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It has been known for a long time that white dwarfs are pulsationally unstable if nuclear burning takes place in their envelopes. Perturbation of energy generation rate promotes pulsational instability and this effect is frequently referred to as ε -mechanism. In recent years, with the advent of high-speed photometry, many rapidly varying white dwarfs have been discovered. However, periods of variability were found to be significantly longer than the periods of radial pulsations which were the only type of oscillations considered before the discovery. Furthermore, the case of ε -mechanism as being responsible for the observed variability has never been made strong for any of the observed objects.

Variable white dwarfs are found among: 1° single DA-type objects in the effective temperature range 10000-15000K; 2° members of close, usually but not always, cataclysmic binary systems. Although, following an early suggestion by Warner and Robinson (1972), the excitation of nonradial oscillation is postulated in both cases, the two types represent very different physical situations and they will be discussed here separately.

There are 12 single variable white dwarfs known to date, and they form a well defined type of variable stars called ZZ Ceti. Comprehensive surveys of the properties of these stars are available in review articles by Nather (1979) and Robinson (1979). Incidence of ZZ Ceti among white dwarfs is similar to incidence of pulsating variables among early post-main sequence or horizontal branch stars. Namely, it is confined to a narrow range of effective temperatures and it is not related to any peculiarity of the objects. ZZ Ceti stars with their low amplitudes and multiperiodicity bear most resemblance to δ Scuti-type variables. Amplitude changes occurring on the time scales of hours or days are frequently observed among ZZ Ceti stars. Also in the case of δ Scuti stars relatively rapid amplitude variations have been reported, but their reality was questioned by Fitch (1976). The variable white dwarfs, however, differ sharply from other pulsating stars in the modes they choose to pulsate. Periods of variability

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W. DZIEMBOWSKI

found in ZZ Ceti stars, ranging from about 100 to 1000s, are by at least two orders of magnitude longer than the periods of radial pulsations and correspond to lower-order gravity modes.

The location of ZZ Ceti stars on the H-R diagram led McGraw and Robinson (1976) to suggest that these stars owe their variability to the same opacity-mechanism that causes Cepheid pulsations. assessment, with some modification, has been supported by recent stability analyses made for relevant stellar models. It has been found by Dziembowski (1977) that opacity mechanism acting on HeII ionization zone cannot provide sufficient driving for the slow modes that are observed because during such motion this zone hardly deviates from the thermal equilibrium and therefore has almost no effect on stability. However, more recent opacity data reveal that there is a driving zone below, at the temperature range $1 - 2x10^5 K$, which as found by Stellingwerf (1978, 1979) plays an important role in destabilizing δ Scuti and possibly & Cep-type stars. Then, it was promptly noted by Cox and Hansen (1979) that driving in this zone may cause instability of ZZ Ceti stars to gravity mode. Indeed, strong instability to modes in the observed period range was found by Dziembowski (1979) and Keeley (1979). This result is still somewhat uncertain because it depends on the existence of helium with sufficient abundance in this rather shallow layer (10-10 - 10-9 $\mathrm{M}_{\Theta})$ and on details of the opacity behavior.

In addition to gravity modes, the same mechanism should drive also acoustic modes with periods less than 1s as it was found by Cox et al. (1979a,b) for the case of radial pulsation and by Dziembowski (1977, 1979) for the case of nonradial p-modes. This instability exists in a very wide range of frequencies and horizontal wave-numbers. However, in spite of the efforts (McGraw and Starrfield, private communication) no significant power was found in periodograms for ZZ Ceti stars at these short periods. A possible explanation may be connected to the fact that nonlinear coupling between modes is in the case of these stars important already at quite low amplitudes, $\Delta M_{\rm bol}$ ~ 10⁻³ as found by Dziembowski (1979). As a result of consecutive coupling with higher and higher frequency modes shock waves may be formed in which energy gained from the radiative flux via opacity mechanism is effectively dissipated. On the other hand, coupling between the gravity modes may easily account for rapid amplitude changes that are observed.

The determination of rotational velocity for some ZZ Ceti stars (McGraw, 1977; Chlebowski, 1978) has been so far only an attempt to diagnose white dwarf properties with the use of pulsation data. Prospects exist for a much wider use of such data, particularly for diagnosing the chemical structure of white dwarf envelopes.

Turning to white dwarfs in close binary systems, let us first notice that they represent much less homogeneous type of rapidly varying objects and the interpretation of their variability in terms of nonradial oscillations is by no means generally accepted. There are about 20 variables of this type (for a complete survey of their properties see Nather, 1979; Warner, 1979). Variability periods in these stars range from about 10 to 70s; in most cases, however, the periods are between 20 and 30s. Also in most cases, oscillations are seen only occasionally and if the system is a cataclysmic one this

WHITE DWARF PULSATIONS 471

happens during the outburst. There are two continuous variables that belong to this type: DQ Her, in which case models involving rotating magnetized white dwarf are now favored (see Papaloizou and Pringle, 1978 and references therein); and WZ Sge, which sometimes exhibits two periodicities and therefore models involving oscillations are preferred in this case.

In systems which exhibit transient oscillations, rapid period variations are observed and this seemed to invalidate the original interpretation based on gravity-mode excitation. On the other hand, Bath's (1973) suggestion that variability is caused by luminous blobs of gas revolving on Keplerian orbits around white dwarf is difficult to accept in those cases in which oscillations are of larger amplitudes and are relatively stable.

Papaloizou and Pringle (1978) noted that original interpretation based on gravity-mode excitation was incorrect as the effects of white dwarf rotation were either ignored or treated as a small perturbation, while in reality rotation may play a dominant role in restoring force for these modes. The reason is that we are dealing with white dwarfs undergoing disc accretion and therefore rotation rate in their outer layers should be very high.

Moreover, Coriolis forces give rise to a new set of modes which are almost completely toroidal and are known in geophysics as Rossby-waves or r-modes. Both g-modes in rapidly rotating white dwarfs and r-modes can be effectively trapped in the region containing only a very small fraction of stellar mass (10^{-8} – 10^{-9} M_{$_{\odot}$}). Consequently, rapid period variations can easily be understood. Furthermore, Papaloizou and Pringle found that Kelvin-Helmholtz instability in the boundary layer between accreting disc and the star may cause excitation of r-modes on the white-dwarf surface. Their recent numerical calculation (Papaloizou and Pringle, 1979) seem to support this idea. The calculations which are quite difficult in this case involve some arbitrary assumptions and certainly much more work is needed to confirm the plausibility of this hypothesis.

Finally, let us address the question of under what circumstances and what kind of white dwarf oscillations may be driven by ε -mechanism. Independently, DeGregoria (1977) and Sienkiewicz and Dziembowski (1977) found that gravity modes can be excited by ε -mechanism with rates which are by at least three orders of magnitude higher than those found for radial pulsations. However, thermal stability analysis made for the same models (Sienkiewicz and Dziembowski 1977, Sienkiewicz, 1979) shows that in most cases these models are thermally unstable too. Thermal instability which usually is the most violent one is expected (Paczyński and Zytkow, 1978) to move the star on the H-R diagram far away from its original location to the place where pulsational instability does not occur.

Only hot white dwarfs having effective temperatures confined to a very narrow range may be simultaneously thermally stable and pulsationally unstable. This occurs at log T_e $^{\approx}$ 5.8 and log L/L $_{\Theta}$ $^{\approx}$ 4 for M = 1.39 M $_{\Theta}$ white dwarf model and at log T_e $^{\approx}$ 4.5 and log L/L $_{\Theta}$ $^{\approx}$ 2 for M = 0.4 M $_{\Theta}$ white dwarf model. The models show strongest instability to low-order gravity modes with periods being of the order of a few seconds for M = 1.39 M $_{\Theta}$ model and in the range 70-150s for M = 0.4 M $_{\Theta}$

W. DZIEMBOWSKI

model. It was concluded that one may expect to find such oscillations in luminous white dwarfs such as nuclei of planetary nebulae or hot components of symbiotic stars.

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