

THE OORT LIMIT

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1. Introduction

During galaxy formation non-baryonic dark matter is not expected to pack down into a thin disk. Hence the mass-to-light ratio Υ_d of the Galactic disk is a key number as the mass-to-light ratio of a purely baryonic system. If Υ_d proves to be large, it suggests that the large values of Υ that are detected elsewhere in the Universe are due to dark baryonic matter rather than some exotic form of matter. Conversely, a small value Υ_d lends credence to the idea that most of the mass in the Universe is non-baryonic by suggesting that baryonic systems are well endowed with luminous stars.

2. Basic formulae

In cylindrical polar coordinates (R, ϕ, z) Poisson's equation may be written

$$4\pi G\rho = \frac{1}{R} \frac{\partial v_c^2}{\partial R} - \frac{\partial K_z}{\partial z}, \quad (1)$$

where v_c is the circular speed and K_z is the vertical component of the gravitational field. The term $\partial v_c^2/\partial R$ can be evaluated in terms of the Oort constants and proves to be negligible. The Jeans equation governing the vertical distribution of a tracer population of number density ν can be written

$$\nu K_z - \frac{\partial}{\partial z}(\nu \langle v_z^2 \rangle) = \frac{\partial}{\partial R}(\nu \langle v_R v_z \rangle) + \frac{\nu}{R} \langle v_R v_z \rangle \quad (2)$$

Near the plane we expect that $\langle v_R v_z \rangle \simeq \alpha(v_R^2 - v_z^2)z/R$ with $0 < \alpha < 1$ and $\nu \langle v_z^2 \rangle \sim e^{-2R/R_d}$, so the r.h.s. of (2) evaluates to $-(2\alpha/R_d - 1/R)(v_R^2 - v_z^2)\nu z/R$. For plausible values of α and R_d ($\alpha = 0.5, R_d = R_0/3$) the ratio of the r.h.s. of (2) to a typical term on its l.h.s. is r.h.s./l.h.s. $\sim hz/R_0^2$, where $h \simeq 300$ pc is the scale height of the disk. This argument suggests that the r.h.s. of (2) can be neglected. Combining the resulting approximate equation with (1) and integrating the result through the disk we obtain successively

$$4\pi G\rho = -\frac{\partial}{\partial z} \left(\frac{1}{\nu} \frac{\partial}{\partial z} (\nu \langle v_z^2 \rangle) \right) \quad \text{and} \quad 2\pi G\Sigma(z) \simeq -\frac{1}{\nu} \frac{\partial}{\partial z} (\nu \langle v_z^2 \rangle), \quad (3)$$

where $\Sigma(z) = \int_{-z}^z dz \rho$.

The bad news is that these equations require differentiation of the measured quantities ν and $\langle v_z^2 \rangle$. In the approximation in which motions in and perpendicular to the plane are decoupled, the good news is that *both* $\nu(z)$ and $\langle v_z^2 \rangle(z)$ are determined by a single unknown function, the distribution of tracers with respect to vertical energy $E = \frac{1}{2}v_z^2 + \Phi$. Kuijken & Gilmore (1989) and Créze et al. (1997) exploit this connection: they adopt a parametric form of $\Phi(z)$ and use it and the measured form of $\nu(z)$ to determine $f(E)$, from which they can predict the distribution of velocities at any z . The parameters in $\Phi(z)$ are chosen to optimize the fit between the predicted and measured velocity distributions.

Kuijken & Gilmore applied this technique to a sample of 512 K-dwarfs that were identified in a photometric survey of fields near the SGP. Since such tracer stars lie in a cone with the Sun at

the apex, there are few stars near $z = 0$, and the sample cannot constrain ρ_0 . Kuijken & Gilmore (1991) concluded that the sample most strongly constrains

$$\Sigma(1.1 \text{ kpc}) = 71 \pm 6 M_{\odot} \text{ pc}^{-2}. \tag{4}$$

The samples of Créze et al. are formed by stars with $M_V < 2.5$ in the Hipparcos Catalogue. Depending on the distance out to which they include stars, their samples contain 3000 – 2300 stars. Since they require v_z but do not have radial velocities for most stars, their velocity data are confined to the proper motions of the 17% of the samples that lie at $|b| \leq 10^\circ$. They conclude that $\rho_0 = 0.076 \pm 0.015 M_{\odot} \text{ pc}^{-3}$.

Bahcall (1984) and Bahcall, Flynn & Gould (1992) chose rather to parametrize $f(E)$. Specifically, they assumed that the tracer population was isothermal: $f \propto e^{-E/\sigma^2}$. Indeed, they assumed that every mass-bearing population is isothermal with some measured velocity dispersion σ_i . Then $\rho_i(z) = \Upsilon_i \nu_i(z) = \rho_{i0} e^{-\Phi/\sigma_i^2}$ and Φ has to solve the non-linear o.d.e.

$$\frac{d^2\Phi}{dz^2} = 4\pi G \sum_i \rho_i = 4\pi G \sum_i \rho_{i0} e^{-\Phi/\sigma_i^2}. \tag{5}$$

This equation was repeatedly solved for assumed values of the parameters ρ_{i0} until the predicted density law of a given tracer, $\nu_i(z) = (\rho_{i0}/\Upsilon_i) e^{-\Phi/\sigma_i^2}$, agreed with the measured density profile.

This technique was applied to old samples of F and K giants by Bahcall (1984), and to a new sample of K giants by Bahcall et al. (1992). In each study at least one population required an unexpectedly large value of Υ . Different solutions $\Phi(z)$ were obtained depending on the population that was assigned the anomalous value of Υ . Bahcall et al. (1992) concluded that

$$\Sigma(1.1 \text{ kpc}) = 83 M_{\odot} \text{ pc}^{-2} \quad \text{and} \quad \rho(0) = 0.26 M_{\odot} \text{ pc}^{-3} \quad \text{with errors} \sim \pm 0.1 \text{ dex} \tag{6}$$

This estimate of $\Sigma(1.1 \text{ kpc})$ agrees with that of Kuijken & Gilmore within the errors, while the Bahcall et al. value of $\rho(0)$ is incompatible with that of Crézé et al. Since the Bahcall et al. samples, like those of Kuijken & Gilmore, are confined to small patches of the sky, while those of Crézé et al. are all-sky, the newer value, $\rho(0) \simeq 0.076 M_{\odot} \text{ pc}^{-3}$ is surely to be preferred.

3. Estimating Υ_d

Once $\Sigma(1.1 \text{ kpc})$ has been estimated, two things remain to be done before Υ_d can be evaluated: (i) estimate how much of $\Sigma(1.1 \text{ kpc})$ is attributable to the disk rather than any spheroidal halo; (ii) estimate the disk's surface luminosity density L_d .

Kuijken & Gilmore argued that, near $z = 1.1 \text{ kpc}$, Φ could be approximated by the first two terms in its Maclaurin series: $\Phi(z) = Kz + Fz^2$. Physically, K quantifies the mass within $|z|$ of the plane while F quantifies the equatorial halo density. The Kuijken & Gilmore sample imposes a single constraint on K and F : $K_z = K + 2z_0 F \simeq \text{const.}$ with $z_0 \simeq 1.1 \text{ kpc}$. If the halo were spherical, the Galaxy's circular-speed curve would impose a second constraint: $0.094K + F \times 1 \text{ kpc} = 15 \pm 3 M_{\odot} \text{ pc}^{-2}$. Combining these constraints Kuijken & Gilmore concluded that $\Sigma_d = 48 \pm 9 M_{\odot} \text{ pc}^{-2}$.

Dehnen & Binney (1997) reached an essentially identical conclusion by a rather different route: by an analysis of global Galactic mass models similar to that of Caldwell & Ostriker (1981), they concluded that (4) implies that $\Sigma_d = 50 \pm 2 M_{\odot} \text{ pc}^{-2}$ unless either R_d is unexpectedly short ($\leq R_0/4$) or the halo is highly flattened ($b/a \leq 0.3$).

The interstellar medium is thought to contribute $\Sigma_{\text{ISM}} \simeq 13 M_{\odot} \text{ pc}^{-2}$ to the local surface density (Bahcall et al. 1992), so dynamical estimates of the local disk mass imply that stars contribute $\Sigma_{\star} \simeq 37 M_{\odot} \text{ pc}^{-2}$ to the local surface density.

From K -band *SPACELAB* data Kent et al. (1991) infer that the Galaxy's surface luminosity density at R_0 is $I(R_0) = 70 L_{\odot} \text{ pc}^{-2}$ for a solar K -band absolute magnitude $M_K(\odot) = 3.41$. By comparison, from *COBE/DIRBE* data Malhotra et al. (1996) find that the Galaxy has $M_K = -24.06$, or $L_K = 9.7 \times 10^{10} L_{\odot}$, 44% larger than the value $L_K = 6.7 \times 10^{10} L_{\odot}$ inferred by Kent et al. Thus it seems unlikely that $I(R_0)$ is significantly smaller than $70 L_{\odot} \text{ pc}^{-2}$. In view of the estimate given above of Σ_{\star} we conclude that the local K -band mass-to-light ratio is $\Upsilon_K \lesssim 0.52$.

Bruzual & Charlot (1993) estimate that a system in which the star-formation rate declines exponentially with time constant $\tau = 5 \text{ Gyr}$ and the IMF has Salpeter's form down to $0.1 M_{\odot}$, has

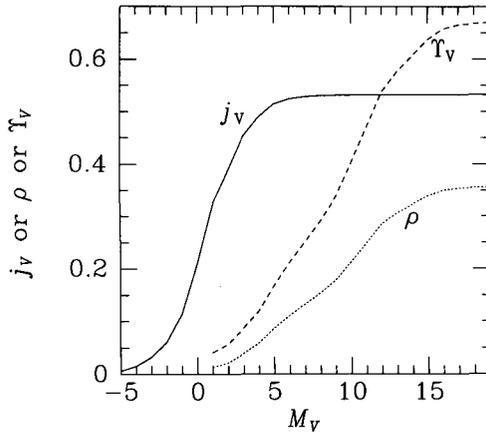


Figure 1. The full curve shows the V -band luminosity (in $10^{-1} L_{\odot} \text{pc}^{-3}$) in stars brighter than M_V . The dotted curve shows the mass (in $10^{-1} M_{\odot} \text{pc}^{-3}$) in these stars. The dashed curve shows the corresponding values of Υ_V .

$\Upsilon_K \simeq 0.74$ after 8 Gyr and $\Upsilon_K = 0.87$ after 12 Gyr. Thus the local mass-to-light ratio is lower than a Salpeter IMF implies, and the IMF should flatten significantly above $0.1 M_{\odot}$.

Recently Gould et al. (1996) used HST to determine the luminosity function and the vertical density profile of low-mass stars. Their luminosity function agrees with that of Stobie et al. (1989) in turning down sharply faintward of $M_V = 12$, with the result that they concluded that M stars contribute only $\Sigma_M \simeq 13.5 \pm 3 M_{\odot} \text{pc}^{-2}$ to the local surface density. When this figure is added to the surface density in stars other than M stars, $\sim 15 M_{\odot} \text{pc}^{-2}$, we obtain $\Sigma_* \simeq 28.5 M_{\odot} \text{pc}^{-2}$, which is smaller than the corresponding dynamical estimate.

The luminosity functions of Stobie et al. and Gould et al. both rely on photometric distances. At $M_V > 12$ luminosity functions that are based on trigonometric parallaxes of high-proper-motion stars lie consistently above those based on photometric parallaxes and therefore predict that there is more mass in M stars. For example, the luminosity function that Jahreiß and Wielen (1997) obtain from Hipparcos parallaxes of stars within 25 pc of the Sun is flat from $M_V \simeq 12$ to $M_V \simeq 16.5$. When this luminosity function is combined with older data for luminous stars and the main-sequence mass-luminosity relation of Kroupa et al. (1993), one finds that stars brighter than $M_V = 18$ have a cumulative mass-to-light ratio $\Upsilon_V = 0.62$ – see Fig. 1. Observations of external galaxies that are similar to our own (de Jong 1996) suggest that the local disk has $V - K \simeq 2.8$. If the disk is this red, then $\Upsilon_V = 0.62$ corresponds to $\Upsilon_K = 0.17$, which is smaller even than the work of Malhotra et al. would suggest. Hence even with the larger number of low-mass stars that are found by parallax-based luminosity functions, the local mass-to-light ratio is surprisingly small.

In summary: estimates of the local luminosity surface density combined with dynamical estimates of the local mass surface-density yield a mass-to-light ratio Υ_K that is smaller than expected from a population of appropriate age with Salpeter IMF right down to $0.1 M_{\odot}$. On the other hand, counts of faint stars yield an even smaller value of Υ_K than is observed. This suggests that there are more low-luminosity stars than even parallax-based surveys find, but fewer than a Salpeter IMF predicts.

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