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VOLUME 19, 2002

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*An international journal of
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Helium-star Mass Loss and Its Implications for Black Hole Formation and Supernova Progenitors

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Received 2001 November 12, accepted 2002 March 15

Abstract: Recently the observationally derived stellar-wind mass-loss rates for Wolf-Rayet stars, or massive naked helium stars, have been revised downwards by a substantial amount. We present evolutionary calculations of helium stars incorporating such revised mass-loss rates, as well as mass transfer to a close compact binary companion. Our models reach final masses well in excess of $10 M_{\odot}$, consistent with the observed masses of black holes in X-ray binaries. This resolves the discrepancy found with previously assumed high mass-loss rates between the final masses of stars which spend most of their helium-burning lifetime as Wolf-Rayet stars ($\sim 3 M_{\odot}$) and the minimum observed black hole masses ($6 M_{\odot}$). Our calculations also suggest that there are two distinct classes of progenitors for Type Ic supernovae: one with very large initial masses ($\gtrsim 35 M_{\odot}$), which are still massive when they explode and leave black hole remnants, and one with moderate initial masses ($\sim 12\text{--}20 M_{\odot}$) undergoing binary interaction, which end up with small pre-explosion masses and leave neutron star remnants.

Keywords: binaries: close — black hole physics — stars: evolution — stars: mass loss — stars: Wolf-Rayet — supernovae: general

1 Introduction

A helium star is the naked core of a star that has lost its H-rich envelope, as a result of either a strong stellar wind or binary interaction. In a very massive star (initial mass $M_i \gtrsim 40 M_{\odot}$) the stellar wind is strong enough to remove the envelope before or during the central He-burning phase of evolution. Such stars can thus leave single naked He-burning stars with masses larger than about $15 M_{\odot}$. Less massive helium stars can be produced by mass transfer in a close binary system, if the primary (more massive) component of a binary has $M_i > 2\text{--}3 M_{\odot}$ and the orbital dimensions are such that Roche lobe overflow (RLOF) starts during the main sequence (case A mass transfer) or during the Hertzsprung gap or the first giant branch (case B). The remnant of mass transfer will then be an almost naked helium star with $M > 0.32 M_{\odot}$ (the minimum mass for helium burning) orbiting a more massive main sequence star (van den Heuvel 1994). This allows quite a large range in orbital periods as well as initial masses, and helium stars in binaries are therefore expected to be quite common.¹ Here we will consider helium stars that are massive enough to undergo core collapse and end their lives as neutron stars or black holes, i.e. $M_{\text{He}} \gtrsim 2\text{--}2.5 M_{\odot}$. This requires initial masses of at least $8\text{--}10 M_{\odot}$.

¹The fact that not many such systems are known can be understood by considering that the main sequence star completely dominates the spectrum at optical wavelengths (Pols et al. 1991).

1.1 Mass Loss and the Formation of Black Holes

Helium stars with $M \gtrsim 5\text{--}10 M_{\odot}$ have strong mass loss themselves; they are identified with hydrogen-free Wolf-Rayet (WR) stars of type WN or WC. Mass loss strongly influences the evolution of these stars, as well as their final masses and fate (Langer 1989). Unfortunately, the observationally derived mass-loss rates for WR stars are very uncertain. Evolution models for He stars computed with the mass-loss parametrisations suggested by Langer (1989) or Hamann, Koesterke, & Wessolowski (1995) result in strong mass convergence: even for the largest initial masses the final mass before core collapse is no more than $3\text{--}4 M_{\odot}$ (e.g. see Woosley, Langer, & Weaver 1995; Wellstein & Langer 1999). However, recent WR wind models that take into account the inhomogeneous structure of the wind have led to a downward revision of the mass-loss rate by a factor of 3–5 (Hamann & Koesterke 1998; Nugis & Lamers 2000). In fact, the Nugis & Lamers rate is nearly an order of magnitude smaller than the Hamann et al. (1995) rate for WN stars.

Whether the helium star leaves a neutron star or a black hole remnant will be determined to a large extent by its final mass before core collapse, or rather, by its final core mass. However, the outcome depends on the details of the explosion mechanism and the limiting mass is very uncertain (e.g. see Fryer et al. 2002). Important observational constraints come from X-ray binaries with low-mass companions (LMXB) in which the dynamically inferred mass of the compact star exceeds $3 M_{\odot}$, the maximum possible

neutron star mass. A strict lower limit to the black hole (BH) mass is set by the mass function, which in several LMXB is at least $6 M_{\odot}$ (e.g. Casares, Charles, & Naylor 1992; McClintock et al. 2001). In a few cases, the inferred BH mass is very likely to be $\gtrsim 10 M_{\odot}$ (McClintock 1998; Orosz et al. 2001). In the evolutionary scenarios for the formation of BH–LMXB the immediate progenitor of the black hole is a naked He star. Because the black holes in these systems can hardly have accreted any mass (King & Kolb 1999), the pre-explosion mass must have exceeded the BH mass, probably by a substantial amount if the collapse was accompanied by a supernova explosion. Clearly, if all naked He stars reach final masses of only $3\text{--}4 M_{\odot}$ these facts cannot be accounted for. One possible solution is that the progenitors of the observed systems result from case C mass transfer (i.e. RLOF started after central He exhaustion) rather than case A/B (Brown, Lee, & Bethe 1999). In that case the naked He star has already gone through core He burning and is close to core collapse when it forms, so there is insufficient time to lose a significant amount of mass in a stellar wind. Whether this scenario can explain all (or any) of the observed BH–LMXBs depends critically on the range of initial masses and orbital periods for which case C mass transfer is possible, which in turn depends quite sensitively on uncertain details of stellar evolution models. However, in the light of the revised WR mass-loss rates, it is worthwhile to reconsider whether case A/B mass transfer cannot after all produce massive black holes. Nelemans & van den Heuvel (2001), using a simple analytic estimate, suggest that this is indeed possible. In this paper we present full-scale evolutionary calculations incorporating the Nugis & Lamers (2000) mass-loss rate in order to investigate this question.

1.2 Supernovae of Types Ib and Ic

The amount of mass loss from a helium star, and hence its final mass, also has important consequences for the type of supernova (SN) explosion it produces. Type Ib supernovae show helium lines in their spectra, and their connection to the core collapse of helium stars is quite straightforward. Type Ic supernovae show little or no evidence for helium, and their progenitors are less obvious. However, the similarity of their late-time spectra indicates that the progenitors of SN Ib and SN Ic are related, and in fact small amounts of helium may be present in SNe Ic (Filippenko, Barth, & Matheson 1995). Hence helium cores that have been stripped of all or most of their helium layers are the best candidates for SNe Ic. As with the formation of helium stars themselves, either strong stellar-wind mass loss or mass transfer in a binary may be responsible for this additional stripping. If the initial mass is large enough, the strong WR wind can expose the C-rich core and remove most of the helium. In binary systems that have gone through case A/B mass transfer, a second phase of mass transfer is possible from the helium star to its companion (case BB mass transfer), if the orbit is close enough and the helium star is not too massive ($M \lesssim 6 M_{\odot}$).

In this paper we attempt to identify likely progenitors of Type Ic supernovae and of black holes in X-ray binaries in the light of revised mass-loss rates for Wolf-Rayet stars. To this end we present evolutionary calculations of massive helium stars, losing mass both through a stellar wind and by mass transfer to a nearby compact binary companion. We describe the calculations and their initial conditions in Section 2, and the relevant results in Section 3. The implications are discussed in Section 4.

2 Evolutionary Calculations and Initial Conditions

The stellar evolution calculations on which this paper is based are described in detail by O. R. Pols (in preparation) and Dewi et al. (2002). The initial configuration at the start of each calculation is a homogeneous star of almost pure helium with a solar fraction of heavier elements ($Y = 0.98$, $Z = 0.02$) which we evolve through central helium burning and through carbon burning. We consider initial helium star masses $M_{\text{He},i}$ between 2 and $32 M_{\odot}$. The underlying assumption is that such a He star has formed by case A or case B mass transfer in a binary, and we consider two extreme cases.

In the first case (I) we assume that the companion is a main sequence star and that mass transfer was conservative, which leads to a widening of the orbit (van den Heuvel 1994). For this case we computed the evolution of the He star in isolation, i.e. not considering the possibility of a second (case BB) mass transfer phase as the He star expands. This will be correct except perhaps for He stars less massive than $2.5 M_{\odot}$ which can attain radii of $100 R_{\odot}$ or more. In the calculations we apply the stellar-wind mass-loss rate of Nugis & Lamers (2000), which depends on the luminosity L and on the surface abundances, thus:

$$\dot{M} = 1.0 \times 10^{-11} (L/L_{\odot})^{1.29} Y^{1.7} Z^{0.5} M_{\odot}/\text{yr}. \quad (1)$$

The composition dependence causes mass loss to accelerate when C and O are exposed at the stellar surface, which has interesting consequences. This set of single-star models applies both to sufficiently wide case A/B binaries, and to actually single He stars with $M_{\text{He},i} \gtrsim 15 M_{\odot}$, which can be formed by stellar-wind mass loss. These calculations are described by O. R. Pols (in preparation).

In the second case (II) we assume that the companion is a neutron star (NS) in a close orbit, and we compute the non-conservative mass transfer that ensues when the He star fills its Roche lobe. We assume that the NS accretes up to its Eddington limit and that the excess mass is lost from the system with the specific orbital angular momentum of the NS. Such a system can form as a result of the spiral-in of the neutron star in the envelope of a massive star evolving off the main sequence in an initially wide orbit, i.e. as a remnant of a Be/X-ray binary (van den Heuvel 1994). We consider a range of orbital periods for the He-star + NS binary, consistent with the (very uncertain) periods expected after such a spiral-in (Dewi et al. 2002). This case can be considered as an advanced evolutionary stage of case I, i.e. after the first-formed He

star has undergone core collapse and become a neutron star. In these calculations we apply a somewhat different mass-loss rate, namely one fourth of the L -dependent rate proposed by Hamann et al. (1995). This rate is larger (by roughly a factor of two) than the Nugis & Lamers rate, but there is no real discrepancy with case I because the final mass in case II is determined by RLOF and not by the stellar wind for the masses considered (up to $6.6 M_{\odot}$; more massive He stars expand very little and usually avoid RLOF). We take full account of the orbital evolution resulting from non-conservative RLOF, the stellar wind, and gravitational wave radiation (for details see Dewi et al. 2002).

We note that other cases are possible, e.g. non-conservative case B mass transfer leading to a He-star + main sequence binary in a fairly close orbit. In such a system we also expect the He star to undergo case BB mass transfer, but this time to a main sequence star and — in all likelihood — conservatively. The result would probably be intermediate between cases I and II, but this should be borne out by actual calculations.

3 Results: Final Masses and Helium Amounts

In this section we concentrate on the final configurations resulting from the calculations: the stellar mass $M_{\text{He},f}$ and the amount of helium left in the envelope $\Delta M_{\text{He},f}$ just before core collapse,² and we discuss their implications for supernovae and black hole formation. For other aspects of the models we refer to the papers by O. R. Pols (in preparation) and Dewi et al. (2002).

3.1 Case I

The final masses of the single He-star models (i.e. remnants of conservative case A/B mass transfer) are plotted in Figure 1. At the top of the figure, the initial zero-age main sequence (ZAMS) mass has been indicated, under the assumption that the initial mass of the He star equals the core mass at helium ignition. This is approximately correct for case B, but underestimates the ZAMS mass if the He star formed by case A. The relation $M_{\text{He},i} = 0.098 M_{\text{ZAMS}}^{1.35}$ has been used to estimate the ZAMS mass (Hurley, Pols, & Tout 2000, equation 44). Final masses up to $13 M_{\odot}$ are reached for the most massive stars considered. Although the final–initial mass relation levels off, no mass convergence is apparent. We see that for case B binaries starting with $M_{\text{ZAMS}} > 27 M_{\odot}$, final masses in excess of $6 M_{\odot}$ are reached. This is consistent with the largest observed black hole masses if no additional mass is lost during the collapse of the black hole. On the other hand if we assume, for example, that 25 per cent of the mass is ejected when the black hole forms, the ZAMS mass needs to be $>40 M_{\odot}$. Under this assumption it is still possible to obtain even a

²The values given refer to the end of the calculations, which in most cases extended well into carbon shell burning. During the very short remaining time until core collapse, the masses cannot change significantly and can indeed be considered as final.

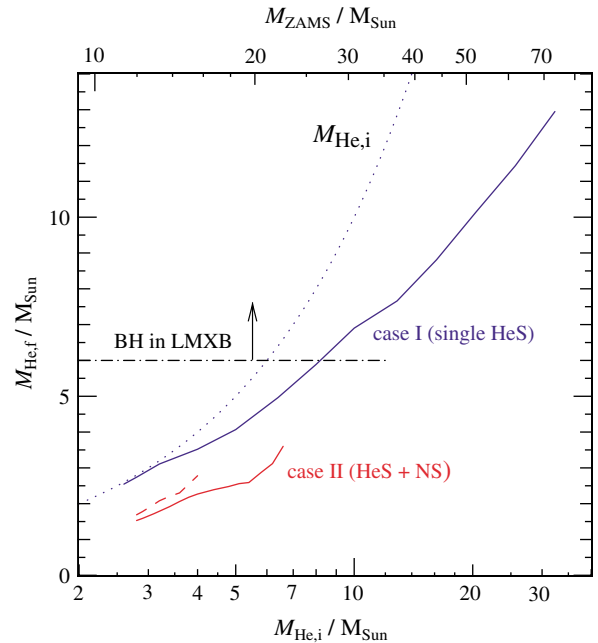


Figure 1 The relation between initial helium star mass and final mass, for wide binaries (effectively single He stars, case I) and close binaries with additional case BB mass transfer to a NS companion (case II). Stellar-wind mass loss according to the prescription by Nugis & Lamers (2000) has been applied for case I (upper, blue solid line). The lower, red solid line is for case II with binary periods of 0.08–0.09 days, the dashed red line is for periods of 0.4–0.5 days. The observed lower mass limit for the most massive black holes in LMXBs is shown as a dashed–dotted line. The dotted line reproduces the initial He-star mass (note that the horizontal scale is logarithmic). Along the top the ZAMS mass has been indicated, assuming that the He star formed as a result of case B mass transfer.

$10 M_{\odot}$ black hole after case B mass transfer, starting from a $32 M_{\odot}$ He star (or $M_{\text{ZAMS}} \approx 70 M_{\odot}$).

A slight break in the initial–final mass relation is apparent between $M_{\text{He}} = 10$ and $12 M_{\odot}$. This is caused by the composition dependence in the Nugis & Lamers mass-loss rate which increases with Z , the fraction of heavy elements. For initial mass up to $10 M_{\odot}$, the products of He burning are never exposed to the surface, while for larger masses the surface becomes carbon enriched (these stars make the transition from WN to WC stars). As a result the mass loss accelerates when this transition is made.

In Figure 2 we show the final mass of ${}^4\text{He}$ left in the envelope prior to core collapse, as a function of initial He star mass. For case I two regimes can be distinguished. For $M_{\text{He},i} \lesssim 10 M_{\odot}$ more than $1 M_{\odot}$ of He remains in the envelope, while for $M_{\text{He},i} \gtrsim 12 M_{\odot}$ only $\approx 0.4 M_{\odot}$ of He is left. The transition is quite sharp, for the same reason as indicated above: mass loss accelerates when the star becomes a WC star and removes a large fraction of the remaining helium. The transition is smoother if mass loss does not depend on composition but only on luminosity, as in the calculations of Wellstein & Langer (1999, their Figure 6). Following the arguments in that paper, it is tempting to identify progenitors of Type Ib supernovae with $M_{\text{He},i} \lesssim 10 M_{\odot}$ and those of Type Ic SNe with

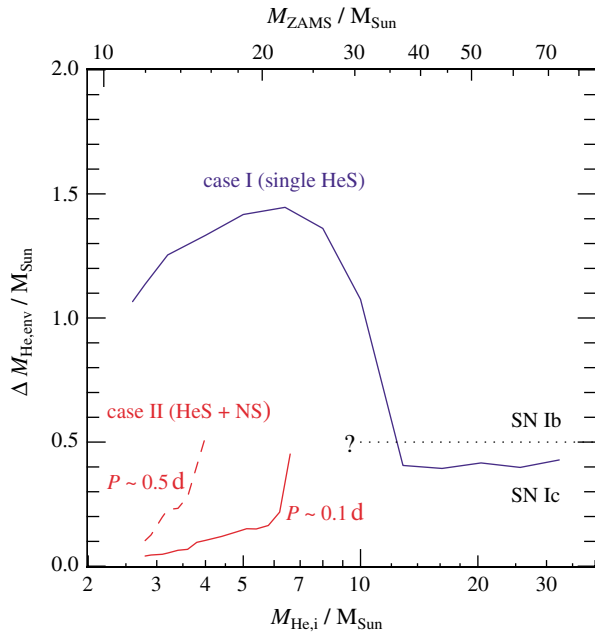


Figure 2 Final mass of helium left in the envelope of the He star before core collapse. The curve styles are the same as in Figure 1. The dotted line suggests a possible critical mass of He, below which the explosion would appear as a Type Ic supernova.

$M_{\text{He},i} \gtrsim 12 M_{\odot}$. Although the latter still have a substantial amount of helium, this may not show up in the SN spectrum if it is not mixed with the radioactive ^{56}Ni (Woosley & Eastman 1997). Such Type Ic SN progenitors would be massive ($>7 M_{\odot}$), and probably leave black hole remnants, although in view of the large uncertainties in the core-collapse mechanism, neutron star remnants cannot be excluded. Although rather massive black holes may form by direct collapse (Fryer 1999), in at least one LMXB there is strong evidence that the formation of the black hole was indeed accompanied by a supernova explosion (Israelian et al. 1999). These explosions can possibly be identified with hypernovae such as SN 1998bw, which was of Type Ic and associated with a γ -ray burst. A massive, almost bare CO core in combination with a large explosion energy can explain the bright and slowly declining lightcurve and broad spectral features of such hypernovae (Iwamoto et al. 2000).

3.2 Case II

Helium stars in close orbits around a NS companion initially also lose mass in a stellar wind. When non-conservative RLOF starts, much higher mass-loss rates are reached, up to $10^{-4} M_{\odot}/\text{yr}$, because mass transfer occurs on the thermal timescale of the He star (see Dewi et al. 2002 for more details). At the end of the calculations most of the envelope has been transferred and lost from the binary system, unless RLOF starts when the He star is already close to carbon burning. As a consequence, the final masses, shown in Figure 1, depend somewhat on the initial orbital period of the He-star + NS system, but are usually close to the CO core mass except for the widest

orbits and most massive He stars. In all cases they are substantially smaller ($1.5\text{--}3 M_{\odot}$) than in case I, where only the stellar wind operates. For $M_{\text{He},i} > 6.6 M_{\odot}$ RLOF becomes dynamically unstable, but such systems are rare because He stars of such masses hardly expand. The small final mass implies core collapse will result in neutron star formation. Unless the explosion disrupts the binary, the remnants of these systems are therefore double NS binaries which, if close enough to merge in a Hubble time, are candidate progenitors of γ -ray bursts.

The final mass of He in the envelope, as shown in Figure 2, is also much smaller than in case I but, like the final stellar mass, depends on the initial He star mass and on the orbital period. In most cases it is $\lesssim 0.2 M_{\odot}$, and for the shortest orbital periods, $P \approx 0.1$ day, it can be as small as $0.04 M_{\odot}$. This is much less than is achieved for massive He stars by stellar wind only (case I, see above). It is also less than in the conservative case BB mass-transfer models by Wellstein & Langer (1999). We conclude that non-conservative case BB mass transfer to a compact companion is the most efficient way to produce almost bare CO cores prior to explosion. These stars will almost certainly produce a Type Ic supernova. The small progenitor mass would result in a relatively faint, fast declining lightcurve. Such a model was first suggested by Nomoto et al. (1994) as the progenitor of the Type Ic SN 1994I, and shown to match the observed lightcurve (Iwamoto et al. 1994).

4 Summary and Conclusions

If the reduced and composition-dependent mass-loss rate for WR stars of Nugis & Lamers (2000) is adopted, the following picture emerges. Binary components with $M \gtrsim 35 M_{\odot}$ that form WR stars through case A or B mass transfer, as well as single stars massive enough to form WR stars directly, reach final He-star masses in excess of $7 M_{\odot}$ and have only a small amount of He ($\sim 0.4 M_{\odot}$) left in their envelopes. Such stars probably leave black hole remnants, and the final masses are large enough to be consistent with the observed BH masses in X-ray binaries. Unless such black holes form by direct collapse, these stars very likely undergo a Type Ic supernova explosion which can possibly be identified with bright, slowly declining hypernovae such as SN 1998bw.

Less massive stars in binaries undergoing case A/B mass transfer, but wide enough to avoid case BB mass transfer, have smaller final masses and at least $1 M_{\odot}$ of He left in their envelope. These stars undergo a Type Ib supernova and leave either neutron stars or, possibly, black holes for stars at the upper end of the mass range.

Binary components with initial mass $\lesssim 20 M_{\odot}$ and in close enough orbits undergo case BB mass transfer. Independently of the adopted WR mass-loss rate, this results in even smaller final masses and a smaller amount of He left in the envelope. The most efficient way to end up with an almost He-free star prior to core collapse is by case BB mass transfer to a neutron star in a very close orbit (i.e. RLOF from the initial secondary component of the binary, after the primary has already collapsed). For small initial

masses and orbits with $P \sim 0.1$ day it is possible to have as little as $0.04 M_{\odot}$ of He left in the envelope. Such stars almost certainly explode as a SN Ic, and have very small pre-SN masses ($1.5\text{--}3 M_{\odot}$). They can be identified with faint, fast declining Type Ic supernovae like SN 1994I.

In summary, our most important conclusions are:

- Adoption of the Nugis & Lamers mass-loss rate leads to final He-star masses after case A/B mass transfer that are consistent with the observed black hole masses. There is no need to resort to case C mass transfer to explain the observed BH binaries, although such a scenario remains a possibility.
- Massive binary evolution leads to two distinct classes of SN Ic progenitors: one with large pre-explosion mass, $M > 7 M_{\odot}$, formed from initial masses $M > 35 M_{\odot}$, which probably leave BH remnants and can possibly be identified with hypernovae, and the other with small pre-explosion mass, $1.5\text{--}3 M_{\odot}$, formed by case BB mass transfer in binaries with $M \sim 12\text{--}20 M_{\odot}$, which leave NS remnants.

The latter conclusion is similar to that of Wellstein & Langer (1999), but in their models massive SN Ic progenitors have much smaller pre-explosion masses, $3\text{--}4 M_{\odot}$.

We note that other consequences of the mass-loss rate of WR stars also need to be explored, in particular for the properties of WR stars themselves. Their luminosity and abundance distributions and the number ratio of WN to WC stars all depend on the adopted mass-loss rate. This is beyond the scope of this paper, but we emphasise that in order to draw definite conclusions about WR mass loss and BH formation, all these aspects have to be considered in conjunction.

Acknowledgments

It is a pleasure to thank Norbert Langer, Ed van den Heuvel, Gijs Nelemans, and Gert-Jan Savonije for stimulating discussions and valuable suggestions. Jasinta Dewi

acknowledges sponsorship by NWO Spinoza Grant 08-0 to E. P. J. van den Heuvel.

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