Annals of Glaciology 17 1993

Dielectric anisotropy in ice Ih at 9.7 GHz

SHUJI FUJITA, SHINJI MAE, TAKESHI MATSUOKA
Department of Applied Physics, Faculty of Engineering, Hokkaido University, Sapporo 060, Japan

ABSTRACT. Dielectric anisotropy in ice Ih was investigated at 9.7 GHz with the waveguide method. The measurement of dielectric permittivity was made using single crystals collected from Mendenhall Glacier, Alaska. The result of the measurement shows that $\epsilon'_{\parallel c}$, the real part of dielectric permittivity parallel to the *c* axis, is larger than $\epsilon'_{\perp c}$, the real part of dielectric permittivity perpendicular to the *c* axis. This tendency is similar to that at low frequencies in the region of the Debye relaxation dispersion. It can be proposed that $\epsilon'_{\parallel c} > \epsilon'_{\perp c}$ in the HF, VHF and microwave frequency range. $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ depend slightly upon temperature but the dielectric anisotropy, $\Delta\epsilon' = \epsilon'_{\parallel c} - \epsilon'_{\perp c}$, is constant and becomes 0.037 (±0.007). Based on the present results, a simple caculation indicates that the maximum power reflection coefficient caused by the dielectric anisotropy is about −50 ~ −80 dB, which is significantly larger than the power reflection coefficient observed in the ice sheet by radio-echo sounding, about −70 ~ −80 dB. This leads to a conclusion that dielectric anisotropy is one of the dominant causes of internal reflections.

INTRODUCTION

The relative dielectric permittivity of ice at frequencies from HF to microwave is of importance for the analysis of remote sensing data on the cryosphere because electromagnetic waves with such frequencies can penetrate into the depths of large ice masses. The crystal structure of ice has a uniaxial symmetry and ice is well known to be uniaxially birefringent at optical frequencies. The static dielectric permittivity also indicates the crystal orientation dependence (Humbel and others, 1953; Kawada, 1978). Natural ice is a polycrystalline aggregate and shows various fabric patterns which represent the distribution of the crystal orientations of grains in it. Harrison (1973) proposed that variation of fabric with depth in ice sheets was one of the major causes of internal reflections observed by radio-echo soundings, RES.

The real part of the relative permittivity of ice obtained experimentally is shown schematically in Figure 1. Dispersion at the lower frequencies is caused by the Debye relaxation mechanism. Another dispersion is observed in the infrared region. The real part of permittivity at frequencies between the two dispersions is often denoted as ϵ_∞ as shown in Figure 1, and it is a constant. Here the subscript $_\infty$ refers to frequencies well above the Debye relaxation frequency only, and not any higher frequency dispersions. Therefore, the real part of permittivity at frequences from HF to microwave becomes ϵ_∞ in the temperature range of the cryosphere.

It has been accepted that ϵ_∞ is approximately 3.17 (e.g. Evans, 1965) which depends slightly on temperature. This value has been used to analyze the data of RES obtained on the ice sheets at frequencies between ten and a few hundred MHz and the data of microwave remote sensing. Johari and Charette (1975) measured the real part of permittivity perpendicular to the *c* axis $\epsilon'_{\perp c}$, using artificial ice single crystals and that of artificial polycrystalline ice, ϵ'_{poly}, at 35 and 60 MHz. They did not detect any difference between $\epsilon'_{\perp c}$ and ϵ'_{poly} and proposed that anisotropy of ϵ_∞ was less than about 0.032. Johari and Jones (1978) measured the permittivity of single crystals of zone-refined ice at frequencies between 0.5 Hz and 0.2 MHz and at temperatures between −73°C and −2°C. The obtained result showed that the difference between $\epsilon'_{\perp c}$ and the real part of permittivity parallel to the *c* axis, $\epsilon'_{\parallel c}$, was not detectable within experimental error, 0.5%.

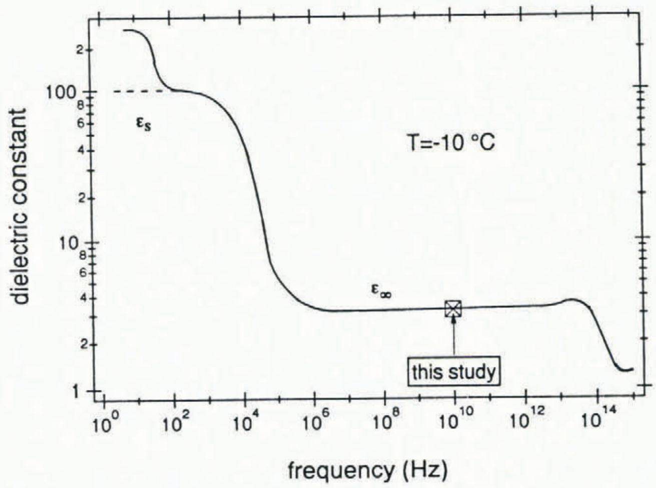

Fig. 1. Schematic dispersion spectrum of ice at −10°C.

We performed new measurements of $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ of single crystals of ice at 9.7 GHz. Although the studies described above did not show any anisotropy in the real part of permittivity, we found a clear difference that $\epsilon'_{\parallel c} - \epsilon'_{\perp c} = 0.04\ (\pm 0.01)$. We reported it as our preliminary result in our previous paper (Fujita and others, 1992). Taking into account the ice fabric variation with depth measured using ice cores, this value is large enough to raise the internal reflection of radio waves in the ice sheets observed by RES. In the present paper we report the detailed results of the measurements of $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ at 9.7 GHz. The effect of ice fabric to the propagation of radio waves is reported in another paper (Fujita and Mae, this volume).

EXPERIMENT

Measurement in this study is by the "two point method", which is one of the standing wave methods (Sucher and Fox, 1963; Muzil and Zácek, 1986). Figure 2 shows the experimental arrangement for measuring the dielectric constants of ice. In this method, an ice sample is set in an empty waveguide terminated by a short circuit. When the incident wave in TE_{10} mode is transmitted to the waveguide, the incident and reflective wave forms the standing wave in it. The input impedance of the short-circuited waveguide is measured with and without the ice

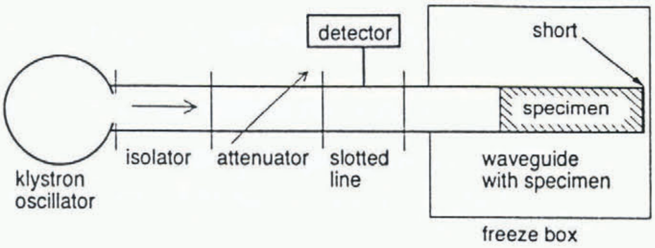

Fig. 2. Experimental arrangement for measuring dielectric constants in waveguide.

Table 1. Ice samples used for the measurements

Sample no.	*Angle between the electric field and the c axis* ±5°	*Density* kg m^{-3} ±2
1	10	–
2	0	913
3	1	914
4	1	915
5	1	912
6	1	913
7	84	–
8	84	915
9	88	913
10	88	911

sample using the slotted line, and a transcendental equation is solved to derive the dielectric constant. Since in TE_{10} mode, the electric field vector in the waveguide is always in the plane transverse to the waveguide axis and is parallel to the shorter side of the waveguide wall, one can investigate the anisotropy of permittivity in materials with this method. Although the system could be used to measure the complex permittivity, we restrict ourselves to measure the real part in this paper. The imaginary part was discussed in our earlier paper (Fujita and others, 1992).

Ice samples

To investigate the real part of permittivity as a function crystal orientation, ten specimens of single crystal collected from Mendenhall Glacier, Alaska, were used as shown in Table 1. Six of them were used to measure $\epsilon'_{\parallel c}$. Four of them were used to measure $\epsilon'_{\perp c}$. The orientation of the *c* axis in each single crystal specimen was determined using a universal stage, which measures the orinetation of the *c* axis of ice by applying optical birefringence of the ice crystal. A report by Langway (1958) gave details of the equipment and the technique to determine orientation of the *c* axis. We confirmed that the orientation of the *c* axis determined with the universal stage was the same as that determined by X-ray analysis in our laboratory. The procedures to cut the ice specimens and how it was placed in the waveguide are described in the next section. All ice specimens were bubble-free and transparent. Their densities are shown in Table 1. Average value of the density is 913 kg m^{-3}. The angle between the *c* axis of the samples and the applied electric field vector is also shown in Table 1. Here, the angle is defined as the angle between two axes in the plane which contains them. Concentration of total impurity ions in ice was of the order of 0.1 ppm (Fujita and others, 1992). Such very low impurity does not influence the real value of permittivity.

Experimental procedures

Measurements were made at temperatures between about –30°C and melting point. 9.7 GHz was used for all measurements. Experimental procedures are described as the following three steps: (1) preparation of the ice sample; (2) setting up the sample to the dielectric measurement system; (3) measurement and calculation. These procedures and precautions are described below.

The specimen prepared for each measurement was a rectangular prism with cross-section 10.9 × 22.9 mm^2 and length 30.0 mm, cut from an original ice block of single crystal with diameter of the order of 10 cm. The transverse dimension of the rectangular specimen was the same as the inner dimension of the rectangular waveguide. When each specimen was cut from the original single crystal ice block, the orientation of its *c* axis was investigated first. The error of this investigation is ±5°. Next, each rectangular ice specimen was cut out with a band saw. Then the *c* axis of each specimen was made to orient parallel to the shortest or longest side of the rectangular prism so that the *c* axis was parallel or perpendicular to the electric field when placed in the waveguide. After making each rectangular prism, the orientation of the *c* axis in each rectangular prism was measured again for confirmation. Then the angle between the *c* axis of each specimen and the electric field vector, when placed in the waveguide, was calculated. This is shown in Table 1. Finally, the surface of each prism was finished with a microtome, which smoothed the surface precisely. Once a specimen was prepared, it was inserted into 30 mm long waveguide whose standard was WRJ-10. The waveguide was made from brass with inner wall coated in silver. One side was short-circuited by the metal plate and the other side was terminated with a flange.

In inserting the sample into the waveguide, care was taken to eliminate the small air gap between the inner wall of the waveguide and the ice sample because it causes serious error in the measured result of permittivity. However, such a small air gap does not affect the orientation of the *c* axis in the waveguide, because the ice specimen was cut very precisely. To eliminate the small air gap, each ice sample was inserted into the waveguide in a bath filled with ion-exchanged water at melting point: the specimen and the water which filled the gap were frozen together. This procedure ensured that permittivity measured was that of the specimen itself and the negligible layer of ice which filled the gap, at most a few per cent of the whole ice measured. After freezing, ice outside the waveguide was completely removed and the surface of the specimen at the flange was cut to a plane with roughness less than 0.1 mm. Next the 30 mm long waveguide containing the sample was connected to the waveguide which comes from slotted line, and was set in a freezer in which temperature could be varied between –70°C and 25°C (Fig. 2). In connecting the waveguides, a polyethylene sheet was put between them to prevent vapour leaking from a small slit in the slotted line. The influence of the polyethylene on the experimental results was calibrated and corrected in calculation.

Measurements were carried out varying temperatures at a rate of 10°C (h^{-1}). When permittivity was measured with varying temperature at a slower rate, (e.g. when temperature was fixed for a few hours), no difference was found in the experimental results. In addition, no difference was found in either lowering or raising the temperature. To obtain the permittivity, the input impedance of the empty waveguide was first measured, then the input impedance of the specimen-filled waveguide. The input impedance of the specimen-filled waveguide varies with temperature, not only by the temeprature-dependence of permittivity in ice, but also by the thermal expansion of the waveguide and temperature dependence of resistivity at the inner wall of the waveguide. Their influences were corrected by measurements of the input impedance of the empty waveguide at various temperatures. Estimated error of the measurement was at most ± 0.01.

RESULTS

$\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ measured at 9.7 GHz are shown in Figure 3. Since the direction of *c* axis of samples is not exactly parallel nor exactly perpendicular to the electric field vector as shown in Table 1, a small correction is made to obtain correct $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$. As shown in Figure 3, $\epsilon'_{\parallel c}$ is clearly larger than $\epsilon'_{\perp c}$. Both $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ increase slightly with increasing temperature as follows:

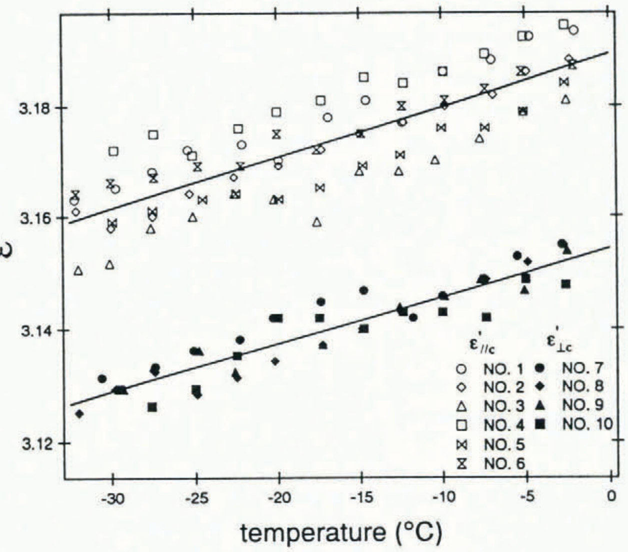

Fig. 3. The dielectric permittivity of single crystals of ice at 9.7 GHz when the c axis is parallel to the electric field (sample no. 1–6) and perpendicular (sample no. 7–10).

$$\epsilon'_{\parallel c} = 3.189(\pm 0.006) + 0.00092(\pm 0.00007)T \quad (1)$$

$$\epsilon'_{\perp c} = 3.152(\pm 0.003) + 0.00086(\pm 0.00005)T \quad (2)$$

where T is temperature expressed by °C.

From Equations (1) and (2), the difference between $\epsilon'_{\parallel c}$ and $\epsilon'_{\perp c}$ is given by

$$\Delta\epsilon' = \epsilon'_{\parallel c} - \epsilon'_{\perp c} = 0.037(\pm 0.007) + 0.00006(\pm 0.00009)T. \quad (3)$$

The second term of Equation (3) is negligibly small compared with the first. Since below about –120°C, the second term of Equation (3) exceeds the error of the first one, we consider that $\Delta\epsilon'$ is constant in the temperature range of the cryosphere.

DISCUSSION

Anisotropy of ϵ_∞

Evans (1965) showed that most of ϵ_∞ measured in earlier studies is expressed by

$$\epsilon_\infty = 3.17 \pm 0.07. \quad (4)$$

Taking into account the error term of Equation (3), the real parts measured in this study are included in ϵ_∞ given by Equation (4). Johari and Charette (1975) measured the complex permittivity of ice at 35 and 60 MHz above –25°C. As shown in Figure 4, the real part at 35 MHz is slightly larger than our result, especially at higher temperatures above –7°C, but that at 60 MHz is very similar to $\epsilon'_{\parallel c}$. Mätzler and Wegmüller (1987) measured the complex permittivity at frequencies between 2.4 and

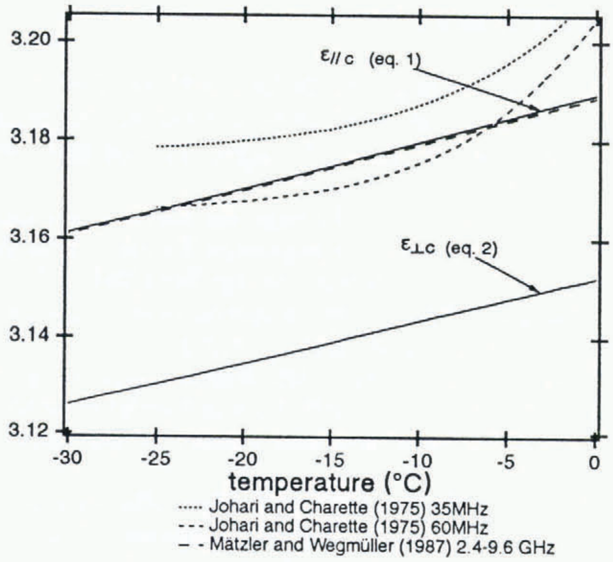

Fig. 4. $\epsilon'_{\|c}$ and $\epsilon'_{\perp c}$ compared with the results of ϵ_∞ in the earlier studies.

9.6 GHz at temperatures above −30°C using the cavity-resonator method. They assumed ice to be isotropic and concluded that the real part is expressed as:

$$\epsilon_\infty = 3.1884 + 0.00091T. \qquad (5)$$

Equation (5) is shown in Figure 4. ϵ_∞ of Equation (5) is completely equal to $\epsilon'_{\|c}$. Then we can conclude that the real parts obtained in this study agree approximately with these earlier results. It is not clear why ϵ_∞ of Equation (5) is completely equal to $\epsilon'_{\|c}$, although the former is assumed to be isotropic. Possible causes of this strange coincidence are as follows: (1) there are small systematic errors in the results; (2) the assumption that ice was isotropic was not correct in Equation (5). Even if the former is the case, and even if the small systematic error exists in the results of this study, $\Delta\epsilon'$ in Equation (3) is not influenced by such error at all.

Johari and Charette (1975) measured $\epsilon'_{\perp c}$ of artificial single-crystal ice and ϵ_{poly} of polycrystalline ice at 35 and 60 MHz. They could not detect any difference between $\epsilon'_{\perp c}$ and ϵ_{poly}. Since they did not observe the ice fabric of the polycrystalline ice, it is assumed that the ice fabric shows isotropy. In this case, $\epsilon_{\text{poly}} = \epsilon'_{\perp c} + (\epsilon'_{\|c} - \epsilon'_{\perp c})/3$. Using Equation (3), we obtain $\epsilon_{\text{poly}} - \epsilon'_{\perp c} = 0.012$, which is approximately equal to their experimental error, 0.010. This may be the reason why they could not obtain the anisotropy.

Humbel and others (1953) measured the complex permittivity of single crystal ice at low frequencies in the kHz region and found that $\epsilon'_{\|c}$ was clearly larger than $\epsilon'_{\perp c}$. Based on their result, anisotropy between static permittivities, $\epsilon'_{s\|c}$ and $\epsilon'_{s\|c}$ is 15% at −5°C and decreased with increasing frequency. In addition, Kawada (1978) obtained similar results. Although the anisotropy has not been detected at higher frequencies, we conclude now that $\epsilon'_{\|c}$ is larger than $\epsilon'_{\perp c}$ over the frequency range from MHz to GHz.

ϵ_∞ is determined by vibration of the water molecule in ice which represents the higher frequency dispersion in the infrared region (Fig. 1). The precise behaviour of this molecular vibration has not been investigated and anisotropy of the real part cannot be explained. Ehringhaus (1917) measured the anisotropy of the real part at frequency of sodium-D line and found it to be 0.0037 or 0.2% at −1°C.

Reflection induced by dielectric anisotropy

It is well known that ice fabric changes with depth in the ice sheets. Harrison (1973) proposed a hypothesis that variation of ice fabric was one of the major causes of internal reflections observed by RES in the ice sheets. Unfortunately, this has not been supported because the dielectric anisotropy was not considered to be so large as to produce internal reflections. In this study, however, we can obtain the large dielectric anisotropy $\Delta\epsilon' = 0.037$.

When a microwave or a radio wave are normally incident on a plane boundary between two ice media, 1 and 2, characterized by permittivity ϵ'_1 and ϵ'_2, respectively, if the wave travels from ice medium 1 to ice medium 2, the power reflection coefficient PRC is given by

$$\text{PRC} = \left|\frac{Z_1 - Z_2}{Z_1 + Z_2}\right|^2 \qquad (6)$$

where Z_1 and Z_2 are the intrinsic impedance of the two ice media, respectively. If change in impedance is due to only the real part of the complex permittivity, PRC, given by Paren and Robin (1975) is:

$$\text{PRC} = \left|\frac{\delta\epsilon'}{4\epsilon'_2}\right|^2 \qquad (7)$$

where $\delta\epsilon'$ is the difference in permittivity between two media ($= \epsilon'_1 - \epsilon'_2$). Equations (6) and (7) hold only for reflection at a single plane boundary, separating two media infinitely extended regions. Equation (7) holds only when $\delta\epsilon' << \epsilon'_1, \epsilon'_2$. The present discussion satisfies this condition because maximum $\delta\epsilon'$ is only 1.2% of ϵ'_1 and ϵ'_2. To discuss exactly, we should consider the influence of birefringence, i.e. reflection of the ordinary wave and the extraordinary wave in ice should be considered separately. However, since the purpose of this discussion is to estimate the order of the magnitude of reflection coefficient caused by the dielectric anisotropy, the effect of birefringence is ignored. When change in the real part of permittivity is due only to the contribution of dielectric anisotropy, Equation (7) can be expressed as

$$\text{PRC} = \left|\frac{D_a \Delta\epsilon'}{4\epsilon'_2}\right|^2. \qquad (8)$$

Here, D_a is a coefficient which shows the degree of contribution of $\Delta\epsilon'$ to change in permittivity between media 1 and 2. D_a is a number between 0 and 1.

Figure 5 shows the relation between D_a and PRC. In the extreme case, i.e. when we assume that $\epsilon'_1 = \epsilon'_{\|c}$ and $\epsilon^1_2 = \epsilon'_{\perp c}$, D_a is 1 and PRC is about −50 dB. If $\epsilon'_1 = \epsilon'_{\perp c}$ and medium 2 is isotropic, we obtain $D_a = \frac{1}{3}$. Then PRC is about −60 dB. Moreover, even when D_a is as small as $0.03 \sim 0.1$, PRC is about $-70 \sim -80$ dB. Since PRC

https://doi.org/10.3189/S0260305500012969 Published online by Cambridge University Press

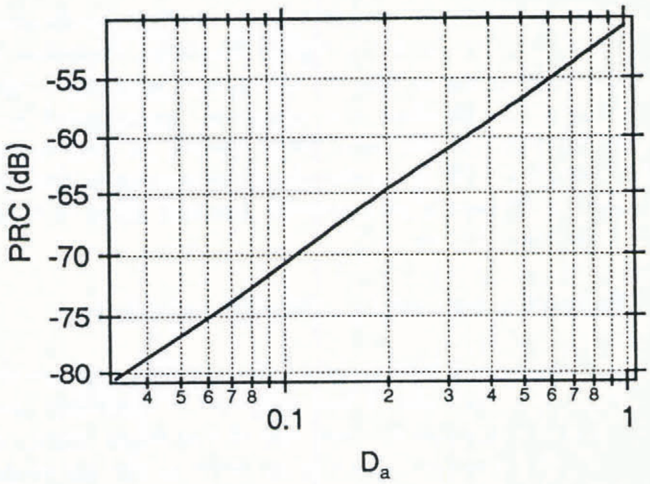

Fig. 5. Power Reflection Coefficient (PRC) as a function of D_a.

observed by RES in the ice sheets is about $-70 \sim -80$ dB, it is concluded that the measured dielectric anisotropy can be one of the dominant causes of internal reflections. Because the temperature dependence of $\Delta\epsilon'$ is negligibly small in the temperature range of the cryosphere, PRC is not influenced by temperature. Details of the relation between the ice-fabric variation and PRC are reported in another paper (Fujita and Mae, this volume).

Ackley and Keliher (1979) computed a depth profile of PRC using the measured physical properties of core to the bedrock (324 m) taken at Cape Folger, East Antarctica, in order to compare with observed radio-echo reflections. They used the variation of density, bubble size and shape, and ice fabric as parameters. Adopting $\Delta\epsilon' = 0.0037$ as the dielectric anisotropy, they calculated PRC to be about –90 dB which was negligibly small compared with values deduced using other parameters ($\gtrsim$–80 dB). Applying our result, $\Delta\epsilon' = 0.037$, as the correct dielectric anisotropy, we obtain PRC $\cong$–70 dB and, even in the case of Ackley and Keliher, the dielectric anisotropy becomes a dominant cause of the internal reflection.

ACKNOWLEDGEMENT

This study was supported by the Special Coordination Funds for Promoting Science and Technology provided by Science and Technology Agency.

REFERENCES

Ackley, S. F. and T. E. Keliher. 1979. Ice sheet internal radio-echo reflections and associated physical property changes with depth. *J. Geophys. Res.*, **84**(B10), 5675–5680.

Ehringhaus, A. 1917. Beiträge zur Kenntnis der Dispersion der Doppelbrechung einiger Kristalle. *Neues Jahrb. Mineral. Geol. Paläontol.*, Beilage Band **B41**, 342–419.

Evans, S. 1965. Dielectric properties of ice and snow — a review. *J. Glaciol.*, **5**(42), 773–792.

Fujita, S. and S. Mae. 1993. Relation between ice sheet internal radio-echo reflections and ice fabric at Mizuho Station, Antarctica. *Ann. Glaciol.*, **17** (see paper this volume).

Fujita, S., M. Shiraishi and S. Mae. 1992. Measurement on the microwave dielectric constant of ice by the standing wave method. *In* Maeno, N. and T. Hondoh, *eds. Proceedings of the International Symposium on the Physics and Chemistry of Ice.* Hokkaido, Hokkaido University Press, 415–421.

Harrison, C. H. 1973. Radio echo sounding of horizontal layers in ice. *J. Glaciol.*, **12**(66), 383–397.

Humbel, H., F. Jona and P. Scherrer. 1953. Anisotropie der Dielektrizitätskonstante des Eises. *Helv. Phys. Acta.*, **26**(1), 17–32.

Johari, G. P. and P. A. Charette. 1975. The permittivity and attenuation in polycrystalline and single-crystal ice Ih at 35 and 60 MHz. *J. Glaciol.*, **14**(71), 293–303.

Johari, G. P. and S. J. Jones. 1978. The orientation polarization in hexagonal ice parallel and perpendicular to the *c*-axis. *J. Glaciol.*, **21**(85), 259–276.

Kawada, S. 1978. Dielectric anisotropy in ice Ih. *J. Phys. Soc. Jpn*, **44**(6), 1881–1886.

Langway, C. C., Jr. 1958. Ice fabrics and the universal stage. *SIPRE Tech. Rep.* 62.

Mätzler, C. and U. Wegmüller. 1987. Dielectric properties of fresh-water ice at microwave frequencies. *J. Phys. D: Appl. Phys.*, **20**, 1623–1630.

Muzil, J. and F. Zácek. 1986. *Microwave measurement of complex permittivity by free space methods and their applications.* New York, Elsevier.

Paren, J. G. and G. de Q. Robin. 1975. Internal reflections in polar ice sheets. *J. Glaciol.*, **14**(71), 251–259.

Sucher, M. and J. Fox, *eds.* 1963. *Handbook of microwave measurements.* New York, John Wiley and Sons. (Polytechnic Institute of Brooklyn Series, Vol. II.)

The accuracy of references in the text and in this list is the responsibility of the authors, to whom queries should be addressed.