# Transformations from WISE to 2MASS, SDSS and BVI Photometric Systems: II. Transformation Equations for Red-Clump Stars 

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#### Abstract

We present colour transformations for the conversion of Wide-Field Survey Explorer W1, W2, and $W 3$ magnitudes to the Johnson-Cousins $B V I_{\mathrm{c}}$, Sloan Digital Sky Survey gri, and Two Micron All Sky Survey $J H K_{\mathrm{s}}$ photometric systems, for red clump (RC) stars. RC stars were selected from the Third Radial Velocity Experiment Data Release. The apparent magnitudes were collected by matching the coordinates of this sample with different photometric catalogues. The final sample ( 355 RC stars) was used to obtain metallicitydependent and free-of-metallicity transformations. These transformations combined with known absolute magnitudes at shorter wavelengths can be used in space density determinations for the Galactic (thin and thick) discs at distances larger than the ones evaluated with $J H K_{\mathrm{s}}$ photometry alone, hence providing a powerful tool in the analysis of Galactic structure.


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## 1 Introduction

Red clump (RC) stars are core helium-burning stars, in an identical evolutionary phase to that of stars on the horizontal branch in globular clusters. However, in intermediate- and higher-metallicity systems only the red end of the distribution is seen, forming a clump of stars in the colour-magnitude diagram. In recent years much work has been devoted to studying the suitability of RC stars as a distance indicator. Their absolute magnitude in the optical ranges from $M_{\mathrm{V}}=+0.70 \mathrm{mag}$ for those of spectral type G8 III to $M_{\mathrm{V}}=+1.0 \mathrm{mag}$ for the K2 III ones (Keenan \& Barnbaumet 1999). The absolute magnitude of these stars in the $K_{\mathrm{s}}$ band is $M_{K_{\mathrm{s}}}-1.54 \pm 0.04 \mathrm{mag}$ with negligible dependence in metallicity (Groenewegen 2008). The optical and infrared colour ranges for these stars are $0.8 \leq(B-V)_{0} \leq 1.3$ and $0.29 \leq(J-H)_{0} \leq 0.65$, respectively, and they have a limited surface gravity, i.e. $2.1 \leq \log g \leq 2.7 \mathrm{~cm} \mathrm{~s}^{-2}$ (Puzeras et al. 2010).

It should be added that RC stars are different in structure than the ones in late transitional phases of evolution off the main sequence or immediately before a supernova, which have circumstellar material, i.e. red supergiants, yellow hypergiants, luminous blue variables,
$B[e]$ supergiants and equatorial rings, interacting binaries and Wolf-Rayet stars.

In a former paper (Bilir et al. 2011a, hereafter Paper I) we presented the transformation equations from WideField Survey Explorer WISE to Two-Micron All-Sky Survery (2MASS), Sloan Digital Sky Survey (SDSS) and Johnson-Cousins photometric systems for dwarf stars. Here, our aim is to obtain similar transformations between the same photometric systems but for RC stars. The galactic model parameters can be obtained more precisely using WISE absolute magnitudes calculated from these transformations. In the next paragraphs, we give a short definition for the mentioned photometric systems and the Wide-Field Survey Explorer (WISE) survey, which provides the data used in our study. However, we refer the reader to Paper I for a more complete information.

The SDSS obtains images almost simultaneously in five broad bands ( $u, g, r, i$, and $z$ ) centered at 3560,4680 , 6180,7500 and 8870 A, respectively, (York et al. 2000). The magnitudes derived from fitting a point spread function (PSF) are currently accurate to about $1 \%$ in $g, r, i, z$ and $2 \%$ in $u$ for point sources (Padmanabhan
et al. 2008). The data have been made public in a series of yearly data release where the eighth data release (DR8, Aihara et al. 2011) covers $14555 \mathrm{deg}^{2}$ of imaging area. The limiting magnitudes are $(u, g, r, i, z)=(22,22.2,22.2$, 21.3, 20.5).

The 2MASS (Skrutskie et al. 2006) provides the most complete data base of near infrared (NIR) Galactic point sources available to date. Observations cover 99.998\% (Skrutskie et al. 2006) of the sky with simultaneous detections in $J(1.25 \mu \mathrm{~m}), H(1.65 \mu \mathrm{~m})$, and $K_{\mathrm{s}}(2.17 \mu \mathrm{~m})$ bands up to the limiting magnitudes of $15.8,15.1$, and 14.3, respectively. Bright source extractions have $1 \sigma$ photometric uncertainty of $<0.03 \mathrm{mag}$ and astrometric accuracy on the order of 100 mas.

The WISE (Wright et al. 2010), an up-to-date infrared (IR) satellite, began surveying the sky on 2010 January 14 and completed its first full coverage of the sky on 2010 July 17 with much higher sensitivity than comparable previous IR survey missions. WISE has four IR filters W1, $W 2, W 3$, and $W 4$ centered at $3.4,4.6,12$, and $22 \mu \mathrm{~m}$, and with the angular resolutions $6.1,6.4,6.5$, and 12 arcsec , respectively and has a $40-\mathrm{cm}$ telescope feeding array with a total of four million pixels. WISE has achieved $5 \sigma$ pointsource sensitivities better than $0.08,0.11,1$ and 6 mJy at $3.4,4.6,12$, and $22 \mu \mathrm{~m}$, respectively. These sensitivities correspond to the Vega magnitudes $16.5,15.5,11.2$, and 7.9. Thus WISE will go a magnitude deeper than the 2MASS $K_{\mathrm{s}}$ data in $W 1$ for sources with spectra close to that of an A0 star, and even deeper for moderately red sources like K stars or galaxies with old stellar populations.

The RAVE (Steinmetz et al. 2006) measures radial velocities and stellar atmospheric parameters from spectra using the 6 dF multi-object spectrometer on the Anglo-Australian Astronomical Observatory's $1.2-\mathrm{m}$ UK Schmidt Telescope. The survey looks in the Ca-triplet region ( $8410-8795$ A), has a resolution of $\sim 7500$, and is magnitude limited. The targets chosen are Southern Hemisphere stars taken from the Tycho-2, SuperCOSMOS and the Deep Near Infrared Survey of the Southern Sky (DENIS, Fouque et al. 2000) surveys with $I$-band magnitudes between 9 and 13. The average internal errors in radial velocity are $\sim 2 \mathrm{~km} \mathrm{~s}^{-1}$, and the approximate radial velocity offset between the RAVE and the literature is smaller than $\sim 1 \mathrm{~km} \mathrm{~s}^{-1}$. The catalogue also includes 2MASS photometry and proper motions from Starnet 2.0, Tycho-2, SuperCOSMOS, and UCAC2 (for more information about RAVE, see Zwitter et al. 2008).

The passband profiles for the Johnson-Cousins, SDSS, 2MASS, and WISE photometric systems are shown in Figure 1. With respect to the same figure in Paper I, we omitted here the passband for $R$ which is not used in our transformations but we added the DENIS passband $I_{\mathrm{d}}$ with which we evaluated the Cousins optical magnitudes (I). $W 3$ and $W 4$ could not be used in Paper I in the transformations for dwarfs due to the faintness of dwarfs in both bands. In this study of RC stars, the W3 magnitudes could
be used but the $W 4$ magnitudes were too faint. Hence, as in the inverse transformations for dwarfs, the $J-H$ colour of the 2MASS photometric system is used as a second colour combined linearly with $W 1-W 2$. Figure 2 plots the fields available with the WISE and RAVE surveys.

In Section 2 we present the sources of our sample and the criteria applied to the chosen stars. The transformation equations are given in Section 3 and finally, we give a summary and conclusions in Section 4.


Figure 1 Normalised passbands of the Johnson-Cousins-DENIS filters (a), the SDSS filters (b), the 2MASS filters (c), and the WISE filters (d).


Figure 2 Equatorial coordinates of the stars observed in WISE (grey regions) and RAVE (black squares) surveys.

## 2 The Data

### 2.1 RAVE Sample with Tycho-2, DENIS, and 2MASS Data

The main source of our data is the RAVE Data Release (DR3) catalogue (Siebert et al. 2011). The advantage of the RAVE catalogue is that it includes the atmospheric parameters $\left(T_{\text {eff }}, \log g,[M / H]\right)$ with high accuracy. This is important, as the surface gravity is used to separate the dwarfs and the RC stars and the transformations are derived for different metallicity bins. We initially applied two constraints: $2<\log g\left(\mathrm{~cm} \mathrm{~s}^{-2}\right) \leq 3$ and $J-H>0.4$, and obtained a sample of 8003 stars from the RAVE DR3. The reason of these constraints is due to the fact that most of the RC stars lie in this $\log g$ interval and that they are much larger in number in the $J-H>0.4$ colour interval (Bilir et al. 2011b). We then included the following additional but necessary constraints.

1. We selected stars for which Tycho-2 ( $B_{\mathrm{T}}, V_{\mathrm{T}}$ ), DENIS $\left(I_{\mathrm{d}}\right)$, and 2MASS $\left(J H K_{\mathrm{s}}\right)$ magnitudes were available (3103 stars).
2. We matched the reduced RAVE DR3 catalogue with the WISE Preliminary Data Release (PDR) Catalogue ${ }^{1}$ and chose the stars which were available with $W 1, W 2$, $W 3$, and $W 4$ magnitudes ( 954 stars).
3. We used magnitudes, labeled with 'AAA' flags, which means $S / N \geq 10$, i.e. they have the highest quality measurements, for the 2MASS and WISE magnitudes (918 stars).
4. We limited $B_{\mathrm{T}}-V_{\mathrm{T}}$ colours with $0.8<\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right) \leq 1.7$ and excluded the stars with $B_{\mathrm{T}}-V_{\mathrm{T}}$ error larger than 0.2 . Thus the complete sample reduced to 355 stars. The $(J-H)_{0}-(B-V)_{0}$ two colour diagram and the spectral distribution of the final sample in three metallicity categories are given in Figure 3.


Figure 3 Two colour diagram of the sample stars. The symbols give: ( 0 ) $[M / H]>-0.4,(+)-1<[M / H] \leq-0.4$, and ( $\triangle$ ) $[M /$ $H] \leq-1$ dex.

[^0]
### 2.2 Evaluating the $\mathrm{BVI}_{c}$ Magnitudes

The RAVE survey does not involve any star observed in the Johnson-Cousins ( $B V I$ ) system. Hence, we have to use the following procedure to obtain $B, V$, and $I$ magnitudes for our sample: We revealed that 370 stars in the Landolt's (2009) UBVRI Photometric Standard Stars catalogue were observed in the DENIS survey (Fouque et al. 2000). We excluded stars with errors in $I_{\mathrm{d}}$ larger than 0.1 mag, thus the sample reduced to 355 . We matched this sample with the 2MASS catalogue and used magnitudes, labeled with 'AAA' flags, for obtaining the magnitudes of highest quality. This constraint reduced the number of stars to 344 .

Finally, we plotted the $V$ magnitudes of Johnson versus the $J$ magnitudes of 2MASS in a two magnitude diagram and eliminated the dwarfs from the sample in Figure 4 (see Bilir et al. 2006, for a description of the elimination method). By doing this, the final giant sample consists of 128 stars.

Figure 4 b compares the optical magnitudes of DENIS $\left(I_{\mathrm{d}}\right)$ and Cousins ( $I$ ) supplied from Landolt (2009). After rejecting four stars which showed large scattering, we obtained the following equation which is used for evaluation of the $I$ (hereafter $I$ ) magnitudes of the sample:

$$
\begin{equation*}
I=1.040( \pm 0.007) I_{\mathrm{d}}-0.501( \pm 0.085) \tag{1}
\end{equation*}
$$

The $V$ magnitudes and $B-V$ colours were evaluated by the following equations taken from the Hipparcos and Tycho catalogue (ESA 1997):

$$
\begin{align*}
V= & V_{\mathrm{T}}+0.0036-0.1284\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right) \\
& +0.0442\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)^{2}-0.015\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)^{3}, \tag{2}
\end{align*}
$$

where

$$
(B-V)=\left\{\begin{array}{c}
\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)-0.113-0.258 z+0.40 z^{3}  \tag{3}\\
\text { if } \quad 0.65<\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)<1.1 \\
\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)-0.173-0.220 z-0.01 z^{3}, \\
\text { if } \quad 1.1<\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)
\end{array}\right.
$$

and

$$
z=\left\{\begin{array}{lll}
\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)-0.95, & \text { if } & 0.65<\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)<1.1  \tag{4}\\
\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)-1.20, & \text { if } & 1.1<\left(B_{\mathrm{T}}-V_{\mathrm{T}}\right)
\end{array} .\right.
$$

### 2.3 Reddening and Metallicity

The $E(B-V)$ colour excess of the stars has been evaluated in two steps. First, we used the maps taken from Schlegel, Finkbeiner \& Davis (1998) and evaluated a $E_{\infty}(B-V)$ colour excess for each star. We then reduced


Figure 4 The $J$ versus $V$ two-magnitude diagram for 344 stars observed by Landolt (2009) and the 2MASS survey (panel a) and the two magnitude diagram of Johnson's and DENIS optical magnitudes, $I$ and $I_{\mathrm{d}}$, for the 124 giants (panel b) identified in panel (a).
them using the following procedure (Bahcall \& Soneira 1980):

$$
\begin{equation*}
A_{\mathrm{d}}(b)=A_{\infty}(b)\left[1-\exp \left(\frac{-|d \times \sin b|}{H}\right)\right] \tag{5}
\end{equation*}
$$

Here, $b$ and $d$ are the Galactic latitude and distance of the star, respectively. $H$ is the scaleheight for the interstellar dust, which is adopted as 125 pc (Marshall et al. 2006) and $A_{\infty}(b)$ and $A_{\mathrm{d}}(b)$ are the total absorptions for the model and for the distance to the star, respectively. $A_{\infty}(b)$ can be evaluated by means of the following equation:

$$
\begin{equation*}
A_{\infty}(b)=3.1 E_{\infty}(B-V) \tag{6}
\end{equation*}
$$

$E_{\infty}(B-V)$ is the colour excess for the model taken from the Schlegel et al. (1998). Then, $E_{\mathrm{d}}(B-V)$, i.e. the colour excess for the corresponding star at the distance $d$, can be evaluated by Equation 7 adopted for distance $d$,

$$
\begin{equation*}
E_{\mathrm{d}}(B-V)=\frac{A_{\mathrm{d}}(b)}{3.1} \tag{7}
\end{equation*}
$$

As explained in Section 2.1, our sample consists of RC stars. Hence, we adopted the absolute magnitude $M_{K_{\mathrm{s}}}=$ $-1.54 \pm 0.04$ cited by (Groenewegen 2008) and substituted it into the following equation to obtain the distances of the sample stars:

$$
\begin{equation*}
\left(K_{\mathrm{s}}-M_{K_{\mathrm{s}}}\right)_{0}=5 \log d-5 \tag{8}
\end{equation*}
$$

This value has also a small dependence on metallicity and age, hence it can be used accurately in determining the


Figure 5 Distribution of colour excess $E(B-V)$.
distances to the sources (see Cabrera-Lavers et al. (2007) and references therein for a complete description about using the red clump sources as distance estimators). As the total absorptions for the model and distance to a star are different, $A_{\mathrm{d}}(b)$, and colour excess, $E_{\mathrm{d}}(B-V)$, could be evaluated by iterating Equations 6 to 8 .

We have omitted the indices $\infty$ and $d$ from the colour excess $E(B-V)$ in the equations. However, we use the terms model for the colour excess of Schlegel et al. (1998) and 'reduced' the colour excess corresponding to distance $d$. The total absorption $A_{\mathrm{d}}$ used in the section and classical total absorption $A_{\mathrm{V}}$ have the same meaning.

We de-reddened the colours and magnitudes by using the $E_{\mathrm{d}}(B-V)$ colour excesses of the stars evaluated using the procedures explained above and the equations of Fan
Table 1. Johnson-Cousins, SDSS, 2MASS and WISE magnitudes and colours of the sample stars ${ }^{\text {a }}$

| Star name | $l(\mathrm{deg})^{\mathrm{b}}$ | $b(\mathrm{deg})^{\text {b }}$ | $V_{0}$ | $(B-V)_{0}$ | $(V-I){ }_{0}$ | $g_{0}$ | $(g-r)_{0}$ | $(r-i)_{0}$ | $J_{0}$ | $(J-H)_{0}$ | $\left(H-K_{\mathrm{s}}\right)_{0}$ | $W 1_{0}$ | $(W 1-W 2)_{0}$ | $(W 2-W 3)_{0}$ | $E_{\mathrm{d}}(B-V)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| T7934_31569_1 | 2.10152 | -28.69877 | 10.236 | 1.068 | 0.917 | 10.920 | 0.784 | 0.260 | 8.635 | 0.513 | 0.118 | 7.933 | -0.097 | 0.116 | 0.085 |
| T5031_00326_1 | 2.46213 | +34.06951 | 15.566 | 0.804 | 0.890 | 11.296 | 0.749 | 0.247 | 9.095 | 0.503 | 0.113 | 8.478 | -0.087 | 0.135 | 0.169 |
| T7949_00356_1 | 3.90360 | -34.07636 | 10.651 | 0.991 | 1.143 | 11.082 | 0.850 | 0.288 | 8.646 | 0.555 | 0.133 | 8.024 | 0.015 | 0.147 | 0.047 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | ... | $\ldots$ | $\cdots$ | ... | ... | ... |
|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| T8396_01656_1 | 351.88421 | -33.13869 | 10.517 | 0.921 | 1.165 | 11.143 | 0.874 | 0.299 | 8.647 | 0.579 | 0.130 | 7.910 | -0.118 | 0.201 | 0.041 |
| T8409_01526_1 | 352.34396 | -34.52388 | 10.510 | 1.012 | 1.287 | 10.864 | 0.746 | 0.247 | 8.651 | 0.544 | 0.075 | 7.939 | -0.100 | 0.158 | 0.038 |
| T5008_00508_1 | 354.27874 | +45.28820 | 9.669 | 0.703 | 1.053 | 9.993 | 0.656 | 0.209 | 7.974 | 0.532 | 0.023 | 7.343 | $-0.031$ | -0.042 | 0.183 |

[^1]Table 2. Mean errors and standard deviations

| Filter | Mean error (mag) | $s$ | Photometry |
| :--- | :---: | :---: | :--- |
| $B$ | 0.1155 | 0.0370 | Johnson-Cousins |
| $V$ | 0.0604 | 0.0161 |  |
| $I$ | 0.0349 | 0.0531 |  |
| $J$ | 0.0246 | 0.0044 | 2 MASS |
| $H$ | 0.0382 | 0.0123 |  |
| $K_{\mathrm{s}}$ | 0.0244 | 0.0046 |  |
| $W 1$ | 0.0246 | 0.0043 | WISE |
| $W 2$ | 0.0216 | 0.0032 |  |
| $W 3$ | 0.0393 | 0.0098 |  |

(1999) and Fiorucci \& Munari (2003) for $V-I$ colour and for the 2MASS photometry:

$$
\begin{align*}
V_{0} & =V-3.1 E_{\mathrm{d}}(B-V) \\
(B-V)_{0} & =(B-V)-E_{\mathrm{d}}(B-V) \\
(V-I)_{0} & =(V-I)-1.250 E_{\mathrm{d}}(B-V) \\
J_{0} & =J-0.887 E_{\mathrm{d}}(B-V)  \tag{9}\\
(J-H)_{0} & =(J-H)-0.322 E_{\mathrm{d}}(B-V) \\
\left(H-K_{\mathrm{s}}\right)_{0} & =\left(H-K_{\mathrm{s}}\right)-0.183 E_{\mathrm{d}}(B-V) .
\end{align*}
$$

As the intrinsic gri magnitudes were transformed from the equations of Yaz et al. (2010), no de-reddening was necessary. For de-reddening the magnitudes $W 1, W 2$ and $W 3$, we adopted the corresponding total absorptions cited by Bilir et al. (2011a) i.e. $A_{W 1} / A_{\mathrm{V}}=0.051$, $A_{W 2} / A_{\mathrm{V}}=0.030$, and $A_{W 3} / A_{\mathrm{V}}=0.028$, evaluated by means of a spline function fitted to the data of Cox (2000) which cover a range of $0.002 \leq \lambda \leq 250 \mu \mathrm{~m}$. Figure 5 shows that the colour excess, $E(B-V)$, is less than 0.1 for most of the stars, and that their distribution peaks at $E(B-V)=0.05 \mathrm{mag}$.

The complete data for the sample of 355 stars are given in Table 1, while the errors for the magnitudes and colours for $B V I, J H K_{\mathrm{s}}$, and $W 1 W 2 W 3$ photometric systems are given in Table 2 and Figure 6. As the SDSS magnitudes were transformed from Yaz et al. (2010), we have not shown the corresponding errors in this study. The metallicities are the calibrated values of RAVE DR3. Figure 7 shows that our sample consists mostly of thin and thick disc stars, that present mean metallicities of -0.4 dex (Rocha-Pinto et al. 2006), and -0.7 dex (Cabrera-Lavers, Garzón \& Hammersley 2005), respectively. There are only very few stars with $[M / H]<-1$ and $[M / H]>$ 0.2 dex, and the mode is at $[M / H]=-0.35$ dex.

## 3 Results

### 3.1 Metallicity-Dependent Transformations

We adopted the procedure in Yaz et al. (2010) and used the following general equations to derive nine sets of transformations from WISE to Johnson-Cousins, SDSS and 2MASS photometries. As explained in Yaz et al. (2010), this approach, that includes a metallicity term instead of deriving transformations for a set of stars with a metallicity range but omitting the metallicity term, can be


Figure 6 The error distributions for the Johnson-Cousins (BVI), 2MASS $\left(J H K_{\mathrm{s}}\right)$ and $\operatorname{WISE}(W 1, W 2, W 3)$ magnitudes.


Figure 7 Metallicity distribution for the sample stars.
explained by the fact that stars change their positions in two colour diagrams by shifting an amount proportional to their metallicities. The general equations are as follows:

$$
\begin{align*}
(V-W 1)_{0}= & a_{1}(B-V)_{0}+b_{1}(V-I)_{0} \\
& +c_{1}[M / H]+d_{1}, \\
(V-W 2)_{0}= & a_{2}(B-V)_{0}+b_{2}(V-I)_{0} \\
& +c_{2}[M / H]+d_{2}, \\
(V-W 3)_{0}= & a_{3}(B-V)_{0}+b_{3}(V-I)_{0} \\
& +c_{3}[M / H]+d_{3}, \\
(g-W 1)_{0}= & a_{4}(g-r)_{0}+b_{4}(r-i)_{0} \\
& +c_{4}[M / H]+d_{4}, \\
(g-W 2)_{0}= & a_{5}(g-r)_{0}+b_{5}(r-i)_{0} \\
& +c_{5}[M / H]+d_{5},  \tag{10}\\
(g-W 3)_{0}= & a_{6}(g-r)_{0}+b_{6}(r-i)_{0} \\
& +c_{6}[M / H]+d_{6}, \\
(J-W 1)_{0}= & a_{7}(J-H)_{0}+b_{7}\left(H-K_{\mathrm{s}}\right)_{0} \\
& +c_{7}[M / H]+d_{7}, \\
(J-W 2)_{0}= & a_{8}(J-H)_{0}+b_{8}\left(H-K_{\mathrm{s}}\right)_{0} \\
& +c_{8}[M / H]+d_{8}, \\
(J-W 3)_{0}= & a_{9}(J-H)_{0}+b_{9}\left(H-K_{\mathrm{s}}\right)_{0} \\
& +c_{9}[M / H]+d_{9} .
\end{align*}
$$

The first three sets correspond to the Johnson-Cousins photometry, while the second three and third three sets were derived for SDSS and 2MASS photometries. The numerical values of the coefficients in Equations 10 are given in Table 3.

### 3.2 Metal-Free Transformations

We also derived metal-free transformations from WISE to Johnson-Cousins, SDSS and 2MASS photometries. These can be used to transform the BVI, gri, and $J H K_{\mathrm{s}}$ data of RC stars with unknown metallicities. Thus we give the chance to the researchers to transfer their BVI, gri, and $J H K_{\mathrm{s}}$ data with lack of metallicities for RC stars to the $W 1 W 2 W 3$ ones. The general equations are as follows:

$$
\begin{align*}
(V-W 1)_{0} & =\alpha_{1}(B-V)_{0}+\beta_{1}(V-I)_{0}+\gamma_{1}, \\
(V-W 2)_{0} & =\alpha_{2}(B-V)_{0}+\beta_{2}(V-I)_{0}+\gamma_{2}, \\
(V-W 3)_{0} & =\alpha_{3}(B-V)_{0}+\beta_{3}(V-I)_{0}+\gamma_{3}, \\
(g-W 1)_{0} & =\alpha_{4}(g-r)_{0}+\beta_{4}(r-i)_{0}+\gamma_{4}, \\
(g-W 2)_{0} & =\alpha_{5}(g-r)_{0}+\beta_{5}(r-i)_{0}+\gamma_{5},  \tag{11}\\
(g-W 3)_{0} & =\alpha_{6}(g-r)_{0}+\beta_{6}(r-i)_{0}+\gamma_{6}, \\
(J-W 1)_{0} & =\alpha_{7}(J-H)_{0}+\beta_{7}\left(H-K_{\mathrm{s}}\right)_{0}+\gamma_{7}, \\
(J-W 2)_{0} & =\alpha_{8}(J-H)_{0}+\beta_{8}\left(H-K_{\mathrm{s}}\right)_{0}+\gamma_{8}, \\
(J-W 3)_{0} & =\alpha_{9}(J-H)_{0}+\beta_{9}\left(H-K_{\mathrm{s}}\right)_{0}+\gamma_{9} .
\end{align*}
$$

The numerical values of the coefficients in Equations 11 are given in Table 3. The comparison between the correlation coefficients $R$ and the standard deviations $s$ for
Table 3. Coefficients for the transformations

|  | $\begin{gathered} i=1 \\ (V-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=2 \\ (V-W 2)_{0} \end{gathered}$ | $\begin{gathered} i=3 \\ (V-W 3)_{0} \end{gathered}$ | $\begin{gathered} i=4 \\ (g-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=5 \\ (g-W 2)_{0} \end{gathered}$ | $\begin{gathered} i=6 \\ (g-W 3)_{0} \end{gathered}$ | $\begin{gathered} i=7 \\ (J-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=8 \\ (J-W 2)_{0} \end{gathered}$ | $\begin{gathered} i=9 \\ (J-W 3)_{0} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Metal-dependent transformations |  |  |  |  |  |  |  |  |  |
| $a_{i}$ | $0.231 \pm 0.056$ | $0.206 \pm 0.054$ | $0.228 \pm 0.056$ | $12.781 \pm 1.272$ | $11.618 \pm 0.973$ | $11.513 \pm 1.453$ | $1.081 \pm 0.056$ | $0.946 \pm 0.041$ | $1.004 \pm 0.066$ |
| $b_{i}$ | $0.859 \pm 0.049$ | $0.822 \pm 0.047$ | $0.859 \pm 0.049$ | $-23.488 \pm 3.074$ | $-20.908 \pm 2.351$ | $-20.510 \pm 3.510$ | $0.695 \pm 0.076$ | $0.618 \pm 0.056$ | $0.731 \pm 0.089$ |
| $c_{i}$ | $0.177 \pm 0.031$ | $0.161 \pm 0.030$ | $0.179 \pm 0.031$ | $-0.067 \pm 0.013$ | $-0.077 \pm 0.010$ | $-0.054 \pm 0.015$ | $0.016 \pm 0.010$ | $-0.003 \pm 0.007$ | $0.019 \pm 0.012$ |
| $d_{i}$ | $1.367 \pm 0.086$ | $1.330 \pm 0.083$ | $1.410 \pm 0.086$ | $-0.911 \pm 0.197$ | $-0.774 \pm 0.151$ | $-0.654 \pm 0.225$ | $0.057 \pm 0.034$ | $0.035 \pm 0.025$ | $0.133 \pm 0.039$ |
| $R$ | 0.758 | 0.754 | 0.759 | 0.974 | 0.983 | 0.965 | 0.749 | 0.795 | 0.678 |
| $s$ | 0.127 | 0.121 | 0.127 | 0.045 | 0.035 | 0.052 | 0.043 | 0.031 | 0.050 |
| $<\Delta_{\text {res }}>^{\text {a }}$ | -0.00052 | 0.00014 | -0.00017 | 0.00002 | 0.00062 | 0.00084 | -0.00009 | -0.00016 | -0.00022 |
| Metal-free transformations |  |  |  |  |  |  |  |  |  |
| $\alpha_{i}$ | $0.362 \pm 0.053$ | $0.326 \pm 0.051$ | $0.362 \pm 0.053$ | $9.301 \pm 1.126$ | $7.620 \pm 0.900$ | $8.685 \pm 1.263$ | $1.102 \pm 0.055$ | $0.942 \pm 0.040$ | $1.029 \pm 0.064$ |
| $\beta_{i}$ | $0.941 \pm 0.049$ | $0.897 \pm 0.047$ | $0.943 \pm 0.049$ | $-15.287 \pm 2.745$ | $-11.485 \pm 2.193$ | $-13.843 \pm 3.078$ | $0.737 \pm 0.072$ | $0.610 \pm 0.053$ | $0.780 \pm 0.084$ |
| $\gamma_{i}$ | $1.082 \pm 0.073$ | $1.071 \pm 0.070$ | $1.121 \pm 0.073$ | $-0.306 \pm 0.166$ | $-0.079 \pm 0.133$ | $-0.162 \pm 0.186$ | $0.035 \pm 0.031$ | $0.039 \pm 0.022$ | $0.106 \pm 0.036$ |
| $R$ | 0.731 | 0.729 | 0.731 | 0.972 | 0.980 | 0.963 | 0.746 | 0.794 | 0.675 |
| $s$ | 0.132 | 0.126 | 0.132 | 0.047 | 0.038 | 0.053 | 0.043 | 0.031 | 0.050 |
| $<\Delta_{\text {res }}>^{\text {a }}$ | 0.00128 | 0.00030 | -0.00011 | -0.00010 | 0.00024 | -0.00042 | -0.00031 | 0.00010 | 0.00048 |

[^2]Table 4. Coefficients for the inverse transformations

|  | $\begin{gathered} i=1 \\ (B-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=2 \\ (V-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=3 \\ (I-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=4 \\ (g-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=5 \\ (r-W 1)_{0} \end{gathered}$ | $\begin{gathered} i=6 \\ (i-W 1)_{0} \end{gathered}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Metal-dependent inverse transformations |  |  |  |  |  |  |
| $a_{i}$ | $2.980 \pm 0.212$ | $2.380 \pm 0.167$ | $1.480 \pm 1.140$ | $3.352 \pm 0.124$ | $2.350 \pm 0.080$ | $1.945 \pm 0.063$ |
| $b_{i}$ | $-0.911 \pm 0.194$ | $-0.845 \pm 0.153$ | $-0.633 \pm 0.128$ | $-0.804 \pm 0.113$ | $-0.764 \pm 0.073$ | $-0.747 \pm 0.058$ |
| $c_{i}$ | $0.452 \pm 0.037$ | $0.264 \pm 0.029$ | $0.146 \pm 0.024$ | $0.223 \pm 0.022$ | $0.136 \pm 0.014$ | $0.102 \pm 0.011$ |
| $d_{i}$ | $1.940 \pm 0.111$ | $1.220 \pm 0.088$ | $0.620 \pm 0.073$ | $1.224 \pm 0.065$ | $0.941 \pm 0.042$ | $0.882 \pm 0.033$ |
| $R$ | 0.758 | 0.732 | 0.624 | 0.870 | 0.888 | 0.899 |
| $s$ | 0.168 | 0.132 | 0.110 | 0.098 | 0.064 | 0.050 |
| $<\Delta_{\text {res }}>^{\text {a }}$ | -0.00070 | -0.00109 | -0.00121 | 0.00057 | -0.00012 | -0.00019 |
| Metal-free inverse transformations |  |  |  |  |  |  |
| $\alpha_{i}$ | $3.353 \pm 0.251$ | $2.597 \pm 0.184$ | $1.598 \pm 0.145$ | $3.536 \pm 0.140$ | $2.462 \pm 0.090$ | $2.129 \pm 0.070$ |
| $\beta_{i}$ | $-1.199 \pm 0.230$ | $-1.014 \pm 0.168$ | $-0.726 \pm 0.133$ | $-0.947 \pm 0.128$ | $-0.851 \pm 0.082$ | $-0.812 \pm 0.064$ |
| $\gamma_{i}$ | $1.534 \pm 0.127$ | $0.983 \pm 0.093$ | $0.489 \pm 0.073$ | $1.025 \pm 0.071$ | $0.819 \pm 0.045$ | $0.791 \pm 0.035$ |
| $R$ | 0.627 | 0.653 | 0.570 | 0.825 | 0.854 | 0.872 |
| $s$ | 0.200 | 0.147 | 0.116 | 0.112 | 0.072 | 0.056 |
| $<\Delta_{\text {res }}>^{\text {a }}$ | 0.00000 | -0.00048 | 0.00016 | -0.00046 | 0.00005 | -0.00055 |

[^3]Equations 10 and 11 show that the metallicity dependent transformations are the preferred ones.

### 3.3 Metallicity-Dependent Inverse Transformations

As was explained before, $W 4$ magnitudes cannot be used for the RC star sample. Hence, we adapted the procedure used for dwarfs (Bilir et al. 2011a) to get the metallicity dependent inverse transformations with two colours: By combining linearly the near and mid-infrared colours, $(J-H)_{0}$ and $(W 1-W 2)_{0}$, we transformed them to the optical colours: $(B-V)_{0}$ and $(V-I)_{0},(g-r)_{0}$ and $(r-i)_{0}$. The general equations are as follows:

$$
\begin{align*}
(B-W 1)_{0}= & a_{1}(J-H)_{0}+b_{1}(W 1-W 2)_{0} \\
& +c_{1}[M / H]+d_{1} \\
(V-W 1)_{0}= & a_{2}(J-H)_{0}+b_{2}(W 1-W 2)_{0} \\
& +c_{2}[M / H]+d_{2} \\
(I-W 1)_{0}= & a_{3}(J-H)_{0}+b_{3}(W 1-W 2)_{0} \\
& +c_{3}[M / H]+d_{3}, \\
(g-W 1)_{0}= & a_{4}(J-H)_{0}+b_{4}(W 1-W 2)_{0}  \tag{12}\\
& +c_{4}[M / H]+d_{4} \\
(r-W 1)_{0}= & a_{5}(J-H)_{0}+b_{5}(W 1-W 2)_{0} \\
& +c_{5}[M / H]+d_{5} \\
(i-W 1)_{0}= & a_{6}(J-H)_{0}+b_{6}(W 1-W 2)_{0} \\
& +c_{6}[M / H]+d_{6} .
\end{align*}
$$

The numerical values of the coefficients in Equations 12 are given in Table 4.

### 3.4 Metal-Free Inverse Transformations

We adapted the procedure explained in Section 3.3 and derived metal-free inverse transformations between WISE and Johnson-Cousins, SDSS photometries. The general equations are as follows, and the numerical values of the coefficients in these equations are given in Table 4:

$$
\begin{align*}
(B-W 1)_{0} & =\alpha_{1}(J-H)_{0}+\beta_{1}(W 1-W 2)_{0}+\gamma_{1}, \\
(V-W 1)_{0} & =\alpha_{2}(J-H)_{0}+\beta_{2}(W 1-W 2)_{0}+\gamma_{2}, \\
(I-W 1)_{0} & =\alpha_{3}(J-H)_{0}+\beta_{3}(W 1-W 2)_{0}+\gamma_{3},  \tag{13}\\
(g-W 1)_{0} & =\alpha_{4}(J-H)_{0}+\beta_{4}(W 1-W 2)_{0}+\gamma_{4}, \\
(r-W 1)_{0} & =\alpha_{5}(J-H)_{0}+\beta_{5}(W 1-W 2)_{0}+\gamma_{5}, \\
(i-W 1)_{0} & =\alpha_{6}(J-H)_{0}+\beta_{6}(W 1-W 2)_{0}+\gamma_{6} .
\end{align*}
$$

Comparison of the correlation coefficients and the standard deviations for Equations 12 and 13 show that the inverse transformations are recommended especially when they are used with a metallicity term as in the direct transformations.

### 3.5 Residuals

We compared the observed colours with those evaluated by means of the transformations. The residuals corresponding to the Equations 10 and 11 are plotted versus observed colours $(B-V)_{0},(g-r)_{0}$, and $(J-H)_{0}$ in the
same figure (Figure 8) with different symbols. For the observed optical colours, the residuals corresponding to the equations just cited are different and they favour the ones with the metallicity term. Whereas for the observed near-infrared colour, i.e. $(J-H)_{0}$, the two sets of residuals overlap, diminishing the effect of the metallicity term. The same result can be deduced from comparison of the metallicity term $c_{i}(i=1, \ldots, 9)$ in Table 3 , where $c_{i}$ decreases from $0.179 \pm 0.031$ for $(V-W 3)_{0}$ to $-0.003 \pm 0.007$ for $(J-W 2)_{0}$. The residuals corresponding to the Equations 12 and 13 are plotted versus observed $(W 1-W 2)_{0}$ in the same figure with different symbols (Figure 9). The difference between the residuals of two sets are much larger for the ones corresponding to the BVI magnitudes relative to the residuals for gri. The numerical values of the metallicity term, $c_{i}(i=1, \ldots, 6)$, in Table 4 confirm this suggestion. Actually, $c_{1}=0.452$ $\pm 0.037$ for $(B-W 1)_{0}$ whereas it is only $c_{6}=0.102$ $\pm 0.011$ for $(i-W 1)_{0}$. Hence, we conclude that the metallicity term provides more accurate inverse transformations for BVI magnitudes, but that its contribution to gri is rather limited.

## 4 Summary and Conclusions

We have obtained colour transformations for the conversion of WISE ( $W 1 W 2 W 3$ ) magnitudes to the Johnson-Cousins (BVI), SDSS (gri), and 2MASS ( $J H K_{\mathrm{s}}$ ) photometric systems, for RC stars. The sample was selected by applying two constraints to the RAVE DR3 data (resulting a sample of 8003 giants): 1) $2<\log g<3 \mathrm{~cm} \mathrm{~s}^{-2}$ and 2) $J-H>0.4$. Matching the coordinates of this sample with the Tycho-2, DENIS, 2MASS, and WISE catalogues we produced a reduced sample with available magnitudes that is the one used in the transformations. In order to obtain the most accurate transformations, we included four additional constraints: 3) the data were de-reddened, 4) only the stars with high quality were selected, 5) a metallicity term was added to the transformation equations and 6) transformation equations are two-colour dependent; that reduced the total sample to 355 stars.

The transformation equations, and the inverse ones, were designed in two sheets: one with a metallicity term and the other metallicity-free. Comparison between the correlation coefficients and the standard deviations for the two sets promotes the use of the metallicity dependent transformation equations. It is noticeable that even when the procedure used for the transformations for dwarfs was different in Bilir et al. (2008, 2011a), we separated the dwarf sample into different metallicity sub-samples instead of adding a metallicity term to the transformation equations, we obtained here the same result, that is they were metallicity dependent. This dependence of the transformations on metallicity had been also confirmed in Yaz et al. (2010).

As in the case of dwarfs, WISE has an advantage relative to the 2MASS photometric system due to its deeper magnitudes. Actually, $W 1$ is a magnitude deeper than $K_{\mathrm{s}}$ for sources with spectra close to an A0 star and

Figure 8 Colour residuals for the metallicity-dependent (O) and metal free ( $\mathbf{\Delta}$ ) transformations. The notation used is $\triangle$ (colour) $=$ (evaluated colour) - (observed colour). The horizontal dashed lines correspond to $1 \sigma$ residuals. (a) - (c) for $B V I_{\mathrm{c}}$, (d) - (f) for SDSS, and (g) - (i) for 2MASS photometric system.


Figure 9 Colour residuals for the metallicity dependent $(O)$ and metal free $(\mathbf{\Delta})$ inverse transformations. The notation used is $\triangle$ (colour) $=$ (evaluated colour) - (observed colour). The horizontal dashed lines correspond to $1 \sigma$ residuals. (a) - (c) for $B V I_{\mathrm{c}}$, and (d) - (f) for SDSS photometric system.
even deeper for $K$ and $M$ spectral stars. The present transformations can be applied to stars with known absolute $V$, $g$, or $J$ magnitudes, when absolute magnitudes for $W 1$ can be also provided. These two advantages can be used to investigate the RC stars in the thin and thick discs more accurately, and combining this study with the one carried out for dwarfs would be even more fruitful. A possible interesting application of the transformations presented here would be the comparison of the (new) Galactic model parameters and the ones estimated in situ, but the transformations also can be used in a wide variety of research fields.

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[^0]:    ${ }^{1}$ http://irsa.ipac.caltech.edu/cgi-bin/Gator/nphscan?mission=irsa\&submit=Select\&projshort=WISE_ PRELIM

[^1]:    ${ }^{\text {a }}$ The complete table is available in electronic format (see Supporting Information).

[^2]:    ${ }^{\mathrm{a}}$ Mean of residual

[^3]:    ${ }^{\mathrm{a}}$ Mean of residual.

[^4]:    Aihara, H., et al., 2011, ApJS, 193, 29
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