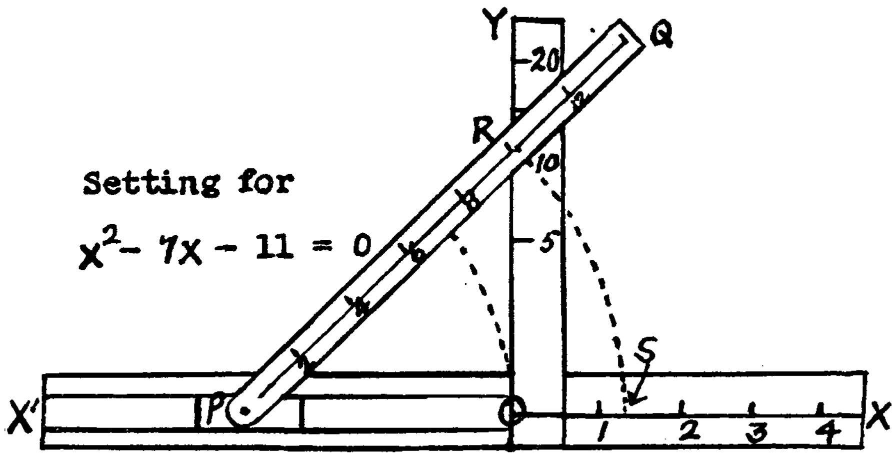
Mechanical devices for solving quadratic and cubic equations

By G. D. C. Stokes.

Quadratic Equations. The device is designed to give the numerical value of the numerically smaller root of the equation $x^2 - px + q = 0$, the other root and the signs of the roots α, β being found thereafter from the relations $\alpha + \beta = p$, $\alpha\,\beta = q$.



The apparatus consists of a base piece with central line $X'OX$ carrying on OX a uniform scale suitably graduated. An upright piece OY is fixed rigidly at right angles to the base piece and carries a scale marked "y" at R, where $OR = \sqrt{y}$. An arm PQ, made of transparent material, can rotate about the pivot P, which is attached to a slide moveable along $X'O$, and carries a uniform scale whose unit is half that of the OX scale.

The instrument is used in different ways according as q is positive or negative. Consider as examples the equations $x^2 - 7x + 11 = 0$ (1) and $x^2 - 7x - 11 = 0$ (2). The numerical values of the smaller roots of these equations are $\frac{7}{2} - \sqrt{\{(\frac{7}{2})^2 - 11\}}$ and $\sqrt{\{(\frac{7}{2})^2 + 11\}} - \frac{7}{2}$ respectively. In the case of (1), by moving the slide the mark "7" on PQ is made to coincide with the reading "11" on OY at the point R. In (2), the reading "7" on PQ is made to coincide with O, and PQ is then rotated to pass through the point R reading "11" on OY. In both cases, PR is then rotated into the position PS, and the numerical

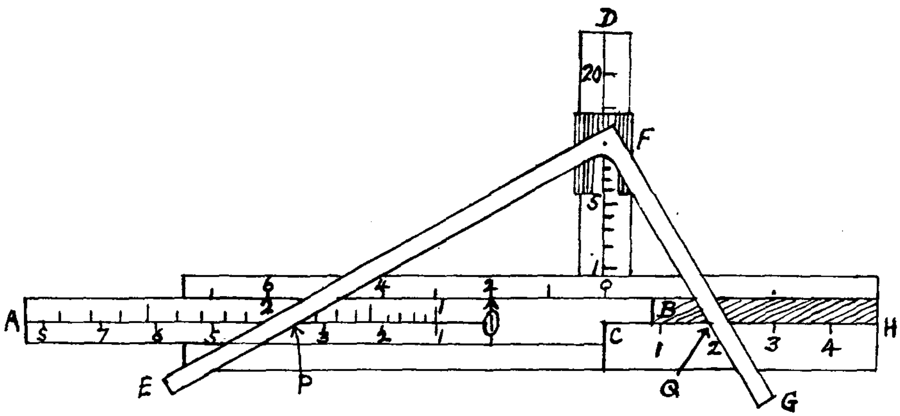
https://doi.org/10.1017/S0950184300003098 Published online by Cambridge University Press

value of the smaller root of the equation is given by OS. The sign of this root and the remaining root of the equation can be readily determined from the coefficients in the equation.

If p is negative, the equation can be transformed to the type considered by putting $x = -y$.

Also, if any reading should fall outside the limited scale of the model in use, the equation should first of all be transformed by putting $x = ky$ with a suitable value of k.

Cubic Equations. The device is in the form of a slide rule AB with an arm CD fixed at right angles to the stock in which the slide moves; a right-angled elbow piece EFG is free to rotate about a pivot fixed at F to a sliding head moveable along CD.



The equation to be solved must be in the reduced form $x^3+px = q$.

CD is graduated to a square root scale, so that when F is set to the reading q, $CF = \sqrt{q}$ ($q = 10$ in the diagram). The slide AB is set to the reading p ($= OC = 2$ in the diagram). OA is graduated to a scale of squares, so that $OP = x^2$, when x is the reading at P. A third scale CH, uniformly graduated, is fixed rigidly to the stock.

The device employs the theorem that, in the right angled triangle PFQ, $PC.CQ = CF^2$. Expressed algebraically, this gives $(x_1{}^2+p)\,x_2=q$, where x_1 is the reading on the x^2 scale at P and x_2 is the reading at Q. When $x_1 = x_2$ a root of the equation is found; this can be effected by rotating the elbow piece EFG about F until the readings agree at P and Q (1·85 in the diagram). The root thus found is necessarily positive.

In the above it has been assumed that q is positive. The various possibilities which may arise are as follows.

https://doi.org/10.1017/S0950184300003098 Published online by Cambridge University Press

(1) $p > 0, q > 0$. The equation has only one real root, which is positive and may be found as above.

(2) $p > 0, q < 0$. The equation has only one real root, which is negative. The root is found by transforming the equation by the substitution $x = -y$ and proceeding as above.

(3) $p < 0, q > 0$. The equation has one positive root which can be found as above and may have two negative roots. The number of real roots can be readily determined from a graph of the curve $(p/3)^3 + (q/2)^2 = 0$, glued on to the back of the stock. As an example, consider the equation $x^3 - 6x = 3$. The positive root $x = 2{\cdot}67$ is found by the above procedure. Then, putting $x = -y$, the equation becomes $y(6 - y^2) = 3$, of which a positive root can be found as follows. Make EF pass through the point K reading $\sqrt{6}$ on OA (so that $OK = 6$) and adjust the slide until the readings at C and Q (where FG crosses CH) are equal. This common reading gives the required root $y = 2{\cdot}12$. Hence $x = 2{\cdot}67$ and $x = -2{\cdot}12$ are roots of the given equation. Since the sum of the roots is zero, the remaining root is $x = -0{\cdot}55$.

(4) $p < 0, q < 0$. This case can be reduced to case (3) by putting $x = -y$.

If q/p is very small, the accuracy of the instrument is low. However, since $x = q/p - x^3/p$, an approximation can be obtained by substituting q/p for x in the right hand side.

Finally, a uniform scale on OA below the central line enables the instrument to be applied to the solution of the quadratic equation $x^2 + px = q$ in the same way.

GREEN GATES,
82 GREENOCK ROAD,
LARGS.

https://doi.org/10.1017/S0950184300003098 Published online by Cambridge University Press