

Cardinal numbers with partition properties

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The first part of this thesis deals with generalizations of the partition properties

$$(1) \quad \kappa \rightarrow (\eta_0, \eta_1, \dots)^n,$$

$$(2) \quad \kappa \rightarrow (\eta)_\nu^{<\omega},$$

extensively studied by Erdős and others (see for example [1]). Chapters II and IV are concerned with the generalization of the relation (1), a generalization obtained by considering not partitions of the n -element subsets of κ , but rather partitions of certain finite sequences of n -element subsets. This is in fact a special case of the polarized partition relation defined in [1]. The discussion in Chapter II is concerned mainly with various forms of the definition, while Chapter IV deals directly with the question of extending the methods and results of [1] to the more general situation.

In Chapter III, an analogous generalization of the relation (2) is considered. Here it is found that many of the properties of the relation (2) extend to cover also the generalized relation. A number of important open questions remain, however.

The final chapter, Chapter V, is unrelated to the earlier ones. This is concerned with a development of the forcing method of P.J. Cohen, in the case when the collection of forcing conditions forms a proper class in the ground model. This construction is used to re-prove, in a common setting, theorems of Easton and Jensen.

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Reference

- [1] P. Erdős, A. Hajnal and R. Rado, "Partition relations for cardinal numbers", *Acta Math. Acad. Sci. Hungar.* 16 (1965), 93-196.