
The first test for divisibility by 7 given by Mr. Clarke (Note 1919) may be generalised and then adapted for numbers greater than 10. Here is the test for divisibility by 19.

Multiply the leading digit by 9 and subtract the next digit; multiply the result by 9 and add the next digit, and so on, subtracting or adding at any stage any multiple of 19. If the final result is a multiple of 19, so is the original number.

Example. 3086322 is a multiple of 19, for:

\[
\begin{align*}
3 \times 9 &= 27 \\
8 \times 9 &= 72 \\
4 \times 9 &= 36 \\
11 \times 9 &= 99 \\
7 \times 9 &= 63 \\
4 \times 9 &= 36
\end{align*}
\]

\[
\begin{align*}
27 - 0 &= 27 \\
72 + 8 &= 80 \\
36 - 6 &= 30 \\
99 + 3 &= 102 \\
63 - 2 &= 61 \\
36 + 2 &= 38
\end{align*}
\]

\[
\begin{align*}
27 - 19 &= 8 \\
80 - 4(19) &= 4 \\
30 - 19 &= 11 \\
102 - 5(19) &= 7 \\
61 - 3(19) &= 4 \\
38 &= 2 \times 19
\end{align*}
\]

The rules for 17 and 13 are similar, writing 7 or 3 in place of 9 and 17 or 13 in place of 19.

The proof is very similar to that given by Mr. Clarke; it depends on \(x + y\) being a factor of \(x^n + y^n\) if \(n\) is odd and of \(x^n - y^n\) if \(n\) is even.

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As I am in the habit of factorising numbers when I cannot get to sleep, I can add to Mr. Parameswaran’s list of tests of divisibility.

Since \(100a + b \equiv 0 \pmod{399}\) implies that \(a + 4b \equiv 0 \pmod{399}\), we can test for divisibility by 3, 7 and 19 simultaneously by multiplying the last two digits of a number by 4, adding them to the next two, and repeating the process till we are left with only two digits. Thus to test 178,431 we proceed:

\[
\begin{align*}
1784 &
\end{align*}
\]

\[
\begin{align*}
124 &
\end{align*}
\]

\[
\begin{align*}
1908 &
\end{align*}
\]

\[
\begin{align*}
19 &
\end{align*}
\]

\[
\begin{align*}
32 &
\end{align*}
\]

51 is divisible by 3, but not by 7 nor by 19. The same is true of 178,431. Again since \(100a + b \equiv 0 \pmod{2001}\) implies that \(2b - a \equiv a - 2b \equiv 0 \pmod{2001}\), and since \(2001 = 3 \times 23 \times 29\), we test for divisibility by 23 and 29 by finding the difference between twice the last three digits and the number of thousands, repeating the process if necessary. Thus 178,431 is divisible by 23 or 29 if and only if \(431 - 2 \times 178 = 75\) is so divisible, which is clearly not the case. Other similar methods can be derived from the facts that \(102 = 2 \times 3 \times 17\), \(201 = 3 \times 67\), \(301 = 7 \times 43\), \(1003 = 17 \times 59\), \(3999 = 3 \times 31 \times 43\), and so on.

Perhaps tests of divisibility in quadratic fields are even more hypnagogic. Thus if \(a + b\) is divisible by 2, \(a + ib\) is divisible by \(1 + i\); if \(2a + b\) is divisible by 5, \(a + ib\) is divisible by \(2 + i\); if \(3a + 2b\) is divisible by 13, \(a + ib\) is divisible by \(3 + 2i\), and so on. In fact, if you can’t achieve sleep by factorising in \(K(1)\), you may hope to do so in \(K(i)\) or \(K(\omega)\).

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The following result of contour integration is to be found in most books on this topic (e.g. Goursat, Cours d’Analyse, II, p. 109):

\[
\int_0^\infty \cos (x^2) \, dx = \int_0^\infty \sin (x^2) \, dx = \frac{1}{\sqrt{2}} \int_0^\infty e^{-x^2} \, dx. \quad \text{..........(I)}
\]

It is established by integrating \(\exp(-x^2)\) round the contour of Fig. 1. The value of the third integral is then assumed; some books (though not Goursat) state that it can not be found by contour methods.

The following integration leads directly to the values of the first two integrals in (I) and hence enables all three to be established by contour integration.*

Consider
\[
\int \frac{e^{iz^2}}{\sin z \sqrt{\pi}} \, dz
\]
taken round the contour of Fig. 2. The sum of the contributions of PQ and HK is
\[
2 \int_{-R}^{R} e^{i(z^2 - y^2)} \cdot i \, dy.
\]
On QH, \( |\sin z \sqrt{\pi}| > \sinh R \sqrt{\pi} \) and \( |e^{iz^2}| = e^{-2Rx} \); thus the contribution of QH is in modulus less than
\[
\left[ \int_{-\sqrt{\pi}}^{\sqrt{\pi}} e^{-2Rx} \, dx \right]/R = \frac{1}{R}.
\]
There is a simple pole of residue \( 1/\sqrt{\pi} \) at the origin and hence
\[
\int_{0}^{\infty} e^{i(z^2 - y^2)} \, dy = \frac{1}{\sqrt{\pi}} ;
\]
thus,
\[
\int_{0}^{\infty} \sin (\frac{1}{2} \pi - y^2) \, dy = 0, \quad \int_{0}^{\infty} \cos (\frac{1}{2} \pi - y^2) \, dy = \frac{1}{\sqrt{\pi}}.
\]
Hence expanding and solving for the first two integrals in (I) their common value is found to be \( 1/2\sqrt{(1/\pi)} \), and that of the third is then \( 1/4 \sqrt{\pi} \).

J. H. Cadwell.


The obvious generalisation of the equilateral triangle for \( n \)-dimensional space is the set of \( n + 1 \) distinct points,
\[
P_j \equiv (x_{j,1}, x_{j,2}, \ldots, x_{j,n}) \quad (j = 1, 2, \ldots, n + 1)
\]
such that the distance of any two is the same. This polygon is represented by the matrix of its coordinates \( X_n \equiv (x_{n,k}) \) of \( n + 1 \) rows and \( n \) columns.

* Since writing this note I am indebted to the Editor for drawing my attention to Mordell’s evaluation of
\[
\int_{-\infty}^{\infty} \frac{e^{at^2 + bt}}{e^{at} + d} \, dt
\]
by contour methods, in Quarterly Journal, 48 (1920), pp. 329–342. It is effected for real and complex values of the coefficients.