loss. T_p/T_w could be taken to be ∿ 1, at least if Case A type mass transfer operates. N_p then turns out to be 530.

The period P_{max} up to which close binaries in the type range considered (F3-8) would need to be drawn from in order to provide such protomorphs can then be expressed by

$$N_p = \Sigma_{P_*}^{P_{max}}\ n_d(P)\ f_1\ f_2(P)\ \Delta P \quad , \qquad (7.4)$$

where $n_d(P)$ is the number of protomorphs in a given period interval about P (in days). From the sample of candidate stars referred to in connection with Figure 4, and matching the observed numbers to the theoretical comparison form we have, when $\Delta P = 0.2$ days, $n_d(P)\ \Delta P \simeq 5.8\ P^{-5/3}$. The selection factor $f_2(P)$ works out at $f_2 = 5.8\ P^{d}$ which reduces to the same value as that used for the W UMa stars at the equal mass unevolved contact period (0.43 days). f_1 is again taken to be unity and though this might overestimate detached protomorph numbers close to P_*, N_W may also have been slightly overestimated as a result of setting $f_1 = 1$ in (7.3).

On these assumptions, it can be found that it would be necessary to look for supposed protomorphs of the GO-6 type contact systems among middle-late F-type primary binaries with periods up to about 10 days. The upper limit period for Case A mass transfer among such systems is only about 1.2 days, however. Only about a third of the considered group of contact binaries could therefore be accounted for in this way, based on the adopted statistics.

Turning to the possibilities of Case B, it can be noted that a number of low mass evolved Algols exist, which if "evolved" backwards (conservatively) must have passed through a common envelope stage. Such systems include R CMa, RW CrB, RZ Dra, AS Eri, DN Ori, RT Per, VV UMa S Vel; all with total mass around 2 $M_\odot$, and which, in a common envelope configuration, must have looked like W UMa type systems. However, a problem now is that, even allowing that angular momentum loss during the common envelope stage might allow that contact persist into the "slow phase" of mass transfer, the entire semi-detached stage of low mass Case B evolution is still only $\sim 10^{-1}$ of the Main Sequence lifetime. The factor T_p/T_w in (7.2) is thus increased appreciably. There are, moreover, other difficulties: if the mass losing star is able to swing in sufficiently close for the final product to look like a contact-system the initial mass ratio seems required to be small. Such systems are believed to represent rather a minority among close binaries (Lucy and Ricco, 1979; Plavec, 1982) and, in any case, do not correspond to the observed candidates on which the comparison statistics are based. Then further ad hoc discussion is required to explain how the supposed relatively small total mass loss fraction carries away the larger angular momentum loss required.

All in all, it seems difficult to account for the large incidence of W UMa type binaries on the basis of a simple uniform distribution of separations for unevolved binaries of all masses, allowing only for the well known mass transfer modes in binary evolution, accompanied by systemic mass and angular momentum loss when transfer commences. When such processes are included, though, the disparity between possible protomorphs and observed W UMa systems need not be as great as previously estimated ($\sim$ factor 10 according to Kraft, 1969). Also it seems likely that certain cases of low-mass semi-detached systems evolving in Case B should have once looked like contact binaries. It is unsatisfactory that no distinguishing mark of such binaries currently in the contact state has been pointed out. As well it should be noted that the failure of our simple trial solution has not proved the impossibility of some appropriate choice of factors in Equations (7.1) and (7.4) from allowing some explanation along these lines. In particular, the assumed constancy in the rate of binary formation may be a weak point in the foregoing comparisons. Also, since the foregoing treatment implies that some of the protomorphs may be drawn from systems evolving relatively rapidly, e.g. by already being in a common envelope phase, the factor $T_p(M - \Delta M)$ would have been overestimated by simply setting it equal to $T_w^p(M)$. In such ways the "slower accumulation" mentioned at the outset could be effected.

Of course, the introduction of magnetic braking in the evolution of cool close binaries, for which there appears to be accumulating evidence (Ruciński et al. 1982, Budding et al. 1982), may help to clarify and remove many problems connected with their incidence. In the simple terms of (7.2) for example, magnetic braking relieves the requirement for a very large N_p, by reducing ΔM, and therefore the volume ratio, as well as possibly reducing the time ratio T_p/T_w, since estimated rates of angular momentum loss can produce coalescence, in some cases, in much less than a nuclear timescale. On the other hand, the same mechanism will entail a breakdown of the uniform distribution idea, producing changes in the factors f_2 and possibly also f_1 in (7.4) which are not obvious. The introduction of the extra degree of freedom associated with angular momentum loss _ab initio_, due to magnetically driven processes, would then detract from the effectiveness of observational evidence, of the kind considered in this paper, in providing unambiguous tests of theory, unless such processes could be separately quantified.

8. SUMMARY

This paper has been aimed at bringing out the peculiar incidence of W UMa type binaries, the essence of which was shown in Section 2. Section 3 pointed out the possibility that this peculiarity could be regarded as a deficit among early type systems of this kind, as well as a surplus at spectral types later than mid F.

Sections 4 and 5 were intended to emphasize ambiguities associated with the photometric evidence alone. The fact that a light curve is classified as of W UMa type does not force us to assume contact, and an analysis of Knipe's (1968) data on RR Cen was used to illustrate the point. Also a comparison was made between the distribution of determined inclination values from analysis of W UMa type light curves and that of the better determinable detached systems. Taking such ambiguities into account, the spatial incidence of contact binaries, though clearly high, need not be as high as that considered by Van't Veer (1975) (Section 6).

The problem posed by the incidence of W UMa systems was found to be open to analysis, on the basis of a number of simplifying assumptions, in the light of what was judged in Section 7 to be the more pragmatic approach to explaining their origin. It may be worthwhile to conclude by indicating such simplifications or limitations, and suggesting possible areas in which the problem could be developed.

Firstly, though the numbers of stars involved in the statistics are moderate, they still could not be regarded as large - large enough, for example, to permit smaller increments than a few spectral type subdivisions or one magnitude range in dealing with the representative stars considered in relation to Equations such as (7.1) or (7.4). In a similar way the latter equation, relating back to Section 3, utilizes the notion of an underlying uniform distribution, which while feasible, cannot be regarded as definitely established by the statistics given in this paper. The trends shown in Figures 3 and 4 could, in fact, be better represented by the truncated Gaussian considered by Farinella and Paolicchi (1978). The form actually used in (7.4) was chosen for reasons of simplicity - but the major results of the discussion are not seriously affected by the particular form chosen.

Throughout Section 7 there was, apart from a qualification added to the discussion of Case B, a concentration on the properties of one star only, which implies fairly constant behaviour or properties of the protomorph's secondary, or that primaries and secondaries in the considered systems tend to have a fixed relationship to each other, such as via a preferred mass ratio, for example. This, like the effects of evolution within the Main Sequence band in relation to the possibility of characterising stable primaries by a single independent variable, has been tacitly associated with small scatter effects, such as that found within the Eggen correlation. (This point does, however, raise an issue which could merit further investigation, namely, the possibility of another "compensating factor" in (7.1) associated with a difference between the range of mass ratios of protomorphs with that of observed W UMa systems. If, for instance, a wider range of mass ratios among the protomorphs is implied, the requirement for a high N_o in (7.4) could be eased, since, as with the small initial mass ratio Case B possibility, some such protomorphs would not easily be observed as binaries.)

Then, of course, details of the supposed scale or mechanism of mass and angular momentum loss that underlie the approach of Section 7 were also dealt with in a purely summary way. The circumstances may differ so much in individual cases as to cast doubts on the reliability of the straightforward linearization of (7.1) into (7.2).

By way of a positive response to such doubts, the main purpose of Section 7 has been not only to offer one approach to observational testing of theories of the origin of contact binaries, but, more generally, to suggest a future possible area of work, in connection with binary evolution. In a parallel way to the use of two dimensional diagrams relating single star evolution to observational data, the distribution of binaries in the plane of, for example, primary type and orbital period could be studied in its dependence on time and in relation to proposed paths of binary evolution starting from a given distribution of initial conditions.

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