Kinetic closure conditions for quasi-stationary collisionless axisymmetric magnetoplasmas

Claudio Cremaschini\textsuperscript{1,2}, John C. Miller\textsuperscript{1,2,3} and Massimo Tessarotto\textsuperscript{4,5}

\textsuperscript{1}International School for Advanced Studies, SISSA, Trieste, Italy
\textsuperscript{2}INFN, Trieste Section, Trieste, Italy
\textsuperscript{3}Department of Physics (Astrophysics), University of Oxford, Oxford, U.K.
\textsuperscript{4}Department of Mathematics and Informatics, University of Trieste, Trieste, Italy
\textsuperscript{5}Consortium for Magnetofluid Dynamics, University of Trieste, Trieste, Italy

\textbf{Abstract.} A characteristic feature of fluid theories concerns the difficulty of uniquely defining consistent closure conditions for the fluid equations. In fact it is well known that fluid theories cannot generally provide a closed system of equations for the fluid fields. This feature is typical of collisionless plasmas where, in contrast to collisional plasmas, asymptotic closure conditions do not follow as a consequence of an H-theorem. This issue is of particular relevance in astrophysics where fluid approaches are usually adopted. On the other hand, it is well known that the determination of the closure conditions is in principle achievable in the context of kinetic theory. In the case of multi-species thermal magnetoplasmas this requires the determination of the species tensor pressure and of the corresponding heat fluxes. In this paper we investigate this problem in the framework of the Vlasov-Maxwell description for collisionless axisymmetric magnetoplasmas arising in astrophysics, with particular reference to accretion discs around compact objects (like black holes and neutron stars). The dynamics of collisionless plasmas in these environments is determined by the simultaneous presence of gravitational and magnetic fields, where the latter may be both externally produced and self-generated by the plasma currents.

Our starting point here is the construction of a solution for the stationary distribution function describing slowly-varying gyrokinetic equilibria. The treatment is applicable to non-relativistic axisymmetric systems characterized by temperature anisotropy and differential rotation flows. It is shown that the kinetic formalism allows one to solve the closure problem and to consistently compute the relevant fluid fields with the inclusion of finite Larmor-radius effects.

The main features of the theory and relevant applications are discussed.

\textbf{Keywords.} kinetic theory, accretion disks, plasmas, magnetic fields

\section{1. Introduction: fluid equations and kinetic closure conditions}

In this paper the issue concerned with the determination of kinetic closure conditions for collisionless magnetized axisymmetric accretion disc (AD) plasmas is discussed. The treatment of this problem is a prerequisite of primary importance for getting correct descriptions for the dynamics of collisionless plasmas in terms of suitable fluid equations and the corresponding fluid fields (Cremaschini \textit{et al.} 2008). The result presented here is obtained within the framework of the Vlasov-Maxwell description and concerns, in particular, collisionless magnetoplasmas in astrophysical accretion discs around compact objects (like black holes and neutron stars). In the present context we focus our attention on the specific case of asymptotic stationary configurations, in the sense defined in Cremaschini \textit{et al.}, (2010). We also assume that the magnetic field has both poloidal and toroidal components and we restrict our analysis to configurations in which the poloidal field admits locally a family of closed nested magnetic surfaces $\psi = \text{const.}$, $\psi$ denoting the poloidal magnetic flux function (Cremaschini \textit{et al.} 2010 and references cited...
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2. Tensor pressure

Concerning the notation adopted, the basic assumptions about the system and the magnetic field configuration, and the details of the construction of the kinetic theory for AD plasmas used here, we refer to the paper by Cremaschini et al. 2010. The species number densities and flow velocities have been computed as fluid moments in the same paper. For this reason, in the following we shall focus attention on the calculation of the tensor pressure, which is required as a closure condition in the Euler fluid equation. The total species pressure tensor is defined as \( \Pi^{tot}_s \equiv \int dv M_s (\mathbf{v} - \mathbf{V}^{tot}_s) \cdot (\mathbf{v} - \mathbf{V}^{tot}_s) \mathbf{f}_s \). Since the AD plasma is collisionless, the KDF is not Maxwellian and we expect to recover some sort of anisotropy in the final form of the pressure tensor. For example, this can be due to the temperature anisotropy, whose origin is related to the conservation of the magnetic moment as an adiabatic invariant. Parallel and perpendicular temperatures are defined with respect to the local direction of the magnetic field. For this reason, it is convenient to introduce the set of orthogonal unitary vectors given by \((\mathbf{b}, \mathbf{e}_1, \mathbf{e}_2)\), where \(\mathbf{b} \equiv \frac{\mathbf{B}}{B}\) is the tangent vector to the magnetic field while \(\mathbf{e}_1\) and \(\mathbf{e}_2\) are two orthogonal vectors in the plane perpendicular to the magnetic field line. Then, from this basis, we can construct the following unitary tensor: \(\mathbf{I} \equiv \mathbf{b} \mathbf{e}_1 + \mathbf{e}_1 \mathbf{e}_2\). The tensor pressure has its simplest representation when expressed in terms of the tensor \(\mathbf{I}\) and the vectors \((\mathbf{b}, \mathbf{e}_1, \mathbf{e}_2)\).

By Taylor expanding the equilibrium KDF (Cremaschini et al. 2010), the species pressure tensor \(\Pi^{tot}_s\) for magnetized plasmas can be written as follows: \(\Pi^{tot}_s \simeq \Pi_s^{(1)} + \Delta \Pi_s^{(3)}\), where \(\Pi_s^{(1)}\) is the leading-order term (with respect to all of the expansion parameters), while \(\Delta \Pi_s^{(3)}\) represents the first-order (i.e., \(O(\varepsilon)\)) correction term. In particular, \(\Pi_s^{(1)}\) is given by

\[
\Pi_s^{(1)} \equiv T_{\perp s} n_s \mathbf{I} + n_s \left[ T_{\parallel s} - T_{\perp s} \right] \mathbf{b} \mathbf{b}, \tag{2.1}
\]

which is expressed in terms of the parallel and perpendicular temperatures. On the other hand, \(\Delta \Pi_s^{(3)}\) can be written as

\[
\Delta \Pi_s^{(3)} \equiv \Delta \Pi_s^{(1)} \mathbf{I} + \Delta \Pi_s^{(3)} \mathbf{b} \mathbf{b} + \Delta \Pi_s^{(3)}, \tag{2.2}
\]

in which \(\Delta \Pi_s^{(1)}\) and \(\Delta \Pi_s^{(2)}\) are the diagonal first-order anisotropic corrections to the pressure tensor, while \(\Delta \Pi_s^{(3)}\) contains all of the non-diagonal contributions. More precisely, \(\Delta \Pi_s^{(1)}\) is given by

\[
\Delta \Pi_s^{(1)} \equiv \gamma_1 \Omega_s R T_{\perp s} n_s - \gamma_2 \frac{2 \Omega_s R T_{\perp s}^2 n_s}{B} + \gamma_3 \frac{\Omega_s R T_{\perp s}^2 n_s}{M_s} \left( 5 + \frac{3}{T_{\parallel s}} \right) + \gamma_3 (\Omega_s R)^3 T_{\perp s} n_s + \gamma_3 \frac{2 \Omega_s R T_{\perp s}^2 T_{\parallel s} n_s}{B R^2 M_s} + \gamma_3 \frac{8 \Omega_s R T_{\perp s}^2 n_s}{M_s} \left[ \frac{3}{4} - \frac{I^2}{R^2 B^2} \right], \tag{2.3}
\]
while \( \Delta \Pi^2 \) is defined as
\[
\Delta \Pi^2_s \equiv \gamma_1 \Omega_s R n_s \left[ T_{\parallel s} - T_{\perp s} \right] - \gamma_2 \frac{\Omega_s R T^2_{\parallel s} T_{\perp s} n_s}{B} \\
+ \gamma_3 \frac{\Omega_s R n_s T_{\parallel s} T_{\perp s}}{M_s} \left( 5 + 3 \frac{T_{\parallel s}}{\Delta T_s} \right) + \gamma_3 (\Omega_s R)^3 n_s \left[ T_{\parallel s} - T_{\perp s} \right] + \\
\gamma_3 \frac{2 \Omega_s R n_s T_{\parallel s} T_{\perp s}}{M_s} - \gamma_3 \frac{8 \Omega_s R T^2_{\parallel s} n_s}{M_s} \left[ \frac{3}{4} \frac{T^2}{R^2 B^2} + 1 \right] + \gamma_2 \frac{2 \Omega_s R T^2_{\perp s} n_s}{B} \\
- \gamma_3 \frac{\Omega_s R T^2_{\perp s} n_s}{M_s} \left( 5 + \frac{T_{\parallel s}}{\Delta T_s} \right) + \gamma_3 \frac{6 \Omega_s R T_{\parallel s} T^2_{\perp s}}{RB^2 M_s} - \gamma_3 \frac{4 \Omega_s R T^2_{\perp s} T_{\perp s} n_s}{RB^2 M_s}. 
\]

Finally, \( \Delta \Pi^3 \) is symmetric and is given by
\[
\Delta \Pi^3_s \equiv \gamma_3 \frac{16 \Omega_s R T^2_{\parallel s} n_s}{M_s} \left( e_1 \cdot e_2 : e_\gamma \cdot e_\gamma \right) \left[ e_1 e_2 + e_2 e_1 \right] + \\
+ \gamma_3 \frac{4 \Omega_s R T_{\parallel s} T_{\perp s} n_s}{BM_s} \left( e_2 \cdot e_\gamma \right) \left( b e_2 + e_2 b \right) + \left( e_1 \cdot e_\gamma \right) \left( b e_1 + e_1 b \right). 
\]

The following comments about the solution are in order:

- The total tensor pressure \( \Pi^{tot} \) is symmetric in the system defined by the vectors \((b, e_1, e_2)\).
- The leading-order pressure tensor \( \Pi_s \) calculated in this approximation is diagonal but non-isotropic. We notice that the source of this anisotropy in Eq. (2.1) is just the temperature anisotropy. In the limit of isotropic temperature, the leading-order pressure tensor becomes diagonal and isotropic, as can be easily verified.
- The first-order correction \( \Delta \Pi_s \) is non-diagonal and non-isotropic. In particular, non-diagonal terms are carried by the tensor \( \Delta \Pi^3_s \) given in Eq. (2.5). Two main properties of the solution contribute to generating the non-isotropic feature of \( \Delta \Pi_s \). The first is the temperature anisotropy, while the second is the existence of the diamagnetic part of the KDF obtained from the Taylor expansion of \( f_{\gamma s} \) and depending on the thermodynamic forces associated with the gradients of the fluid fields (Cremaschini et al. 2010). Indeed, we notice that taking the limit of isotropic temperature is not enough to make the tensor \( \Delta \Pi_s \) isotropic as well. In fact, even if the parallel and perpendicular temperatures are equal, since the plasma is magnetized and collisionless the KDF will not be perfectly Maxwellian and deviations carried by the diamagnetic part act as a source of anisotropy.

3. Conclusions

In this paper a kinetic solution to the problem concerning the determination of closure conditions for fluid MHD equations has been presented. The theory has been developed within the framework of the Vlasov-Maxwell description and is applicable to non-relativistic axisymmetric collisionless AD plasmas. The specific case of asymptotic stationary configurations has been considered, in which the magnetic field is assumed to admit locally a family of closed nested magnetic surfaces. Some important features of collisionless magnetized plasmas have been included in the treatment. These concern the anisotropies induced by the existence of the magnetic field, like the temperature anisotropy, and the deviation of the equilibrium KDF away from the exact Maxwellian case, expressed through the diamagnetic part of the KDF. In particular, in this study, the calculation of the tensor pressure and the discussion of its physical properties have been considered.

References