

When does a granular material behave like a continuum fluid?

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1 A flowing granular material can behave like a collection of individual interacting
2 grains or like a continuum fluid, depending in large part on the energy imparted to
3 the grains. As yet, however, we have no general understanding of how or under what
4 conditions the fluid limit is reached. Marston, Li & Thoroddsen (*J. Fluid Mech.*, this
5 issue, vol. 704, 2012, pp. 5–36) use high-speed imaging to investigate the ejection
6 of grains from a granular bed due to the impact of a spherical projectile. Their
7 high temporal resolution allows them to study the very fast processes that take place
8 immediately following the impact. They demonstrate that for very fine grains and high
9 impact energies, the dynamics of the ejecta is both qualitatively and quantitatively
10 similar to what is seen in analogous experiments with fluid targets.

11 **Key words:** granular media, fluidized beds, particle/fluid flows

12 1. Introduction

13 Granular materials play a role in almost every aspect of our lives, and are important
14 in agriculture, pharmaceuticals, construction, and many other industries. The properties
15 of a granular material can vary dramatically. Sand on a beach is solid-like, but sand
16 on a hillside can flow – sometimes catastrophically – if the conditions are right. If a
17 container filled with many small particles is shaken, the granular system will behave as
18 a solid, liquid, or gas depending on how hard you shake. To complicate matters,
19 granular flows generally differ from conventional fluid flows because of packing
20 effects and the strong dissipation that results from friction and collisions among the
21 grains. There is as yet no general theory that describes the flow behaviour of granular
22 systems, and the question of when a collection of grains can be adequately described
23 as a continuum fluid is a major open issue in the field. One interesting and complex
24 granular flow results from the impact of a falling projectile into a granular target.
25 Portions of the granular material are fluidized by the impact, with grains initially being
26 forced outwards, then collapsing back inwards as the projectile penetrates below the
27 surface. In addition, some grains are ejected from the target in a process analogous to
28 the familiar formation of a splash in liquids. The end result is an impact crater similar
29 to those seen on the Moon and rocky planets. The flow is transient and evolves very
30 quickly, making it challenging to study.

31 Early work on granular impacts was motivated by an interest in the formation of
32 planetary craters. Several recent papers have studied the morphology and scaling of
granular craters (Uehara *et al.* 2003; Walsh *et al.* 2003; de Vet & de Bruyn 2007)

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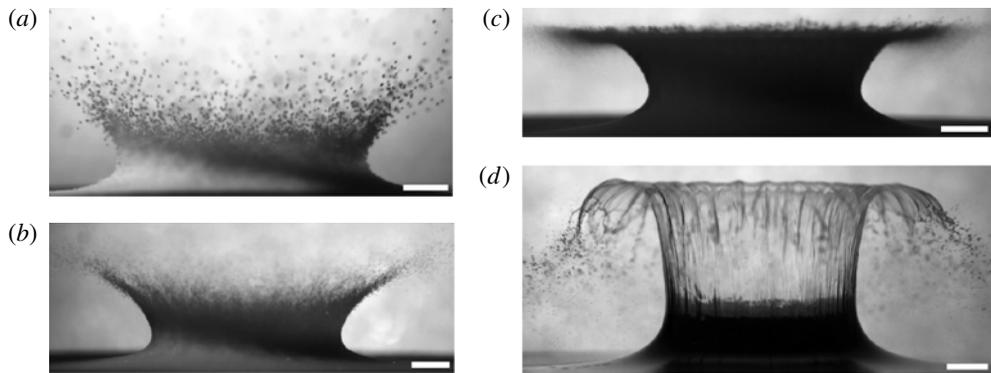


FIGURE 1. Video images showing the ejecta produced by the impact of a solid sphere onto targets of: (a) 520 μm glass beads; (b) 178 μm glass beads; (c) 31 μm glass beads; and (d) water. The size of the sphere and the impact velocity were the same in all cases. While the granular nature of the ejecta is evident in (a), the flow for the finest spheres looks qualitatively very similar to that for water. (From Marston *et al.* 2012; scale bars are 1 cm.)

as well as the granular flows involved in their formation (de Vet *et al.* 2010). The dynamics of the impact and the penetration of the projectile into the target has been studied as a probe of the forces exerted by the granular medium on the projectile (de Bruyn & Walsh 2004; Ambroso, Kamien & Durian 2005; Katsuragi & Durian 2007; Goldman & Umbanhowar 2008). In addition, impact into a target of fine, loosely packed grains can produce impressive granular jets (Thoroddsen & Shen 2001) that are analogous to the well-known Worthington jets seen in fluid impacts.

A recent paper by Marston, Li & Thoroddsen (2012, this issue) reports a detailed investigation of the very early stages of the granular impact process, focusing on the ejection of grains from the target. Marston *et al.* (2012) use high-speed imaging to study the appearance and early evolution of the ejecta with a time resolution of up to 10 μs , much better than that in previous studies of granular ejecta (Boudet, Amarouchene & Kellay 2006; Deboeuf, Gondret & Rabaud 2009). This allows them to view the very rapid events that take place immediately following the impact. In addition to more fully characterizing this complex granular flow, their research helps to address the question of when a granular flow displays truly fluid behaviour.

2. Overview

Marston *et al.* (2012) studied the ejection of material caused by the impact of steel spheres of a range of diameters and impact speeds into targets consisting of small spherical glass beads or sand grains. Images (a–c) in figure 1, which is taken from their paper, are snapshots of the curtain of granular ejecta produced by impact into targets of different sized glass beads. The last image (figure 1d) shows the fluid ejecta following an impact into water. An evolution of the granular ejecta towards more fluid-like appearance and behaviour is evident as the grains get smaller. The Froude number ($Fr = V_0/\sqrt{gD_0}$, where D_0 is the sphere diameter, V_0 its impact speed, and g the acceleration due to gravity) is the same in all four cases illustrated in figure 1, as is the dimensionless time from the sphere's first contact with the target material. The packing fraction of the grains and the ratio of D_0 to the diameter of the grains

61 are different. The shape and size distributions of the grains also play a role. Marston
62 *et al.* systematically characterize the effects of these parameters on the ejecta.

63 Experiments with targets made of larger glass beads display an ‘early stage’ in
64 which individual grains are ejected prior to the development of a coherent ejecta
65 sheet in the ‘main stage’. The first ejected grains appear within a few milliseconds
66 of the initial contact of the sphere with the target. Although seemingly fast, this is
67 an order of magnitude longer than the time for the splash to appear when a sphere
68 hits a fluid (Thoroddsen *et al.* 2004). Marston *et al.* suggest that this delay is due
69 the compressibility of the granular material, which reduces the propagation speed of the
70 disturbance produced by the sphere. This effect is even larger for sand grains, which
71 are less spherical and have a broader shape distribution. When distances are scaled by
72 the sphere diameter D_0 and times by the reciprocal of the local shear rate V_0/D_0 , both
73 the time and the radial position at which the grains emerge are independent of Fr , and
74 largely independent of both the size ratio and the packing fraction.

75 The high-speed imaging technique used by Marston *et al.* gives them access to
76 the previously unobserved early stage of the ejection process. Streakline images of
77 individual grains ejected before the formation of the main ejecta sheet allow direct
78 determination of the angle and speed of ejection for each grain; over the time range
79 studied air drag is almost negligible and the grains travel in straight lines. The fastest
80 grains can be ejected with a speed five times that of the impacting sphere. In a given
81 experiment, the grains ejected at early times are on average faster and are emitted at
82 lower angles than at later times, although there is a large range of both speed and
83 angle due to collisions among the particles as they make their way out of the bulk
84 material. The evolution of the ejection angle is simply explained by the change in the
85 angle between the surface of the sphere and that of the target as the sphere penetrates
86 more deeply, and the range of angles and speeds narrows with time as the ejected
87 grains converge to a fluid-like sheet.

88 In impacts into the finest grains, in contrast, the ejecta forms a coherent sheet,
89 similar in appearance to that seen in impacts into water, from the earliest observable
90 times. The velocity of the emerging sheet is proportional to the impact velocity and
91 increases with the sphere diameter, but is independent of the packing fraction. The tip
92 of the ejecta sheet thickens with time, but air resistance causes individual grains to
93 break off from the tip, forming a hazy cloud around the sheet. The dynamics of this
94 sheet can become quite complex due to an interplay among air entrainment, a vortex
95 ring generated inside the ejecta curtain, and the porosity of the sheet itself. While the
96 packing fraction of the granular target has little effect on the early stages of ejecta
97 formation, it does affect the later evolution of the sheet, as higher packing fractions
98 lead to more porous ejecta sheets.

99 Marston *et al.* tracked the location of the narrowest point of the coherent ejecta
100 sheet as a measure of its expansion with time. They found a power-law scaling of
101 the neck radius at early times, but with a non-universal exponent. Interestingly, the
102 exponents they found are all much lower than the value of 2 predicted by a model due
103 to Deboeuf *et al.* (2009). Marston *et al.* speculate that the early-time behaviour they
104 observe is quite different from the later-time dynamics treated previously, and point
105 to indications that their exponent may in fact be approaching 2 at later times. For
106 the lowest packing fractions, finest grains, and largest impact energies, the exponent
107 measured by Marston *et al.* is close to the value of 0.5 seen in the fluid ejecta
108 sheets produced by impacts into liquid films. This scaling, along with the fluid-like
109 appearance of the sheet seen with the finest grains, suggest that the granular ejecta
110 sheet approaches true fluid-like behaviour in the fine-particle limit.

There are some differences between these granular ejecta sheets and those sheets seen in fluid impact experiments. For example, for similar values of the effective Reynolds number, the velocity of the ejecta sheet is roughly a factor of 2 smaller for granular impacts than in the fluid case. Marston *et al.* (2012) suggest that such differences may be resolved by a better understanding of the effective viscosity of the granular bed, and the changes in packing fraction that occur in response to the impact.

3. Future

The work of Marston *et al.* (2012) gives us a much more complete picture of the ejection of grains in a granular impact and demonstrates the approach of this particular granular flow to fluid-like behaviour as the flow parameters are changed. Many other aspects of granular impacts are not understood in detail, however, including the complex flows involved in excavating the crater (de Vet *et al.* 2010) and the physics that leads to the observed scaling of crater dimensions (de Vet & de Bruyn 2007). In addition, the degree to which the present results apply to other granular systems is unclear, as different granular flows can behave quite differently. Substantial further work will be required to address these challenging issues.

References

- AMBROSO, M. A., KAMIEN, R. D. & DURIAN, D. J. 2005 Dynamics of shallow impact cratering. *Phys. Rev. E* **72**, 041305.
- BOUDET, J. F., AMAROUCHENE, Y. & KELLAY, H. 2006 Dynamics of impact cratering in shallow sand layers. *Phys. Rev. Lett.* **96**, 158001.
- DEBOEUF, S., GONDRET, P. & RABAUD, M. 2009 Dynamics of grain ejection by sphere impact on a granular bed. *Phys. Rev. E* **79**, 041306.
- DE BRUYN, J. R. & WALSH, A. M. 2004 Penetration of spheres into loose granular media. *Can. J. Phys.* **82**, 439–446.
- DE VET, S. J. & DE BRUYN, J. R. 2007 Shape of impact craters in granular media. *Phys. Rev. E* **76**, 041306.
- DE VET, S. J., YOHANNES, B., HILL, K. M. & DE BRUYN, J. R. 2010 Collapse of a rectangular well in a quasi-two-dimensional granular bed. *Phys. Rev. E* **82**, 041304.
- GOLDMAN, D. I. & UMBANHOWAR, P. 2008 Scaling and dynamics of sphere and disk impact into granular media. *Phys. Rev. E* **77**, 021308.
- KATSURAGI, H. & DURIAN, D. J. 2007 Unified force law for granular impact cratering. *Nature Phys.* **3**, 420–423.
- MARSTON, J. O., LI, E. Q. & THORODDSEN, S. T. 2012 Evolution of fluid-like granular ejectas generated by sphere impact. *J. Fluid Mech.* **704**, 5–36.
- THORODDSEN, S. T., ETOH, T. G., TAKEHARA, K. & TAKANO, Y. 2004 Impact jetting by a solid sphere. *J. Fluid Mech.* **499**, 139–148.
- THORODDSEN, S. T. & SHEN, A. Q. 2001 Granular jets. *Phys. Fluids* **13**, 4–6.
- UEHARA, J. S., AMBROSO, M. A., OJHA, R. P. & DURIAN, D. J. 2003 Low-speed impact crater in loose granular media. *Phys. Rev. Lett.* **90**, 194301.
- WALSH, A. M., HOLLOWAY, K. E., HABDAS, P. & DE BRUYN, J. R. 2003 Morphology and scaling of impact craters in granular media. *Phys. Rev. Lett.* **91**, 104301.