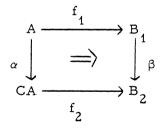
## ON THE EXACTNESS OF THE ECKMANN-HILTON HOMOTOPY SEQUENCE

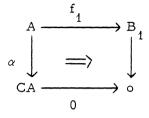
## A.R. Pears

The theorem that the homotopy sequence is exact splits into six statements. Scherk ([4]) obviates the use of homotopy extension in the proof of one of these statements. The purpose of this note is to show that the method can be adapted to give a direct proof of the corresponding statement in the theorem that the Eckmann-Hilton homotopy sequence ([1]) is exact. The note is based on Eckmann's exposition ([2]). We are concerned with the proof of <u>b2</u>, pp. 34-35. Eckmann's notation is used; in particular all base-points are denoted by the symbol o, all constant maps by the symbol 0

Suppose we have a mapping of pairs



where  $\alpha: A \rightarrow CA$  is the natural injection of A into CA. And suppose that the mapping of pairs



is homotopic to 0.

Let 
$$F: A \times I \rightarrow B_1$$
 be a homotopy between  $f_1$  and 0, and 1

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let  $\gamma : A \times I \rightarrow CA$  be the identification mapping. We consider  $G : A \times I \times I \rightarrow B_2$  defined by

$$G(a,s,t) = \begin{cases} f_2 \gamma(a,-\phi) & \text{if } \phi \leq 0\\\\ \beta F(a,\phi) & \text{if } \phi \geq 0 \end{cases}$$

where  $\phi = t - s - st$ .

G is well defined and continuous for

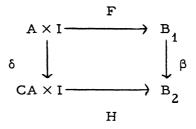
$$f_{2Y}(a,0) = f_{2}\alpha(a) = \beta f_{1}(a) = \beta F(a,0)$$
.

And

$$G(o, s, t) = o$$
,  $G(a, 1, t) = f_2 \gamma(a, 1) = f_2(o) = o$ .

Hence ([3], Lemma 3.4, p. 109) there is a continuous function  $H: CA \times I \rightarrow B_2$  such that  $G(a, s, t) = H(\gamma(a, s), t)$ . Let  $g: CA \rightarrow B_2$  be given by g(c) = H(c, 1). (F, H) is a homotopy between  $(f_1, f_2)$  and (0, g).

For consider the diagram



where  $\delta(a,t) = (\alpha(a),t)$ .

If  $(a,t) \in A \times I$ ,  $H\delta(a,t) = G(a,0,t) = \beta F(a,t)$ 

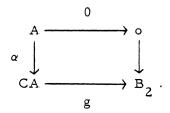
and so the diagram is commutative.

F is a homotopy between  $f_{_{\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!\!}}$  and 0 ; and if  $c\in CA$  ,

$$H(c, 0) = G(a, s, 0) = f_2\gamma(a, s) = f_2(c)$$
,  $H(c, 1) = g(c)$ 

so that H is a homotopy between  $f_2$  and g.

Finally consider



$$g\alpha(a) = G(a, 0, 1) = \beta F(a, 1) = \beta(o) = o$$

Thus the homotopy class of (0, g) is an element of  $\Pi_1(A, B_2)$ . Hence the class of  $(f_1, f_2)$  belongs to the image in  $\Pi_1(A, \beta)$  of  $\Pi_1(A, B_2)$  by J.

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