

A PROCEDURE FOR TWO-DIMENSIONAL ASYMPTOTIC ROTATIONAL-SPLITTING INVERSION

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ABSTRACT Rotational splitting $\Delta\omega(n, l, m)$ of the eigenfrequencies of a star rotating with angular velocity $\Omega(r, \theta)$ about a unique axis can be represented as a weighted integral of Ω over r and θ , (r, θ, ϕ) being spherical polar coordinates about the axis of rotation. For high-frequency acoustic modes, $\Delta\omega/m$ collapses essentially to a function of $w = \omega/(l+1/2)$ and $M = m/(l+1/2)$ alone, and the weighting kernel $K(r, \theta)$ becomes asymptotically degenerate, each factor being of essentially Abel type. Therefore, formally, the splitting integral can be inverted, once a procedure has been found for extending $\Delta\omega$ over the domain of (w, M) such that the turning points (r_t, θ_t) , given by $(c(r_t)/r_t, \sin \theta_t) = (w, M)$ where c is sound speed, span the star. Obtaining that representation is the most difficult stage of the inversion. We report on a procedure that treats the inverted two-dimensional Abel integral as a repeated double integral, representing the data successively along a set of parallel lines $M = \text{constant}$. The method is illustrated by an inversion of artificial data which is compared with the angular velocity from which those data were computed.

ASYMPTOTIC FORMULA FOR INVERSION

We present the results of an initial attempt to measure the angular velocity $\Omega(r, \theta)$, assumed to be about the unique axis of spherical polar coordinates (r, θ, ϕ) , of a solar model from rotational splitting frequencies $f = (\omega_{nlm} - \omega_{n10})/m$, using the asymptotic formula for high-frequency p modes (Kosovichev and Parchevskii, 1988; Gough, 1992):

$$\Omega(\tau(x), \theta(y)) = \frac{1}{2\pi W(x)} \frac{\partial}{\partial x} \int_x^x \frac{d\xi}{\sqrt{x-\xi}} \int_x^\xi \frac{W(\xi') d\xi'}{\sqrt{\xi-\xi'}} \frac{\partial}{\partial y^{1/2}} \int_0^y \frac{f(\xi, \eta) d\eta}{\sqrt{y-\eta}},$$

where

$$x = a^2, \quad y = \cos^2 \theta, \quad \xi = w^2, \quad \eta = M^2 \equiv 1 - m^2/L^2, \quad L = l + 1/2;$$

$$W = -a^{-3} d \ln r / d \ln a, \quad a = c/r \quad \text{and} \quad w = \omega/L,$$

where c is sound speed and ω is the frequency of oscillation of the mode of degree l and azimuthal order m , and the subscript s denotes surface value.

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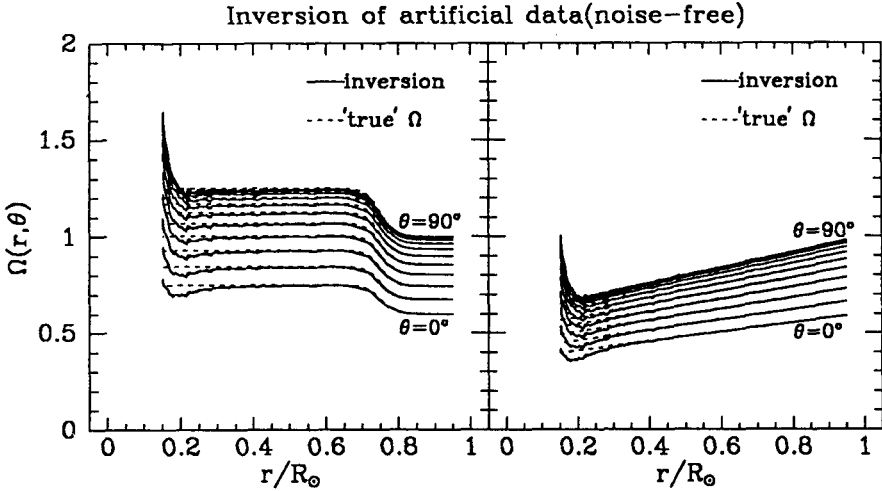


FIGURE 1. Inversions of $\Omega(r, \theta)$, plotted as continuous lines against r/R_{\odot} , where R_{\odot} is the solar radius, at different values of θ distributed uniformly in $\cos \theta$. The actual values of Ω from which the data were computed are represented by the dashed lines.

NUMERICAL METHOD

Artificial data have been computed using the exact linearized expression for the rotational splitting of p modes of a solar model with cyclic frequencies $\omega_i/2\pi$ between 3.0 and 5.1 mHz. To carry out the inversions, first $f(\xi, \eta)$ was smoothed and extrapolated with respect to η in the following fashion. The smoothing was accomplished by representing discrete values f_i by the continuous function \bar{f} obtained by minimizing

$$\Sigma \left(\frac{f_i - \bar{f}_i}{\delta_i} \right)^2 + \alpha \int \left(\frac{d^2 \bar{f}}{d\eta^2} \right)^2 d\eta$$

for a tradeoff parameter α , where f_i is the measured frequency of the i -th mode under consideration and δ_i is the assumed error in f_i . The limits of integration are the extremes of the interval within which the data lie. For modes with $l \geq 5$, extrapolation was accomplished with fourth-degree polynomials that pass through the extreme five mesh points near the ends of the interval. (Polynomials of degree $l - 1$ were used for modes with $l < 5$.) In all, 201 mesh points were used to span the entire interval $[0, 1]$ of η . After differentiation with respect to $y^{1/2}$, smoothing with respect to ξ was carried out in a similar way. Linear extrapolation in ξ to the solar surface was carried out (by regression against the outermost five mesh points) before integration and differentiation with respect to radial variables. Radial mesh points were located at the 1380 different turning points associated with the data.

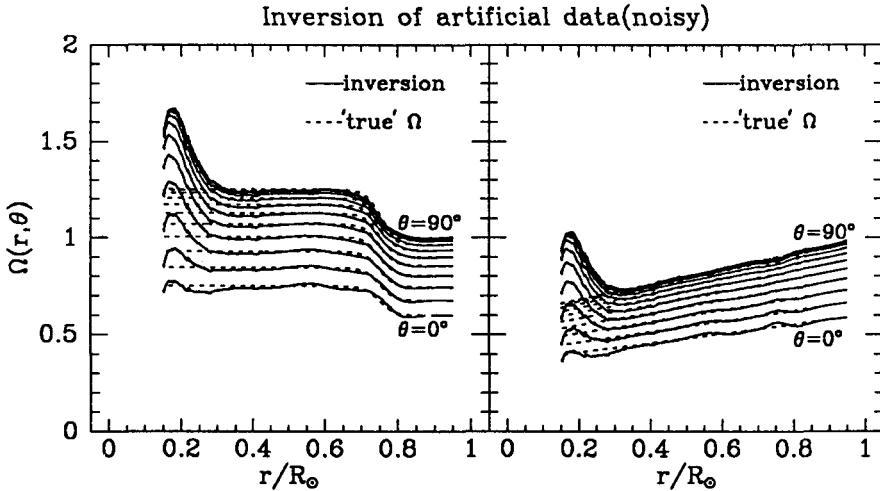


FIGURE II Inversions of the data used for figure I to which were added random errors, normally distributed with zero mean and standard deviation 2 per cent.

RESULTS

Results of inverting precise data from two simple models of Ω are illustrated in Figure I. The continuous curves are $\Omega(r, \theta)$ at eleven different values of colatitude θ from pole (lowest curve) to equator (uppermost curve), uniformly spaced in $\cos \theta$. In the smoothing, δ_i was arbitrarily set to unity for all i . The dashed curves illustrate the actual function $\Omega(r, \theta)$, measured in units of the surface equatorial value, from which the data were computed. The error near the centre arises from a breakdown in the asymptotic representation. The rotation rates obtained at the pole are the result of extrapolation.

In Figure II are shown inversions of data to which two-per-cent errors have been added. As anticipated, they are less accurate, particularly just above the depth at which the previous inversions started to deviate from the true Ω . We plan next to try improving the accuracy of the asymptotic expression at small r/R_\odot .

REFERENCES

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